ARITHMETICK,

VULGAR,
DECIMAL,
INSTRUMENTAL,
ALGEBRAICAL.

In Four PARTS.

By *WILLIAM LEYBOURN.*

The SEVENTH EDITION.

Carefully Corrected; and very much Enlarged by the AUTHOR.

An Account whereof is given in the *Preface* to the READER.

LONDON,
Printed by *J. Matthews*, for *Awnsham* and *John Churchill*, at the *Black-Swan* in *Pater-Noster-Row*. MDCC.

EFFIGIES
GULIELMI LEYBOURN.

Anno — { *Salutis* 1690
Ætatis 64: *Oct.* 18 }

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TO THE
READER.

THis *Treatiſe* of ARITHMETICK hath paſſed ſo many *Impreſſions* that, (what with the ſupine *Negligence* of the *Printers*, and *Careleſſneſs* or *Ignorance* of thoſe to whom the *Reviſal* thereof was committed for want of Application being made to the *Author*;) it hath contracted ſo many *Typographical Sphalmata*, as to disfigure its *Beauty* ſo as to become a meer *Stranger* to its firſt *Parent*: However, in this *Edition*, I have carefully *Examined* every *Rule* and *Example*, and rendred them *Perfect* as at firſt: And whereas, I have in ſome places omitted ſome things of *Leſs* Moment, I have in other places made a *Quadruple* amends, in *Adding* others in lieu thereof, in other places: Of all which, I now come to give you an *Account* of the ſeveral Parts of the *Book*, and in which ſuch *Additional Supplies* are made.

The whole *Treatiſe* is divided into four Parts. The *Firſt* contains *Vulgar ARITHMETICK* in *Whole Numbers* and *Fractions*: And in every *Rule* there are *Examples* for Practice added, and *Queſtions* alſo wrought by thoſe ſingle *Rules*: In *Multiplication* I have added divers *Compendiums*, or brief ways of *Multiplying*, whereby Sums, (having 2 or 3 Figures in the *Multiplier*) may be performed, without any burthen or charge to the Memory, more than in ordinary *Multiplication*, and yet no other (or at moſt very few) Figures

ſet

ſet down but the *Product* it ſelf. And in *Diviſion* (which is the moſt difficult of the four *Species)* there are two ways for the performance of it ſo that every Man may make uſe of that which he beſt underſtands or fancies: And in the working of the *Golden Rule*, &c. I have (to expreſs variety) made uſe ſometimes of one kind of *Diviſion*, and ſometimes of another.

The *Second Part* treateth of DECIMAL ARITHMETICK: Which I have (in this *Edition*) divided into Four *Sections*.

In the Firſt is taught how to *Reduce Vulgar Fractions* into *Decimal Parts*; and thereby to make *Decimal Tables* to expreſs the ſeveral *Denominations* of the *Coyns*, *Weights* and *Meaſures* (of your own or other *Countries*,) in *Decimal Parts*; and how to make uſe of ſuch *Tables* upon all occaſions.

The ſecond *Section* contains *Notation*, and how to work all the *Rules* of *Arithmetick* (treated of in the *Firſt Part) Decimally*: And how to *Extract* the *Square* and *Cube Roots*: in both which (in this Edition) I have been very Copious and Plain.

The Third *Section* treateth of *Simple* and *Compound Intereſt* ---- *Diſcount* or *Rebate* of Money ---- *Æquation* of *Payments* ---- *Purchaſe* of *Annuities*, *Valuation* of *Leaſes* of *Land* or *Houſes*, &c. With *Tables* of all of them ready Calculated.

The Fourth *Section*, Teacheth how to *Meaſure Superficies*, as *Boards*, *Glaſs*, *Land*, *Pavement*, &c. And of *Solids*, as *Timber*, *Stones*, *Spheres*, *Pyramids*, *Cones*, *Cylinders*, &c. ---- And alſo, of the *Works* of the ſeveral *Artificers*, relating to *Building*; as *Carpenters*, *Bricklayers*, *Plaiſterers*, *Maſons*, *Painters*,

Joyners,

Joyners, &c. Whereby this *Decimal Arithmetick* will be as serviceable to all of such *Professions*; as *Vulgar Arithmetick* (in the First Part) was to *Merchants* and other *Traders*.

The *Third Part* treateth of INSTRUMENTAL ARITHMETICK; or *Arithmetick Instrumentally* performed in a *Decimal* way without the help of *Decimal Tables*, by which the whole work of *Reduction* is avoided, there being certain *Scales* of *English Money*, *Weights* and *Measures* divided, and so disposed, that by them (by inspection only) the *Decimal Fraction* of either *Money*, *Weight*, or *Measure*, may be set down as exactly, and in less time than they could have been taken out of the *Decimal Tables* in the Second Part of this *Treatise*. And on the contrary, any *Decimal Fraction*, may be reduced into its proper parts of the integer with the same facility, speed and exactness: A *Figure* of these *Scales* is inserted at the beginning of this *Third Part*.

Unto these *Decimal Scales*, I have now added Two other *Scales*, for the *Extraction* of *Roots*; By the one you may find the *Root* of any *Number* the *Square* thereof being given; Or if the *Root* be given, you may find the *Square Number* answering thereunto, and that by inspection only, without the help of either *Pen* or *Compass* ---- And as this Line doth for extracting the *Square Root*, the other doth the like for the *Cube Root*.

And to make this Third Part of *Instrumental Arithmetick*, yet the more compleat, I have more largely (than in the former *Editions*) insisted upon the *Description* and *Use* of *Nepairs Bones*; large-

ly

ly treating of their *Use* in *Multiplication*, *Division*, and *Extraction* of *Square* and *Cube Roots*. And lastly,

The *Fourth Part* treateth of ALGEBRA, And concerning that, whereas in former *Editions*, there was inserted an *Abridgment* of the *Precepts* of ALGEBRA, written in *French* by *James de Billy*, I have now in this *Edition* (instead of that) added a more *Compleat Treatise* of that *Art*, formerly Published, (at my request) by my worthy Friend Mr. *Thomas Gibson*, among other things of his, in a Book called *Syntaxis Mathematica*, wherein the *Method* which he there follows is the same as in Mr. *Harriot*, in some places, that is, in such *Æquations* as are proposed in *Numbers*: And as in *Des Cartes* in some other *Places*, that is, in such *Æquations* as are *Solid* and not in *Numbers*: Not that the Book is taken out of them, neither does proceed continually with them, but disjunctly, as he thought fit to intermix them among other Things which are not in them.

Thus having given you a full Account of this *Treatise* of ARITHMETICK, and of what is contained in the several *Parts* thereof; I freely offer it to the ingenious Practitioner in the Art of *Numbers*, desiring his Friendly acceptance, and pardon for such *Errors* as may possibly have escaped the *Press* or my self (notwithstanding the great Care I have taken for the prevention on either hand) And in so doing, you will encourage him, who is

Southal in *Middlesex*, *June* 12. 1700.

A Friend to all that are Mathematically affected,

William Leybourn.

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ADVERTISEMENT.

THe Place of the *Author's* Residence is about Ten Miles from *London* West-ward, at a Place called *Southal*, in the Road between *Acton* and *Uxbridge*, and Three Miles from *Brentford*: Where he intends to *Read* the *Mathematicks*, and Instruct young Gentlemen, and others: And to Board such as shall be pleased to make a more close Application to these Studies: Where such Boarders, and others, (during their time of Residence with him) shall have the Use of all *Books*, *Maps*, *Globes*, and other *Mathematical Instruments*, as are necessary for their *Instruction*, till they provide themselves of such as they shall have occasion for afterwards.

ALSO,

IF any would have their Land or Building Surveyed, or Measured, and a Plot thereof made; or any Sun-Dial, or Dials about their House or Garden (of what kind soever, Fixed or Moveable) he will Prepare or Make for them such as shall be desired.

You may hear of him, and have an Account of his Terms, and manner of Proceedings: *By*

Mr. *Robert Morden*, at the Sign of the *Atlas* in *Cornhill*, near the *Royal Exchange*, *Globe-maker*.

Mr. *Henry Wyn*, in *Chancery-Lane*, over-against the *Rolls*, *Mathematical Instrument-maker*.

And where these Books are to be Sold.

ARTS MATHEMATICAL,

Taught by the Author.

- *Arithmetick:*
  - In Whole Numbers, and Fractions.
  - In Decimals, and by Logarithms.
  - Instrumentally, by Decimal Scales, *Napier's Bones:* and to extract the Square and Cube Roots by Inspection.
- *Geometrie:*
  - The Principles thereof with the
    - Practice, and
    - Demonstration.
- *Astronomie:*
  - The Description of the Circles of the Sphere.
  - The Use of the Globes,
    - Celestial, and
    - Terrestrial.
  - To project the Sphere *in Plano* upon any Circle,
    - Right, or
    - Oblique.

And upon these Foundations, the following Superstructures.

- The Use of *Geometrical Instruments*, in the Practice of
  - *Longimetria*, or the *Mensuration* of
    - Heights, as of Trees, Towers, &c.
    - Depths, as of Mines, Wells, Descents, &c.
    - Distances, as of Churches, Towers, &c.
  - *Planometria*, or the *Mensuration* of
    - Board,
    - Glass,
    - Pavement,
    - Tiling, &c.
    - Or any other Superficies.
  - *Stereometria*, or the *Mensuration* of
    - Timber, growing or squared.
    - Stone, regular or irregular.
    - Cask, commonly called Gageing.
  - *Geodæsia*, or the Measuring of Land divers ways.
- *Trigonometria:*
  - Or, the Mensuration of Triangles, both
    - Plain and
    - Spherical.
  - The Application thereof, in the solution of Problems in
    - Geometry.
    - Astronomy.
    - Geography.
    - Navigation.
    - Fortification.
    - Dialling, &c.
- *Navigation:*
  - The Principles thereof, and the manner of Sailing by
    - The Plain Sea-Chart.
    - *Mercator's* Chart.
    - The Arch of a great Cir.
- *Horologiographia* Or *Dialling:*
  - *Arithmetically*, by the Tables of
    - Sines.
    - Tangents.
    - Logarithms.
  - *Geometrically*, by
    - Scale, and
    - Compasses.
  - *Instrumentally*, by the Sector, Quadrants, Scales, &c.

Of

Vulgar Arithmetick.

PART I.

NUMERATION.

NUMERATION, is accounted the First Part of *Arithmetick*, and it is to know how to read a Sum of Figures expreſs'd in Writing; or to write down any Sum to be expreſſed.

To the doing of which, there are Four Things neceſſary.

First, To know their *Number*, which is *Nine*.

Secondly, Their *ſhapes* which are 1. 2. 3. 4. 5. 6. 7. 8. 9. Of which, the firſt toward the left-hand ever ſignifieth *One*, the ſecond *Two*, &c.

Thirdly, To know the *value* of their *places*.

Laſtly, How their proper *ſignification* is altered thereby.

The *value* of their places is thus, when two, three, or more figures ſtand in one Sum, that is, without any *Point*, *Line* or *Comma* betwixt them, as 321, that place next the Right-hand, where the figure 1 ſtandeth, is called the place of *Unity*, or *Unites*, and the figure 1 ſtandeth in that place onely for *One*, and the figure 2 when it is found in that firſt place ſtands only for *Two*; and the like of the reſt.

But in the Sum 321, above expreſſed, the figure 2 is in the ſecond place, and every place contains the value of that place before towards the Right-hand ten times, and therefore the figure 2 doth not ſignifie two, but (in this ſecond place) ten times two, that is *Twenty*, and ſo the figure 3, if it had been in that place had ſignified ten times *Three*, that is *Thirty*; but being here in the third place, it ſignifies ten times *Thirty*, that is *Three*

 hundred

hundred. And ſo the whole Sum 321, is to be read *Three hundred Twenty and one.*

It is hereby ſeen, how their proper ſignifications, which were *Three, Two,* and *One*, are altered by being thus placed, and the Sum, which otherwiſe had been but *Six*, is *Three hundred twenty and one*, as before.

In like ſort if there had been more places, as Seven, the value is quite through increaſed ten times, by being a place more towards the left-hand, as in the Sum $\dot{1}\,1\,1\,\dot{1}\,1\,1\,\dot{1}$. The Figure 1, in the ſecond place, ſtands for ten times one, (that is *Ten*;) in the third for ten times ten, (which is one *Hundred*;) in the fourth for *Ten hundred*, (which is called *One Thouſand*;) in the fifth, for *Ten thouſand*; in the ſixth, for ten times *Ten thouſand*, (which is *One hundred thouſand*;) in the laſt (here the ſeventh) place, for *Ten hundred thouſand*, which is called a *Million*: And ſo on, if there were more places, obſerving the ſame order.

Now to read this readily, make a prick over the place of *Unity*, another the third from it; and over every third, ſtill towards the Left-hand, for ſo thoſe points will be over the places of *Unites, Thouſands*, and *Millions*; and ſo beginning at the laſt, that is, at the Left-hand, read *One Million*; and becauſe the three following towards the Right, ſignifie properly *One hundred* and *eleven*, but the prick belonging to them is in the place of Thouſands, call them *One hundred* and *eleven thouſand*, and the three remaining being under the point over *Unity*, ſignifie only *One hundred* and *eleven*; and all three points read together in one Sum, is *One Million*, *one hundred* and *eleven Thouſand*, *one Hundred* and *eleven.*

In like manner, if this Number 73598624, were given to be read, (according to the former directions) make a prick over every third Figure, beginning with the firſt Figure towards the Right-hand, (which is the place of *Unity*) and then will your Number ſtand thus,

$$7\,\dot{3}\,5\,9\,\dot{8}\,6\,2\,\dot{4}$$

Then for the ready Reading thereof, (becauſe the third prick ſignifies *Millions*) call all the Figures towards the Left-hand, ſtanding from that prick, *Millions*, which in the Example are 7 and 3, ſo then this Number contains 73 *Millions*, 598 *Thouſand*, [624] *Six hundred twenty four*; Which in words at length we read, *Seventy three Millions, Five hundred ninety eight thouſand, Six hundred twenty four.*

Let

Let thus much ſuffice concerning the placing of large numbers, for the ready reading of them; only take theſe four *Tables* following, for illuſtration of what hath been hitherto delivered in words, the very ſight whereof is better than a whole Chapter of information.

The firſt Table is thus to be read.] *One* in the firſt Place ſignifies *One*. *One* in the ſecond Place ſignifies *Ten*. *One* in the third place, ſignifies an *Hundred*, &c. as in the Table.

The ſecond Table is thus to be read.] In this Table you ſhall find the laſt number thereof to conſiſt of theſe figures, 357.846. 903. with a point or a comma betwixt every third figure, for diſtinction ſake, and alſo every three figures in their order are connected together with this brace , which denominates the Places of *Millions*, *Thouſands*, *Hundreds*, ſo that the laſt number of this Table will evidently appear to be 357 *Millions*, 846 *Thouſands*, (903) *nine Hundred and three.*

The third Table] is only the Figures of the ſecond, ſet together, and orderly diſpoſed, having the ſignification or reading of the ſame Numbers in words at length to them annexed, and is here inſerted for the better ſatisfaction of ſuch as ſhall doubt whether they perfectly underſtand what hath been before taught.

The fourth Table] is much like the ſecond, only it conſiſteth but of one number, and extends three Places farther then the greateſt number in the ſecond Table doth: *viz.* to twelve Places; which figures are thus to be read, 736 *Millions of Millions*, 842 *Millions*, 708 *Thouſand*, (645) *Six hundred forty five.*

I. TABLE.

One in the	firſt	place ſignifies	1 one
	ſecond		10 ten
	third		100 a hundred
	fourth		1000 a thouſand
	fifth		10000 ten thouſand
	ſixth		100000 a hundred thouſand
	ſeventh		1000000 a Million
	eighth		10000000 ten Millions
	ninth		100000000 a hundred Millions.

II. TABLE.

								8
							5	4
						7	6	2
					3.	4	8	3
				9	7.	6	2	1
			2	4	3.	7	9	4
		8.	7	4	9.	8	0	7
	5	7.	3	1	6.	2	4	8
3	5	7.	8	4	6.	9	0	3
Millions			Thousands			Hundreds		

III. TABLE.

Number of places.		
1	8	(eight.
2	54	(fifty four,
3	762	(ſeven hundred ſixty two.
4	3483	(three thouſand four hundred eighty three.
5	97621	(ninety ſeven thouſand ſix hundred twenty one.
6	243794	(two hundred 43 thouſand 7 hundred 94.
7	8749807	(eight millions. 749 thouſand 8 hundred and 7.
8	57316248	(fifty ſeven millions, 316 thouſand, 248.
9	357846903	(three hundred 57 millions, 846 thouſ. 903.

IV. TABLE.

7	hundred	Millions of millions
3	ten	
6	one	
8	hundred	Millions
4	ten	
2	one	
7	hundred	Thouſands
0	ten	
8	one	
6	hundred	Hundreds
4	ten	
5	one	

ADDITION.

ADDITION, is the collecting or gathering together of two or more sums, either of one or of divers denominations into one sum, which is called the [*Aggregate*] [*Total*] or [*Gross Sum.*]

In Addition of Numbers of one Denomination, the Order is, to set the Numbers to be added one directly under the other; that is to say; *Unites* under *Unites*, *Tens* under *Tens*, *Hundreds* under *Hundreds*, &c.

RULE.

Having placed your numbers to be added in due order, one under another; draw a line under them, and begin at the lowermost figure towards your right hand, and add that to the next figure above, and the sum of them to the next figure above that; proceeding in this order, till you have added the whole line together: which when you have done, consider how many tens are contained in that line; and for every ten, keep one Unite *in your mind, to be added to the next row; but if there be any odd* Digits, *you must set them beneath the stroak just under the line you added together. Having thus finished the Addition of one line, proceed to the next; and from thence to the third; and so forward, be there never so many.* The examples following will make this plain.

Example I. Let the numbers given to be added together be 7832, 5609, 376, 8547, having thus placed them in order one under another, as in the Margin is done; draw a line under them; then begin your Addition at the lowermost Figure towards your Right-hand; saying, 7 and 6 is 13, and 9 is 22, and 2 is 24: Now (because in 24 there is two tens, and 4 remaining) I place the 4 under the line, and carry the two tens to the next Row of *Tens*; saying, 2 which I carried and 4 make 6, and 7 makes 13, and 3 makes 16; in which Row there is but one ten contained, and 6 remaining, which 6 I set under the line, and carry the ten to the next Row of *Hundreds*; saying, 1 that I carried and 5 makes 6, and 3 makes 9, and 6 makes 15, and 8 makes 23, in which 23 ten is contained two times, and three remaining; the 3 I place under the line, and carry the two tens to the next Row of *Thousands*; saying, 2 which I carried and 8 makes 10, and

	Thousands	Hundreds	Tens	Units
	7	8	3	2
	5	6	0	9
		3	7	6
	8	5	4	7
2	2	3	6	4

and 5 makes 15, and 7 makes 22; in which, ten is contained two times, and two remaining; which 2 I set under the line, and because there is never another Row to be added (to which I should carry the two tens) I therefore set 2 down also under the line towards the Left-hand, as you see done in the Margin: So the *Total* or *gross Summ* of these Numbers, being added together, is 22364.

Example II. *A Man hath in his Orchard* 136 *Apple-Trees*, 76 *Pear-Trees*, 107 *Cherry-Trees, and* 36 *Plum-Trees, and he desires readily to know how many Trees he has in all.*

Place your Numbers one under another, as in the Margin, and then begin to add them together, at your Right-hand; saying, 6 and 7 make 13, and 6 makes 19, and 6 makes 25; place 5 under the line, and carry 2 to the next Row; saying, 2 and 3 is 5, and 7 is 12, and 3 is 15, place 5 under the line and carry 1 to the next Row; saying, 1 and 1 is 2, and 1 is 3; which 3 I set under the line, and (because there was no Tens in that line) the Total is 355, and so many *Trees* are in the *Orchard*.

Apple-Trees	136
Pear-Trees	76
Cherry-Trees	107
Plum-Trees	36
Trees in all	355

Other Examples for Practice.

	95432		321		9161
	76100		1986		235
	2570		23		72
	832		1107		9
Total	174934	Total	3437	Total	9477

Addition of Numbers of divers Denominations.

I. Addition *of English Money*.

The most usual Coins used in *England*, are *Pounds*, *Shillings*, *Pence*, and *Farthings*, of which,

4 Farthings	make	1 Peny	thus character'd.	*d.*
12 Pence		1 Shilling		*s.*
20 Shillings		1 Pound		*l.*

For a Farthing we use *q*.

RULE.

In the Addition of divers Denominations, this Order is to be observ'd, *viz. Place all Numbers of the same Denomination, one directly under another, as* Pounds *under* Pounds, Shillings *under* Shillings,

lings, Pence *under* Pence, *and* Farthings *under* Farthings. *Then draw a line under them, and begin your Addition with the leaſt Denomination firſt; Obſerving, how many times the next greater Denomination is contained in that leaſt: And for every time carry one Unite to the next Denomination, as before you did the Tens, ſetting down the Remainder, if any be; Then adding the next Denomination together, take notice how many times the next greater Denomination is contained in the leſſer; carrying for every time, one to the next greater Denomination.* Thus proceeding till you have gone over all the *Denominations*, be they never ſo many.

Example I. Let the Numbers to be added together be 37 *l.* 16 *s.* 9 *d.* 3 *q.*—21 *l.* 9 *s.* 8 *d.* 1 *q.*—13 *l.* 12 *s.* 9 *d.* 2 *q.* Place the Numbers as in the Margin; draw a line under them, and begin with the leaſt Denomination (which in this Example is *Farthings*) firſt; ſaying, 2 *q.* and 1 *q.* is 3 *q.* and 3 *q.* is 6 *q.* which is one Peny, and 2 *q.* remaining; which 2 *q.* I place under the line, and carry the 1 *d.* to the next Row, which is the place of Pence; ſaying, 1 *d.* and 9 *d.* is 10 *d.* and 8 *d.* is 18 *d.*

l.	*s.*	*d.*	*q.*
37	16	9.	3.
21	9.	8.	1
13	12	9	2
72	19	3	2

which is 1 *s.* and 6 *d.* (Now againſt the 8, make a Prick with your Pen, for your better remembrance, to ſignifie that there is 1 *s.* to be carried to the place of *Shillings*;) then go on, and ſay, 6 *d.* and 9 *d.* is 15 *d.* which is 1 *s.* and 3 *d.* therefore againſt 9, make a prick with your Pen, and (becauſe that is the laſt Number) I ſet down the odd 3 *d.* under the place of *Pence*; and (being I find two pricks in the line of *Pence*, therefore) I carry 2 *s.* to the place of *Shillings*; ſaying, 2 *s.* which I carried, and 12 *s.* is 14 *s.* and 9 *s.* is 23 *s.* which is 1 *l.* and 3 *s.* remaining, make a prick againſt the 9, and going on, ſay, 3 *s.* and 16 *s.* is 19 *s.* which (being there is no more Numbers to be added, and being alſo leſs than 20 *s.*) I ſet under the line, and finding one prick in the line of Shillings, I therefore carry one to the place of Pounds; ſaying, 1 which I carried, and 3 is 4, and 1 is 5, and 7 is 12; ſet down the 2 under the line (as in Addition of Numbers of one Denomination) and carry 1 to the next Row; ſaying, one that I carried, and 1 is 2, and 2 is 4, and 3 is 7, which being the laſt, I ſet down; and ſo the Total or groſs Sum is 72 *l.* 19 *s.* 3 *d.* 2 *q.*

Example II. Let the Numbers to be added be 29 *l.* 16 *s.* 8 *d.*—32 *l.* 17 *s.* 9 *d.*—81 *l.* 13 *s.* 11 *d.* Here in this Example the leaſt Denomination is *Pence*, therefore I begin with them, and ſay, 11 *d.* and 9 *d.* is 20 *d.* which is 1 *s.* and 8 *d.* make a prick againſt the 9, and ſay, 8 *d.* and 8 *d.* is 16 *d.* that is 1 *s.* and 4 *d.* make a prick

prick againſt the 8, and ſet down the odd 4 *d.* Then (becauſe there are two pricks in the line of *Pence*) you muſt carry 2 *s.* to the place of Shillings; ſaying, 2 *s.* which I carry, and 13 *s.* is 15 *s.* and 17 *s.* is 32 *s.* which is 1 *l.* 12 *s.* make a prick againſt 17, and ſay, 12 *s.* and 16 *s.* is 28 *s.* make a prick againſt 16, and (becauſe there is no more Numbers to be added) ſet down the odd 8 *s.* under *Shillings* and (being there are two pricks in the line of *Shillings*) carry two to the place of pounds; ſaying, 2 and 1 is 3, and 2 is 5, and 9 is 14, ſet down 4, and carry 1 to the next line; and ſay, 1 and 8 is 9, and 3 is 12. and 2 is 14, which (becauſe 'tis the laſt) you ſet down; ſo is the *Total*, or groſs Sum, 144 *l.* 8 *s.* 4 *d.*

l.	*s.*	*d.*
29	16.	8.
32	17.	9.
81	13	11
144	8	4

Other Examples for Practice.

l.	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*
29	18.	7	3	36	2	8.
63	11.	2.	1.	29	0	2
229	4	0	2	31	16.	9.
3	7	10	1	6	2	5
326	1	8	3	103	2	0

II. Addition of *Troy-Weight.*

Troy-Weight is a Weight uſed in *England*, by which is weigh'd, *Bread*, *Gold*, *Silver*, *Pearl*, &c. The moſt uſual Denominations of which Weight are, *Pounds*, *Ounces*, *Peny-Weights*, and *Grains*; of which,

24 Grains	make	1 Peny-Weight	thus character'd	*pw.*
20 Peny-Weight	make	1 Ounce	thus character'd	*ou.*
12 Ounces	make	1 Pound	thus character'd	*lb.*

For a Grain we write *gr.*

The Addition of *Troy-Weight* (and conſequently of any other Weight or Meaſure whatſoever, either Domeſtic or Foreign) differeth nothing at all from the Addition of *Engliſh* Coin laſt taught, if the Affinity of one Denomination to another be firſt known; for whereas in Money, becauſe 12 *d.* make 1 *s.* you therefore obſerve how many *twelves* there are in the Addition of your Pence, and for every 12 you add 1 *s.* to the place of *Shillings*; ſo in the Addition of *Troy-Weight*, knowing that 24 *gr.* make one *Peny-Weight*, you muſt therefore in the Addition of *Grains*

Grains of *Troy-Weight*, obſerve how many times 24 you find in your line of *Grains*, and for every 24, carry one to the place of *Peny-weights*; likewiſe, in the addition of *Peny-weights*, you muſt conſider how many times 20 is contain'd in your line, and for every 20 carry one to the place of *Ounces*, (becauſe 20 Peny-weights make an Ounce). Alſo in the Addition of *Ounces* Troy, you muſt obſerve how many times 12 you find in your line of *Ounces*, and for every 12 carry one to the place of *Pounds*; Then laſtly, Add your Pounds together, as numbers of one denomination.

Example. Let the numbers to be added together be 7 *lb.* 11 *ou.* 13 *pw.* 19 *gr.* —6 *lb.* 7 *ou.* 16 *pw.* 19 *gr.* —3 *lb.* 7 *ou.* 9 *pw.* 6 *gr.* Place your numbers as in Addition of Money, each under other, according to their reſpective Denominations as in the Margine: Then draw a Line under them, and begin your Addition with the leaſt Denomination firſt, *viz.* Grains; ſaying, 6 *gr.* and 19 *gr.* is 25 *gr.* which is one *Peny-weight*, and one *Grain*, make a prick againſt 19, and carry the odd Grain to the Number above; ſaying, 1 *gr.* and 19 *gr.* is 20 *gr.* which (becauſe it is leſs than one Peny-Weight) I ſet under the line; then finding one prick in the line of Grains, (I therefore) carry one to the place of Peny-Weights; ſaying, 1 and 9 is 10, and 16 is 26, which is one Ounce, and 6 *pw.* make a prick againſt 16, and ſay, 6 and 13 is 19, which (being leſs than an Ounce) ſet under the line; then for the one prick, carry 1 to the place of Ounces; ſaying, 1 and 7 is 8, and 7 is 15, which is one Pound, and 3 Ounces; make a prick at 7, and ſay, 3 and 11 is 14, which is 1 *lb.* 2 *ou.* make a prick againſt 11, and ſet down the 2 Ounces, and for the 2 pricks carry 2 pounds to the place of *Pounds*; ſaying, 2 and 3 is 5, and 6 is 11, and 7 is 18, which ſet under the place of *Pounds*: So is your Addition ended, and the Sum is 18 *lb.* 2 *ou.* 19 *pw.* 20 *gr.*

lb.	*ou.*	*pw.*	*gr.*
7	11.	13	19
6	7.	16.	19.
3	7	9	6
18	2	19	20

Other Examples for Practice.

lb.	*ou.*	*pw.*	*gr.*	*lb.*	*ou.*	*pw.*	*gr.*
32	9.	12	16	0	10	17	11
17	11.	6	9.	0	6.	0	1
34	8.	15.	10	0	0	19.	8.
8	10	4	7	0	5	2	19
94	3	18	18	1	10	19	15

III. Addition of *Avoirdupois little Weight.*

There is another kind of Weight most commonly used in *England*, called *Avoirdupois little Weight*; by which is weighed all sorts of Ware or Merchandize, Garblable, as *Sugar*, *Pepper*, *Cloves*, &c. This Weight is commonly divided into these Denominations, *Pounds*, *Ounces*, and *Drams*; of which

16 Drams } make { 1 Ounce } thus cha- } *ou.*
16 Ounces } make { 1 Pound } racter'd. } *lb.*

For a Dram we write *dr.*

In the Addition of *Avoirdupois-Weight*, you must observe the very same Method and Order, as in *Money* and *Troy-Weight*, having due respect to the Quantity of the Denominations; as in the Addition of *Drams* to make a prick at every 16, setting down the Remainder, and for every prick carrying an Unite to the next place. The preceding Rules being so copious in this particular, I shall forbear to make any verbal Illustration; but only give you some Examples ready wrought, together with the most usual parts into which the *Weights* and *Measures* now used in *England*, are divided; which to the Ingenious will be sufficient.

Examples of Addition of *Avoirdupois little Weight.*

lb.	*ou.*	*dr.*	*lb.*	*ou.*	*dr.*
12	11.	9	6	13.	7.
76	4	12.	5	9.	12
32	10.	0	6	3	9.
91	7.	13.	10	0	0
32	14.	7	5	7	9
246	00	09	34	02	05

IV. Addition of *Avoirdupois great Weight.*

There is a Weight commonly used in *England*, by which is weighed all Commodities that are sold by the *Hundred*, as *Currans*, *Wooll*, *Flesh*, *Butter*, *Cheese*, and the like; the which Hundred Weight containeth 112 Pounds, and the Hundred Weight is divided into *Quarters*, *Pounds*, and *Ounces*; so that

16 Ounces } make { 1 Pound } thus cha- { *lb.*
28 Pounds } make { 1 Quarter of a *C.* } racter'd. { *qr.*
4 Quarters } make { 1 Hund. weight. } { *C.*

For an Ounce we write *ou.*

Exam-

Examples of Addition of *Avoirdupois great Weight.*

C.	qr.	lb.	ou.	C.	qr.	lb.	ou.
37	3.	21	12.	5	1	7	0
9	1	6	3	3	2.	18.	6.
33	2	20.	0	0	1	6	8
10	0	0	0	11	3.	4	0
12	3.	7	3	6	1	10	5
103	2	27	2	27	1	18	3

I might farther proceed to give you Examples of Addition of common *English Measures*, *viz.* of *Long Measures*, *Liquid Measures*, and *Dry Measures*; as also of *Time*, *Motion*, &c. but the preceding Examples being of sufficient Extent, I shall forbear to trouble either my self or the Reader with that which I conceive superfluous: Only, before I leave *Addition*, I will give you a brief View of the most usual *Measures* used in *England*, which take as follows. And,

V. Of *Liquid Measures.*

Liquid Measures are those by which all Sorts of Liquid Substances are measured; of which (according to the Statute of 12 *Hen.* 7. *chap.* 5.) a *Pint* is the least, from which the greater *Liquid Measures* are deduced; according as is expressed in the following Table.

2 Pints	make	1 Quart
2 Quarts	make	1 Pottle
2 Pottles	make	1 Gallon
8 Gallons	make	1 Firkin of Ale, Soap, or Herrings
9 Gallons	make	1 Firkin of Beer
10½ Gallons	make	1 Firkin of Salmon or Eels
2 Firkins	make	1 Kilderkin
2 Kilderkins	make	1 Barrel
42 Gallons	make	1 Tierce of Wine
63 Gallons	make	1 Hogshead
2 Hogsheads	make	1 Pipe or Butt
2 Pipes or Butts	make	1 Tun of Wine

VI. Of *Dry Measures.*

Dry Measures are those by which all kind of dry Substances are measured, as *Corn*, *Salt*, *Coles*, *Sand*, &c. of which a *Pint* is the least.

2 Pints	make	1 Quart
2 Quarts		1 Pottle
2 Pottles		1 Gallon
2 Gallons		1 Peck
4 Pecks		1 Bushel Land-Measure
5 Pecks		1 Bushel Water-Measure
8 Bushels		1 Quarter
4 Quarters		1 Chaldron
5 Quarters		1 Wey

VII. Of *Long Measures*,

Long Measure is that by which is measured *Cloth*, *Land*, *Board*, *Glass*, *Pavement*, *Tapestry*, &c. of which Measures (according to the Statute of 33 *Edw.* 1. and 25 *Eliz.*) a *Barley-Corn* is the least. So that,

3 Barley-Corns	make	1 Inch
12 Inches		1 Foot
3 Foot		1 Yard
3 Foot 9 Inches		1 Ell
6 Foot		1 Fathom
5 ½ Yard, or 16 ½ Foot		1 Pole or Perch
40 Perches		1 Furlong
8 Furlongs		1 English Mile

VIII. Of *Time*.

Time consisteth of *Years*, *Months*, *Weeks*, *Days*, *Hours*, and *Minutes*. So that,

60 Minutes	make	1 Hour
24 Hours		1 Day Natural
7 Days		1 Week
4 Weeks		1 Month of 28 Days
13 Months, 1 Day, 6 Hours		1 Year

IX. Of *Apothecaries Weights*.

The Weights used by *Apothecaries* are *Grains*, *Scruples*, *Drams*, and *Ounces*; of which,

20 Grains	make	1 Scruple	thus character'd	℈
3 Scruples		1 Dram		ʒ
8 Drams		1 Ounce		℥
12 Ounces		1 Pound		℔

By help of these Tables, and the Rules and Cautions before expressed, any Man may make Addition of any of the abovesaid

Measures

Meaſures one with another, and therefore I ſhall forbear to illuſtrate them by Examples, but leave them to every Man's own practice: And thus I conclude *Addition.*

The Proof of Addition.

Having placed your Numbers in Order, and added them together, and ſet the Total under the Line, Cut off the upper Number by drawing a Line with your Pen betwixt that and the others; then add all the Numbers together except the uppermoſt, and ſet the Total of them under the Total before found; Then add this laſt Total, and the firſt Number, which you cut off with your Pen, together; and if the Sum of thoſe two Numbers be equal with your Total Sum firſt found, then is your Work right, otherwiſe not.

Example. In the firſt Example of whole Numbers, the Sums to be added were 7832, 5609, 376, and 8547; theſe Numbers placed in Order, and added together, the Total or Groſs Sum of them is 22364. Now to prove whether this Total be true or not, I cut off the uppermoſt Number (to wit, 7832) with a Daſh of the Pen, and I add the other three Numbers together, namely, 5609, 376, and 8547, and the Total of them is 14532; which Number being added to 7832, (the Number cut off) the Sum of them is 22364, exactly agreeing with the Total firſt found; clearly evidencing, that the Addition was truly performed: But if they had diſagreed, then the Work had been erroneous. The like Courſe muſt be taken for the Proof of thoſe Sums which have different Denominations, as in *Money* and *Weight*; as by the Examples following will appear.

	7832
	5609
	376
	8547
1 Total	22364
2 Total	14532
Proof	22364

Other Examples proved.

I. *Example of* Money.

	l.	*s.*	*d.*	*q.*
	37	16	9.	3.
	21	9.	8.	1
	13	12	9	2
1 Total	72	19	3	2
2 Total	35	2	5	3
Proof	72	19	3	2

II. *Example of* Troy-Weight.

lb.	*ou.*	*pw.*	*gr.*
33	9.	12	16
25	11	6	9.
34	11.	19.	17
94	8	18	18
60	11	6	2
94	8	18	18

There

There are other ways to prove *Addition*, by casting away all the *Nines* in the Number of one Denomination, and of all the *Twelves*, *Twenties*, and *Nines*, in *Pounds*, *Shillings*, *Pence*, &c. but this, as the most certain and easie, I embrace: And thus much of Addition, and the Proof thereof.

SUBSTRACTION.

SUBSTRACTION is the taking of one or more small Sums out of a greater, as 7 *s.* out of 12 *s.* or 37 *l.* out of 100 *l.* or 137 foot out of 983 foot, and the like.

As in *Addition*, the Sums to be added may be either of one, or divers Denominations; so likewise they may be in *Substraction*, and the manner of placing them is the same; for you must set *Unities* under *Unities*, *Tens* under *Tens*, *Hundreds* under *Hundreds*, &c.

Example I. Of *Substraction* of Numbers of one Denomination. Let it be required to substract 234 out of 986. Place the Numbers one under the other, as you see done in the Margin, draw a Line under them, and begin with the first Figure towards the Right-hand, which is 4, saying, 4 from 6, and there remains 2, place 2 under the Line, and go to the next Figure, which is 3; saying, 3 from 8, and there remains 5, place 5 under the Line, and go to the next Figure, which is 2; saying, 2 out of 9, and there remains 7, place 7 under the Line, and your Substraction is ended; and it is evident by the Work, that if you take 234 out of 986, there will remain 752, which you may thus prove. Add the 234 to 752, and you shall find the Sum of that Addition to be 986, which is equal to the whole Sum from which 234 was substracted.

Number given	986
Number to be substracted	234
Remainder	752

Example II. Let it be required to substract 2976, out of 96527. Place the Numbers one under another, as in the Margin you see done; then draw a Line under them, and beginning with the first Figure towards your Left-hand; say, 6 out of 7, and there remains 1, place 1 under the Line, and proceed to the next Figure; saying, 7 out of 2 I cannot, (wherefore you must always add [10] to the Number above, which in this Example is 2, and it makes 12, therefore) take 7 out of 12, and there

there remains 5, place 5 under the Line, and (because you added 10 to the 2 to make it 12, you must) carry a Unite to the next Figure; saying, one, which I carry, and 9 is 10, take 10 out of 5, which because I cannot, therefore I must add 10 to 5, which makes it 15, and say, 10 out of 15, and there remains 5; place 5 under the Line, and (because you added 10 to 5, to make it 15, you must therefore) carry a Unite to the next Figure; saying, one, which I carry, and 2 is 3, take 3 out of 6, and there remains 3; place 3 under the Line, and because there is no more Figures to be substracted from the Number above, you must say, 0 from 9, and there remains 9; set the 9 under the Line, and your Substraction is ended.

96527
2976
93551

Other Examples for Practice.

	l.		*Reams of Paper.*		*Sheep.*
Lent	5762	Bought	9765	From	1000
Paid	378	Sold	6529	Take	394
Rests to pay	5384	Unsold	3236	Remains	606

Substraction of Numbers of divers Denominations.

RULE.

In Substraction *of Numbers of divers Denominations, you must observe the same Method as in* Addition, *namely, to place every Number in due Order, with respect to the Denomination, as* Pounds *under* Pounds, Shillings *under* Shillings, *&c. the greater Sum always uppermost; and drawing a Line under them, begin with the least Denomination first, substracting it from the Line above, and setting the* Remainder *under the Line, as in whole Numbers; but if the* Pence *or* Shillings *in the upper Row, be smaller than those in the lower Row, you must add* 12 d. *or* 20 s. *to the smaller Number, that so Substraction may be made, as by the* Examples *following will more plainly appear.*

Example I. Let it be required to substract 38 *l.* 12 *s.* 8 *d.* from 269 *l.* 18 *s.* 10 *d.* Place your Numbers as in the Margin; then, beginning with the least Denomination first, (which in this Example is *Pence*) say, 8 *d.* from 10 *d.* and there remains 2 *d.* set the 2 *d.* under the Line, and proceed to the next Denomination, which is *Shillings*; saying, 12 *s.* out of 18 *s.* and there remains 6 *s.* place 6 *s.* under

	l.	*s.*	*d.*
Lent	269	18	10
Paid	38	12	8
Rests	231	6	2

der the Line, and go to the Pounds; ſaying, 8 out of 9, and there remains 1; place 1 under the Line, and ſay, 3 out of 6, and there remains 3; then (becauſe there is no more Figures to be ſubſtracted) ſay, 0 out of 2, and there remains 2, which ſet under the Line. So is you Subſtraction ended, and the Remainder is 231 *l.* 6 *s.* 2 *d.*

Example II. Let it be required to ſubſtract 2628 *l.* 16 *s.* 10 *d.* out of 9320 *l.* 10 *s.* 7 *d.* Place the Numbers in Order, and beginning with the Pence, ſay, 10 *d.* out of 7 *d.* I cannot (therefore I muſt add 12 *d.* (which is 1 *s.*) to 7 *d.* and it makes 19 *d.*) but 10 *d.* out of 19 *d.* and there remains 9 *d.* ſet down the 9 *d.* under the Line, and (becauſe I added 12 *d.* to 7 *d.*) I muſt therefore carry one to the Place of *Shillings*; ſaying, 1 *s.* which I carry, and 16 *s.* is 17 *s.* but 17 *s.* from 10 *s.* I cannot, therefore I muſt add 20 *s.* (which is 1 *l.*) to 10 *s.* and it makes 30 *s.* then 17 *s.* out of 30 *s.* and there remains 13 *s.* ſet 13 under the Line, and carry one to the Place of *Pounds*; ſaying, one which I carry, and 8 is 9, take 9 out of 0, I cannot, but 9 out of 10, and there remains 1; ſet 1 under the Line, and carry an Unite to the next place; ſaying, 1, which I carry, and 2 is 3, now 3 out of 2, I cannot, but 3 out of 12, and there remains 9; place 9 under the Line, and carry 1 to the next place; ſaying, 1 which I carry, and 6 is 7; 7 out of 3 I cannot, but 7 out of 13 and there remains 6; place 6 under the Line, and carry one to the next Row; ſaying, 1 and 2 is 3, take 3 from 9, and there remains 6, place 6 under the Line. So is your Subſtraction ended, and the Remainder is 6691 *l.* 13 *s.* 9 *d.*

Lent	9320	10	7
Paid	2628	16	10
Reſts	6691	13	9

Example III. Suppoſe a Man had lent to another Man 1000 *l.* and that the Borrower had paid thereof at one time 127 *l.* at another time 490 *l.* 10 *s.* and at a third Payment 50 *l.* and the Creditor would know what he hath received, and how much is owing by his Debtor.

Place the Numbers as here you ſee, firſt the Sum of Money lent, and draw a Line under it; then ſet the Sums paid at ſeveral times, one under another, and draw a Line under them: Then add all the Sums which have been paid at ſeveral Times together, which make 667 *l.* 10 *s.* which is the Sum which the Debtor hath paid in all; then ſubſtract this 667 *l.*

	l.	*s.*	*d.*
Money lent	1000	0	0
Paid at ſeveral times.	127	0	0
	490	10	0
	50	0	0
Paid in all	667	10	0
Reſts to pay	332	10	0

667 *l.* 10 *s.* from 1000 *l.* and there will remain 332 *l.* 10 *s.* and ſo much is ſtill owing to the Creditor.

Other Examples *for Practice.*

	l.	*s.*	*d.*
Lent	2601	13	6
Paid	98	7	9
Reſts	2503	5	9

	l.	*s.*	*d.*
Owing in all	100	0	0
Paid in all	36	10	6
Reſts to pay	63	9	6

	l.	*s.*	*d.*	*q.*
Lent	3625	16	8	3
Paid at ſeve-	100	0	6	0
ral times	336	10	0	2
	39	12	9	2
	100	0	0	0
Paid in all	576	3	4	0
Reſts to pay	3049	13	4	3

The Proof of Subſtraction.

The *Proof* of *Subſtraction* is performed by *Addition*, for adding the Number to be ſubſtracted to the Remainder, the Sum of them muſt be equal to the Number given, if you have truly wrought: As in the firſt Example of Numbers of one Denomination.

The Number given is ———————	986
The Number to be ſubſtracted ———	234
The Remainder is ———————	752
Proof —————————————	986

Add the Number to be ſubſtracted, 234, to the Remainder, 752, the Sum of them is 986, equal to the Number given.

Examples for Practice proved.

	l.	s.	d.
Lent	62	18	9
Paid	37	19	6
Reſts	24	19	3
Proof	62	18	9

	l.	s.	d.
Borrowed	100	0	0
Received	36	13	4
Due	63	6	8
Proof	100	0	0

Other Examples *in Weight and Meaſure.*

EXAMPLE I.

Of Troy-Weight.

	lb.	ou.	pw.	gr.
Of Silver	7	11	13	19
Sold	5	7	3	5
Unſold	2	4	10	14
Proof	7	11	13	19

EXAMPLE II.

Avoirdupois Little Weight.

	lb.	ou.	dr.
Bought	84	12	13
Sold	26	8	11
Reſts	58	4	2
Proof	84	12	13

EXAMPLE III.

Avoirdupois Great Weight.

	C.	q.	lb.	ou.
Bought	37	3	22	11
Sold	13	1	23	6
Reſts	24	1	27	5
Proof	37	3	22	11

EXAMPLE IV.

Of Time.

	days	ho.	m.
From	364	23	50
Take	76	9	22
Reſts	288	14	28
Proof	364	23	50

*** *Note.* Becauſe that *Addition* is eaſier than *Subſtraction*, I will ſhew you how to perform both by the Method of the former: Thus,

Begin with the loweſt Denomination, (which in this Example is *Pence*) obſerve whether the uppermoſt Figure be greater than the lower, and conſider what other Figure being added will make an Equality between them, which note down, for the Difference; ſaying, as in this Example, 10 *s.* and 1 *s.* is 11 *s.* ſet down the 1 *s.* and proceed to the next, where I find

	l.	s.	d.
From	143	17	11
Subſtract	79	19	10
Difference	63	18	1

find the uppermost Figure is not greater but less than the undermost, and in such Case you must borrow Unity from the next Denomination, and consider what Number makes an Equality between the undermost Figure and that Unity; which Number add to the upper Figure, noting down the Sum for your Difference, always carrying the Unity forward again to its proper Denomination, as in the rest of this Example, saying, 19 *s.* and 1 *s.* is 20 *s.* this 1 *s.* and 17 *s.* is 18 *s.* put down the 18 *s.* and say, 1 *l.* that I carry, and 9 is 10; now 10 is equal to 10, the next greater Denomination, therefore note down the 3, and say again, 1 that I carry, and 7 is 8, and 6 makes 14, place down the 6, and so you have the Difference between 143 *l.* 17 *s.* 11 *d.* and 79 *l.* 19 *s.* 10 *d.* appearing to be 63 *l.* 18 *s.* 1 *d.*— This with Practice will become much more agreeable and ready, than the usual Way of *Substraction*, the Work being done and proved at one and the same time.

Questions performed by Addition *and* Substraction.

Question 1. *What Number is that which being added to* 376, *shall make* 1000? Substract 376 from 1000, the Remainder is 624, the Number sought.

Question 2. *What Number of Pounds, Shillings, and Pence must be added to* 36 l. 17 s. 3 d. *to make that Sum up* 100 l? Substract 36 *l.* 17 *s.* 3 *d.* from 100 *l.* the Remainder is 63 *l.* 2 *s.* 9 *d.* which added to 36 *l.* 17 *s.* 3 *d.* makes 100 *l.*

Question 3. *In the Year of our Lord* 1440, *the famous Art or Mystery of Printing was invented, I would know how long it is since that Time to this Year of our Lord,* 1699. From 1699, substract 1440, the Remainder is 259; and so many Years are expired since *Printing* was invented.

Question 4. *An Army consisting of* 13721 *Horse*, 26850 *Foot; in an Ingagement there were slain* 3760 *Horse, and* 7523 *Foot; the Question is, How many were slain in all? and how many Horse, and how many Foot escaped?* From the 13721 Horse that went out, substract the 3760 that were slain, there remains 9961, and so many Horse escaped: Also from the 26850 Foot which went out, substract the 7523 which were slain, and there remains 19327, the Number of Foot which escaped; and by adding the 3760 Horse which were slain, to the 7523 Foot that were slain, their Total is 11283, and so many were slain in all.

MULTIPLICATION.

MULTIPLICATION is that Part of *Arithmetic*, which teacheth how to increaſe one Number by another, ſo that the Number produced by their *Multiplication*, ſhall contain one of the Numbers multiplied, ſo many times as there are *Unites* contained in the other. *Multiplication* may fitly be termed a *Compendium* of *Addition*, for that it performeth at one Operation the ſame, which to effect by *Addition*, would require many. For inſtance, if it were required to know how much 7 times 5 is? To perform this by *Addition*, I muſt ſet ſeven *fives*, or five *ſevens* one under another, and adding them together, I ſhall find that either of their Totals ſhall contain 35; but this by *Multiplication* is performed with far more Brevity, as by Examples hereafter ſhall appear.

Before you enter upon the Practice of *Multiplication*, it is neceſſary to remember the Product ariſing by the Multiplication of any one of the nine Digits, by any other of the ſame; as readily to know that 4 times 5 is 20, 6 times 7 is 42, 2 times 9 is 18, 7 times 9 is 63, 8 times 9 is 72, &c. which this Table following will plainly declare, and muſt be perfectly learned by heart, before you attempt to multiply greater Numbers.

The Multiplication-Table,

$$\text{2 times}\left\{\begin{matrix}2\\3\\4\\5\\6\\7\\8\\9\end{matrix}\right\}\text{makes}\left\{\begin{matrix}4\\6\\8\\10\\12\\14\\16\\18\end{matrix}\right\}\quad\text{3 times}\left\{\begin{matrix}3\\4\\5\\6\\7\\8\\9\end{matrix}\right\}\text{makes}\left\{\begin{matrix}9\\12\\15\\18\\21\\24\\27\end{matrix}\right\}\quad\text{4 times}\left\{\begin{matrix}4\\5\\6\\7\\8\\9\end{matrix}\right\}\text{makes}\left\{\begin{matrix}16\\20\\24\\28\\32\\36\end{matrix}\right\}$$

$$\text{5 times}\left\{\begin{matrix}5\\6\\7\\8\\9\end{matrix}\right\}\text{makes}\left\{\begin{matrix}25\\30\\35\\40\\45\end{matrix}\right\}\quad\text{6 times}\left\{\begin{matrix}6\\7\\8\\9\end{matrix}\right\}\text{makes}\left\{\begin{matrix}36\\42\\48\\54\end{matrix}\right\}\quad\text{7 times}\left\{\begin{matrix}7\\8\\9\end{matrix}\right\}\text{makes}\left\{\begin{matrix}49\\56\\63\end{matrix}\right\}$$

$$\text{8 times}\left\{\begin{matrix}8\\9\end{matrix}\right\}\text{makes}\left\{\begin{matrix}64\\72\end{matrix}\right\}\quad\text{9 times }|\,9\,|\text{ makes 81.}$$

The

The Use of the Table of Multiplication, *and the Manner how it is to be read.*

This Table ſheweth what the ſum of any two Digits multiplied one by another doth amount unto, and is thus to be read, 2 times 2 makes 4, 2 times 3 makes 6, 2 times 4 makes 8; Alſo 6 times 4 makes 24, 7 times 8 makes 56, 8 times 8 makes 64, 9 times 9 makes 81, &c.

Another Table of Multiplication.

1	9	8	7	6	5	4	3	2	1
2	18	16	14	12	10	8	6	4	
3	27	24	21	18	15	12	9		
4	36	32	28	24	20	16			
5	45	40	35	30	25				
6	54	48	42	36					
7	63	56	49						
8	72	64							
9	81								

This Table is thus to be read; In the firſt Row, or Column, towards the left-hand, and alſo at the top of the Table, you have the nine Digits in bigger Figures than the reſt; the Figures in the firſt Column beginning with 1, and ſo proceeding by 2, 3, 4, &c. to 9. Thoſe at the top of the Table, beginning with 9 towards the left-hand, and ſo backwards, by 8, 7, 6, &c. to 1, at the right-hand.

Now if by this Table you would know how much 8 times 7 is, find 8 among the great Figures at the head of the Table, and look down that Row or Column, till you come againſt 7 of the great Figures in the firſt *Column*, againſt which you ſhall find 56, and ſo much is 8 times 7, or 8 multiplied by 7.

In

In the ſame manner may you find that 7 times 9 is 63, 5 times 6 is 30, 3 times 4 is 12, and ſo of any two of the nine Digits.

In Multiplication there are three terms commonly uſed, that is to ſay;

The *Multiplicand*,
The *Multiplier*,
The *Product*.

The *Multiplicand* is the Number to be multiplied.

The *Multiplier* is the Number by which the Multiplicand is Multiplied: And,

The *Product* is the Number which is produc'd by the Multiplication of the Multiplicand and the Multiplier together.

Thus, if it were required to multiply 8 by 7, here 8 is the *Multiplicand*, 7 the *Multiplier*, and 56 is the product, for 8 times 7, or 7 times 8, is 56.

In *Multiplication* it mattereth not which of the two Numbers is made the *Multiplicand*, or which the *Multiplier*, for the *Product* produced by either, will be the ſame; but the uſual way is to make the *greater* number the *Multiplicand*, and the *leſſer* number the *Multiplier*.

RULE.

The Numbers to be multiplied muſt be ſet one under another, viz. *the* Multiplicand *(or greater number) above, and the* Multiplier *(or leſſer number) below, the laſt figure of the* Multiplier *under the laſt figure of the* Multiplicand, *then draw a line under them, and (having learned the preceding Tables perfectly by heart) multiply every Digit of the* Multiplier, *into every Digit of the* Multiplicand, *ſetting the ſeveral Products under the line; then having finiſhed your* Multiplication, *draw a line and add all the Products together, and the ſum of thoſe Products is the general* Product *of the whole Multiplication, as by the following* Examples *will appear.*

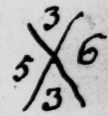
Example I. Let it be required to multiply 736 by 7. *Firſt*, I write down 736 the Multiplicand, and under it 7 the Multiplier, and under them I draw a line; then I multiply 7 into every Digit of the Multiplicand; ſaying, 7 times 6 is 42, place 2 under the line, under 7, and for the 4 Tens keep 4 in mind; then ſay again, 7 times 3 is 21, and 4 which I kept in mind is 25, place 5 under the line, and keep the 2 tens in mind; then ſay again, 7 times 7 is 49, and 2 which I kept in mind is 51; place 1 under the line, and the 5 Tens kept in mind (becauſe there is no more figures to be multiplied) I ſet down under the line, ſo is the work ended, and the Product of this Multiplication is 5152.

736	*Multiplicand*
7	*Multiplier*
5152	*Product*

Example

Example II. Let it be required to multiply 3417 by 5. Place the Numbers one under another, and draw a line under them, as in the Margin; then begin your Multiplication, saying 5 times 7 is 35, place 5 under the line, and keep the 3 tens in mind; then say again, 5 times 1 is 5, and 3 which I kept in mind is 8, place 8 under the line, and (because it is less than 10, I keep nothing in mind) then say again, 5 times 4 is 20, place a Cypher under the the Line, and keep the 2 Tens in mind: Lastly, say, 5 times 3 is 15, and 2 which I kept in mind is 17, which 17 (being the last number) I place under the line, and so is my Multiplication ended, and the Product is 17085.

$$\begin{array}{r} 3417 \\ 5 \\ \hline 17085 \end{array}$$

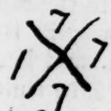


¶ You may be satisfied of the truth of this work, if you will take the pains to set down the Multiplicand 3417, five times one under another, and add them together, as so many several sums, so shall you find the Total of that Addition, to be 17085, exactly the same with the Product of this Multiplication.

Example III. In the two fore-going Examples, the *Multiplier* consisted but of one *Digit*, we are now to shew how *Multiplication* is performed, when the Multiplier consists of more than one figure, therefore in this Example,

Let it be required to Multiply 5704 *by* 37. Place your numbers, and draw a line under them as you see in the Margin. Then begin your Multiplication in this manner; Saying, 7 times 4 is 28, set 8 under the line and keep the 2 tens in mind, then say 7 times nothing is nothing, but the 2 tens in mind is 2, set 2 under the line, then say 7 times 7 is 49, set 9 under the line, and keep 4 in mind, then lastly, say 7 times 5 is 35, and 4 in mind is 39, which being the last Numbers to be multiplied, I set down under the line; so is the Multiplication of one of the Digits (namely 7) finished.

$$\begin{array}{rl} 5704 & \text{Multiplicand} \\ 37 & \text{Multiplier} \\ \hline 39928 & \\ 17112 & \\ \hline 211048 & \text{Product} \end{array}$$



Then begin to multiply the second digit, saying 3 times 4 is 12, place 2 in the second line, one place more towards the left hand, and keep 1 in mind, then say 3 times nothing is nothing, but 1 in mind is 1, set down 1 by the 2 in the second line; thirdly, say 3 times 7 is 21, place 1 in the second line, and keep the 2 tens in mind. Lastly, say 3 times 5 is 15, and 2 is 17, which 17 (because there is no more figures to be multiplied) I place in the second line also.

Having

Having thus done, I draw a line under them, and add these two lines together, as in common Addition of numbers of one denomination; Saying 8 is 8, place 8 under the line; then say 2 and 2 is 4, place 4 under the line; then say 1 and 9 is 10, place a Cypher under the line; and carry 1 to the next place; saying, 1 and 1 is 2, and 9 is 11, place 1 under the line, and carry 1 to the next Row; saying 1 and 7 is 8, and 3 is 11, place 1 under the line, and carry 1 to the next place; saying, 1 which I carry and 1 is 2, place 2 under the line; and so is your Multiplication ended, and the Product is 211048.

Example IV. Let it be required to multiply 57325 by 4032. Place the Multiplicand and the Multiplier one under another, and draw a line as before; then proceed to the Multiplication, as formerly; saying, first, 2 times 5 is 10, set down a Cypher, and keep 1 mind; then 2 times 2 is 4, and 1 in mind is 5, place 5 under the line; then 2 times 3 is 6, set 6 under the line; then 2 times 7 is 14, set down 4 and keep one in mind; then 2 times 5 is 10, and 1 in mind is 11, which 11 (being the last) I set down.

57325	Multiplicand
4032	Multiplier
114650	
171975	
229300	
231134400	Product

The Multiplication of one of the Digits being finished, proceed to the Multiplication of the next; saying, 3 times 5 is 15, set down 5 in the second Line, one place more towards the Left-hand, and keep 1; then 3 times 2 is 6, and 1 kept is 7, set down 7; then 3 times 3 is 9, set down 9; then 3 times 7 is 21, set down 1, and keep 2 in mind; then 3 times 5 is 15, and 2 in mind is 17; which being the last, set down also.

Two of the figures of the Multiplier being finished, proceed to the third, which (in this Example) being a Cypher, you may wholly neglect, and proceed to the Multiplication of the fourth Figure; only remember to remove the Product of the fourth Figure one place more to the Left-hand, as in the Example you may see, for the Cypher, though it be not written down, yet it must keep its place, and the Figures following must be removed a place farther.

Then for the Multiplication of the fourth and last Digit; say, 4 times 5 is 20, set down a Cypher, (under 9) and keep 2 in mind: Then 4 times 2 is 8, and 2 in mind is 10, set down a Cypher, and keep 1 in mind; then 4 times 3 is 12, and 1 is 13, set down 3, and carry 1; then 4 times 7 is 28, and 1 kept is 29, set down 9, and keep 2; then 4 times 5 is 20, and 2 is 22; which (because the Multiplication is ended) set down also.

Having

Having thus multiplied all the Digits severally, draw a line under their Products, and add them altogether, as in the former Example; so shall you find their general Product to be 231134400.

Other Examples for Practice.

73260	50762
45003	4567
219780	355334
366300	304572
293040	253810
	203048
3296919780	231830054

The Proof of Multiplication.

The most certain Proof of Multiplication is by Division; but because Division is not yet known, I will here shew a near Way, by which Multiplication may be proved. Which is thus;

RULE.

Make a Cross, *as in the Margin; then any Sums being multiplied you may prove the Truth of your Work in this manner;* (1) *Cast away all the* Nines *which you can find in the* Multiplicand, *what remaineth set on the right Side of the Cross.* (2) *Cast away also the* Nines *in the* Multiplier, *and what remains set on the left Side of the Cross.* (3) *Multiply the Figure on the right Side of the Cross, by that on the left Side, and out of that* Product *cast away all the* Nines *also, setting the Figure remaining, over the Cross. Then* (4) *cast away all the* Nines *in the* Product; *and if the Figure remaining be the same with that which standeth over the Cross, then is your* Multiplication *truly performed, otherwise not.*

X

Example. Let it be required to prove the Sum in the Margin.

1. Cast away all the *Nines* in the *Multiplicand*; saying, 4 and 3 is 7, and 2 is 9, which being rejected, there remains 4, which I set on the right Side of the Cross. Then,

2. Cast away all the *Nines* in the *Multiplier*; saying, 2 and 3 is 5, (which

4324
23
12972
8648
99452

2
5 X 4
2

being less than 9) I set on the left Side of the Cross. Then,

3. Multiply 4 by 5; saying, 4 times 5 is 20, from which cast away all the *Nines*, which are two, and there remains 2, place 2 over the Cross. And,

4. Cast away all the *Nines* in the *Product*; saying, 2 and 5 is 7, and 4 is 11, cast away 9 and there remains 2: Which exactly agrees with the Figure over the Cross, and demonstrates that the Multiplication is truly performed.

Compendiums in Multiplication.

1. If the *Multiplier* consists of Cyphers in the last place or places, you may omit the Multiplication of them, and place only the Figures of the Multiplier under the Multiplicand: Thus, if it were required to multiply 3257 by 2600; Place the Numbers as you see in the Margin; Then multiplying 3257 by 26, the Product will be 84682, to which if you add two Cyphers, (because there were two Cyphers in the Multiplier) it will be 8468200, which is the true Product of the Multiplication.

$$\begin{array}{r} 3257 \\ 2600 \\ \hline 19542 \\ 6514 \\ \hline 8468200 \end{array}$$

2. If it be required to multiply any Number by 10, 100, 1000, 10000, &c. You have no more to do, but to add so many Cyphers to the *Multiplicand*, as there are Cyphers in the *Multiplier*: Thus, if you were to multiply 365 by 10, the Product will be 3650; or by 100, it would be 36500; or by 1000, it would be 365000; or by 10000, it would be 3650000.

3. If any Number were to be multiplied by 5, you may abbreviate your Work thus. Add a Cypher to the *Multiplicand*, take half that Number, and it shall be the Product required. Thus, if were required to multiply 8627 by 5, add a Cypher to the *Multiplicand*, then it is 86270, the half whereof is 43135, which is the Product required.

To multiply by any of the nine Digits, without burthening the Memory.

To multiply any Number by 2; Either double the Number in your mind, or add it, by setting it twice down; so 57325 produceth 114650.

To multiply any Number by 3; To the Number given add the double thereof, the Sum is the Product; so 57325 produceth 171975.

To multiply any Number by 4; Double the Duplication in your mind; so 57325 produceth 229300.

To

To multiply any Number by 5. Conceive a Cypher added to the given Number, and in your mind take half thereof for the Product, thus a Cypher added to 57325, maketh it 573250, the half whereof is 286625.

To multiply any Number by 6. Add a Cypher to the given Number, and take the half, to which add the given Number, that sum shall be the Product. Thus 57325 produceth 343950.

To multiply any Number by 7. Take half and add it to the double of the former Figure, supposing a Cypher added as before; So 57325 thus ordered, produceth 401275.

To multiply any Number by 8. Double each former figure and substract it from the following, so 57325 produceth 458600.

To multiply any Number by 9. Suppose the Number multiplied by 10, then substract each former Figure from the following, beginning with that next before the Cypher, the Remainder is the Product, so 57325 produceth 515925.

Other Brief Rules of Multiplication.

Having shewed you some *Compendious* ways of *multiplying* by *Article-Numbers*, as by 10, 100, 1000, *&c.* and also by the *nine Digits*, without any trouble or charge to the *Memory*: I will now shew how you may expeditiously and certainly, *multiply* any Sum by divers other Numbers consisting of *two* or *three* places, without setting down many Figures but the Product it self.

To multiply any number by 11.

RULE.

Set the Multiplicand down twice, removing it one place to the left-hand, add them together, the sum is the Product of the Number multiplyed by 11.

Example. *Let it be required to multiply* 97 *by* 11.

Set down 97 twice, removing one of them a place more to the left hand, as you see here in the margin, then add them together, the sum is 1067, which is equal to the Product of 97 multiplied by 11.

$$\begin{array}{r} 97 \\ 97 \\ \hline 1067 \end{array}$$

To multiply any Number by 12, 13, 14, 15, 16, 17, 18, *or* 19.

RULE.

To effect this, You have no more to do, but to *multiply the given number by* 1, 2, 3, 4 5, 6, 7, 8, *or* 9, *and in your Multiplication,*

tiplication, continually to add that figure of the Multiplicand, which ſtandeth on the right hand of the figure you are multiplying by, ſetting down the ſum for the figure of the Product.

Example. *Let it be required to multiply* 3624 *by* 17.

Multiply in this manner by 7, Saying, 7 times 4 is 28, ſet down 8 and carry 2 ——— then 7 times 2 is 14, and 2 which I carryed is 16, and 4 (the figure of the Multiplicand which ſtands on the right-hand of 2) is 20: Set down 0, and carry 2 ——— then 7 times 6 is 42, and 2 which I carried is 44, and 2 (which ſtands on the right-hand of 6) is 46? ſet down 6 and carry 4 ——— then 7 times 3 is 21, and 4 which I carried is 25, and 6 (which ſtands on the right-hand of 3) is 31, ſet down 1, and carry 3, which 3 added to 3 (the left-hand figure of your Multiplicand,) makes 6, which ſet down; ſo the Product of 3624 multiplied 17, is 61608.

In the ſame manner may you multiply by 12, 13, 14, *&c.* as in theſe Examples.

3624	625	4793
17	16	19
61608	10000	91067

To multiply any Number by 102, 103, 104, 105, 106, 107, 108, *or* 109.

RULE.

Multiply the Number given by 2, 3, 4, 5, 6, 7, 8, *or* 9, *ſetting the Product two Places towards the Right-hand of the* Multiplicand, *that* Product *and* Multiplicand *added together in the ſame Order as they ſtand, ſhall be the Product of the whole* Multiplication.

Example. *Let it be required to multiply* 3924 *by* 106,

3624
106
21744
384144

Set them down as in the Margin; then multiply 3624 by 6, it produceth 21744, which added to 3624, in the ſame Order as they ſtand, the Sum of that Addition will be 384144, which is equal to the Product of 3624 multiplied by 106.

Other

Other Examples for Practice.

765			6374
103			107
2295			44618
78795			682018

To multiply any Number by 112, 113, 114 115, 116, 117, 118, *or* 119.

Multiply the given Sum by 12, 13, 14, 15, 16, 17, 18, *or* 19, *as hath been shewn already, setting the Product two Places to the Right-hand of the* Multiplicand; *then add the* Product *and the* Multiplicand *together, in the same Order as they stand, so shall the Sum of that* Addition *be equal to the* Product *of the* Multiplication. As is evident by the Examples following.

Multiply	4065
By	113
The Product multiply'd by 13	52845
Real Product	459345

Multiply	7632
By	119
The Product multiply'd by 19	145008
Real Product	908208

Questions performed by Multiplication *only.*

Question I. *If a Piece of Land be* 236 *Perches long, and* 182 *Perches broad, how many square Perches are contained therein?* Multiply 236 the Length, by 182 the Breadth, the Product is 42952; and so many square Perches are contained in such a square Piece of Land.

Question II. *In a Year there are* 365 *Days Natural, and in every Day* 24 *Hours; How many Hours be there in a Year?* Multiply 365 the Number of Days, by 24 the Number of Hours, the Product is 8760; And so many Hours be there in a Year.

Question

Queſtion III. *From* London *to* Coventry *is accounted 76 miles; How many Yards therefore is it from* London *to* Coventry? Multiply 1760 (which are the Number of Yards contained in one Mile) by 76, the Product is 133760; and ſo many Yards are between *London* and *Coventry*.

DIVISION.

DIVISION is juſt the Contrary to Multiplication, for that turns Small denominations to Greater, as Multiplication turns Greater to Smaller: Or (in whole Numbers, of which only we yet ſpeak) Diviſion is the aſking, how many times one ſum is contained in another; and the number which anſwereth to that queſtion is called the Quotient.

And the Number containing, which is to be divided, is called the *Dividend*.

And the Number contained, or by which the *Dividend* is to be divided, is called the *Diviſor*.

And as often as the *Dividend* contains the *Diviſor*, ſo often doth the *Quotient* contain *Unity*.

So that, As *Multiplication* is a Compendium of many Additions: So *Diviſion* is but a Compendium of many *Subſtractions*.

There are ſeveral ways by which this Rule of *Diviſion* may be wrought; of which, ſome are more eaſie than others, to be performed, and the *Rule* made more intelligible to the *Learner*. Wherefore, rejecting the Old and tedious way, by often ſetting down the *Diviſor* and Cancelling of Figures: I will only inſiſt upon Two or Three of the moſt Plain and *Eaſie Ways*; whereby this *Difficult Rule* of *Diviſion* may be wrought with much eaſe and perſpicuity.

And whereas I ſaid before, in *Multiplication*; that the beſt Proof thereof was by *Diviſion*; So I ſay here, That the beſt *Proof* of *Diviſion* is by *Multiplication*: I will therefore take for my *Examples* in this *Rule*, the Converſe with thoſe which were formerly done in *Multiplication*; whereby the *Proof* of each *Rule* will be evident.

For the Working of *Diviſion* the moſt Plain and Eaſie way, this is the

RULE.

I. *Set down the Dividend* (or number to be divided:) *And on the Left-hand thereof ſet the Diviſor* (or number by which you are to Divide) *with a* Crooked Line between them: *Then, on the Right-hand*

hand of the Dividend make another Crooked Line, *wherein the figures of the Quotient are to be placed: So that, If* 162483 *were a Number given to be Divided by* 1321: *The Numbers muſt be ſet as here you ſee; And the* Diviſion *ended, the* Quotient *will be found to be* 123. *So that* 132 (*the* Diviſor) *will be contained in* 162483 (*the* Dividend) 123 *Times: for that is the Number in the* Quotient.

Diviſor	Dividend	Quotient
1321)	162483	(123

II. *Demand* (or aſk) *how often the* Diviſor *may be had in the* Dividend, *and place that number in the* Quotient: *Then, Multiply the* Diviſor *by the Figure in the* Quotient; *and place that Product under the* Dividend, *and Subſtract it from the* Dividend, *ſetting the* Remainder *under the* Product, *and draw a Line under it.*——— *Then make a Prick under the next figure of the* Dividend *and bring that Figure down to the* Remainder; *And then proceed as before.*

Example I. Let it be required to Divide 5152, by 7.

Firſt, Set the Numbers down according to the former Directions, and as you ſee done in the Margin; And becauſe 7 the *Diviſor*, is greater than 5 the firſt Figure of the *Dividend*, make a prick under the ſecond Figure of the *Dividend*, namely, under 1. Then,

	7)	5152	(736
1 Prod.		49	
		25	1 Rem.
2 Prod.		21	
		42	2 Rem.
3 Prod.		42	
		00	

Secondly, Ask how many times 7, the *Diviſor*, you can have in 51, the two firſt Figures of the *Dividend?* and the Anſwer will be 7 times; wherefore put 7 in the Quotient, and multiply 7 the *Diviſor*, by 7 in the *Quotient*, and the *Product* will be 49; which ſet under 51. Then ſubſtract 49 from 51: and there will remain 2, which ſet under 9, and draw a Line under it.

Thirdly, Make a *Prick* under 5, the third Figure in the *Dividend*, and bring that Figure 5, down to 2 the firſt *Remainder*, making it 25: And aſk again, How many times 7, the *Diviſor*, can you have in 25? the Anſwer will be 3, which ſet in the *Quotient*, and multiply the *Diviſor* 7 by it; ſaying, 3 times 7 is 21, for the ſecond *Product*, which 21 ſet under 25, and ſubſtracting 21 from 25, the *Remainder* will be 4, which ſet under 21, and draw a Line under it.

Fourthly,

Fourthly, Make a *Prick* under 2, the laſt Figure of the *Dividend*, and bring that Figure 2, down to the Second *Remainder* 4, making it 42; and then aſk, How many times 7, the *Diviſor*, you can have in 42? the anſwer will be 6 times, which ſet in the *Quotient*, and multiply the *Diviſor* 7, by it, ſaying, 6 times 7 is 42, for the Third *Remainder*, which 42 ſet under 42, and Subſtracting one from the other, (*viz*. 42 from 42) the *Remainder* will be (00). So is your *Diviſion* ended; and the *Quotient* is 736, which ſhews, that the D*iviſor* 7 is contained 736 times in the *Dividend* 5152; For if you Mu*ltiply* 736 by 7, the *Product* will be 5152: As by the Firſt *Example* in *Multiplication*.

This Firſt *Example* hath but one ſingle Figure to the *Diviſor* and ſo is more eaſie and clear, and will be a good introduction to theſe which follow,

Example II. Let it be required to Divide 211048, by 37.

Set the Numbers down as before, and as in this *Example*. And becauſe 37 the *Diviſor*, is greater than 21 the two firſt Figures

	37) 211048 (5704	
1 Prod.	185	
	260	1 Rem.
2 Prod.	259	
	148	2 and 3 Rem.
3 Prod.	148	
	0	4 Remaind.

of the D*ividend*, make your firſt *Prick* under 1, the third Figure in your D*ividend*: Then,

Firſt, aſk how many times 37, can you have in 211, the anſwer will be 5 times; (or how many times 3 can you have in 21, and then the Anſwer will be 7, for 7 times 3 is 21; but then, although you may have 7 times 3 is 21, you cannot have 7 times 7 which is 49 in 1; and therefore you muſt not put 7 in the *Quotient*, but a leſs Figure, ſuppoſe therefore 6, then 6 times 3 is 18, from 21, and there remains 3; but then again 6 times 7 makes 42, which cannot be taking out of 31, and ſo a leſſer figure than 6 muſt be put in the Quotient, as 5,) And now to proceed: Saying;

How many times 3 can you have in 21, the Anſwer will be 5 times; Put 5 in the *Quotient*, and multiply 37 the *Diviſor* by 5, the *Product* will be 185, which ſet under 211, and ſubſtracting 185 from 211, the *Remainder* will be 26; which ſet under it.

Secondly, Make a *Prick* under 0, the next Figure in the *Dividend* and bring that 0 down to 26, making it 260; and aſk, How

many

many times 37 can you have in 360, (or how many times 3 in 26) The Anſwer will be 7 times; Set 7 in the *Quotient*, and Multiplying the *Diviſor* 37 by 7, the *Product* will be 259; which ſet under 260, and ſubſtract it from 259, ſo will the *Remainder* be 1. Then,

Thirdly, Make a *Prick* under 4, the next Figure in the *Dividend* and bring that 4 down to 1, making it 14, and draw a Line under it: And aſk how many times 37 the *Diviſor*, can you have in 14 the *Remainder*, the anſwer will be never a time, (it being leſs) wherefore put a *Cypher* in the *Quotient*, and

Fourthly, Make a *Prick* under 8, the laſt Figure in the *Dividend*, and bring that 8 down to 14, making it 148, and aſk, how many times 37 can you have in 148, (or how many 3 in 14) the anſwer will be 4 times; Put 4 in the *Quotient*, and multiplying the Diviſor 37, by 4, the *Product* will be 148, which ſet under 148, and ſubſtracting it therefrom, there will remain 0. So is the Diviſion ended, and the *Quotient* is 5704; and ſo many times is 37 the *Diviſor*, contained in the *Dividend* 211048: For if you multiply 5704 the *Quotient*, by 37 the *Diviſor*; the *Product* will be 211048, As in the Third *Example*, of *Multiplication*, which proves your *Diviſion* to be truly performed.

Example III. Let it be required to Divide 231134400, by 57325.

Set the Numbers down as you ſee here done, making a *Prick* under 4, the Sixth Figure of the *Dividend*, and begin your Work as before: ſaying,

	57325) 231134400 (4032	
1 Prod.	229300	
	183440	1 and 2 Rem.
2 and 3 Prod.	171975	
	114650	3 Rem.
4 Prod.	114650	
	0	4 Rem.

First, How many times 57325 (the *Diviſor*) can you have in 231134, (part of the *Dividend*) or more eaſie, How many times 5 can you have in 23, the Anſwer will be 4 times; Put 4 in the *Quotient*; and multiply the *Diviſor* 57325 by 4, the *Product* will be 229300, which ſubſtracted from 231134, there will remain 1834, under which draw a Line.

Secondly, Make a *Prick* under 4, the next Figure in the *Dividend*, and bring that 4 down to 1834, making it 18344; And then aſk how many times 57325 can you have in 18344, the Anſwer is, never a time, it being the greater Number; therefore

fore put a Cypher in the Quotient, and making a prick under the next Figure of the Dividend, which is 0, bring down that 0 to 18344, making it 183440. And then,

Thirdly, Aſk how many times 57325 (the Diviſor) can you have in 183440, or (more eaſie) how many times 5 can you have in 18, the Anſwer will be 3 times; put 3 in the Quotient, and multiply 57325, (the Diviſor) thereby, and the Product will be 171975, which ſubſtract from 183440, and the Remainder will be 11465; under which draw a Line: And,

Fourthly, Make a prick under the laſt Figure of the Dividend, which is 0, and bring that 0 down to the laſt Remainder, making it 114650; And aſk how many times 57325 can you have in 114650, (or how many times 5 in 11) the Anſwer will be 2 times; put 2 in the Quotient, and multiply the Diviſor 57325 thereby, the Product will be 114650, equal to the laſt Remainder. So is the Diviſion ended, the Quotient being 4032; and ſo many times is 57332, the Diviſor, contained in 231134400, the Dividend: As will appear, if you multiply them together, as in the Fourth Example of Multiplication.

Thus you ſee how Diviſion proves Multiplication, and how Multiplication is proved by Diviſion.

But this kind of Diviſion may be proved otherways than by multiplying the Diviſor by the Quotient: By this

RULE.

When your Diviſion *is ended, add all the* Products *reſulting in the whole Work together, in the ſame Order as they ſtand in the Work; the Sum of them (adding the laſt Remainder, if any be) ſhall be equal to the* Dividend.

So in this Third Example.

The *firſt* Product is	229300...
The *ſecond* and *third* Prod.	..171975.
The *fourth* Product	...114650
The Sum equal to the Dividend.	131134400

And thus much for this Way of Diviſion: Another Way, not much unlike it, followeth.

A Second Way of Diviſion.

Example. Let it be required to divide 768325, by 324.

First, Set down the Dividend between two *Parallel Lines*, and the Diviſor on the *Left-hand*, within a *Crooked Line*, and another *Crooked*

Crooked Line on the *Right-hand* for the *Quotient:* All this as in the former way; Then having made a *Prick* under the third Figure, 8, of the Dividend, as you ſee done in the following Example: You may begin your Diviſion thus,

		121		Remain
		445		
		2312·		
		1203··		
Diviſor	324)	768325	(2371	Quotient
		····		
		648···		
		972··		
		2268·		
		324		
Sum		768325		Equal to the Dividend.

Firſt, Aſk how many times 324, the Diviſor, can you have in 768, the three firſt Figures of the Dividend, the Anſwer is 2; ſet 2 in the Quotient, and multiply the Diviſor thereby, and the Product will be 648; which ſet under 768, and ſubſtract it therefrom, ſetting the Remainder 120, over 768.

Secondly, Make a Prick under the next Figure in the Dividend, making the Remainder 120 to be 1203; Then aſk, How many times 324 in 1203, the Anſwer will be 3 times; put 3 in the Quotient, and multiply the Diviſor thereby, and the Product will be 972, which ſet under 1203, and ſubſtract it therefrom, ſetting the Remainder 231, over 1203.

Thirdly, Make a prick under the next Figure of the Dividend 2, making the Remainder 231, to be 2312: Then aſk, How many times 324, can you have in 2312, the Anſwer will be 7 times; put 7 in the Quotient, and multiply the Diviſor 324 thereby, the Product will be 2268; which ſet under 2312, ſubſtracting it therefrom, and ſetting the Remainder 44 over 2312.

Fourthly, Make a Prick under 5, the laſt Figure of the Dividend, making the Remainder 44, to be 445: Then aſk, How often you can have the Diviſor 324 in 445, the Anſwer will be 1; put 1 in the Quotient, and by it multiply the Diviſor 324, which will be the ſame; ſet therefore 324 under 445, and ſubſtract it therefrom, ſetting the Remainder 121 over 445.

And thus is your Diviſion ended, the Quotient being 2371; and ſo many times is 324 contained in 768325, and 121 remaining: Which you may prove, by multiplying 2371 by 324, and adding

adding 121 to the Product; for that Sum will be equal to 768325 the Dividend.

But the Proof of Division may be done more easily in this Way of Division. For

If you add all the Products together, and the last Remainder, if any be, the Sum of that Addition will be equal to the Dividend.

Examples for Practice in both these Ways of Division ready wrought do here follow;

Examples of the First Way.

```
Divisor)  Divid.   (Quotient.
  2325)  763258   (328
            ...
        ---------
          6975
           6575
        ---------
           4650
           19258
        ---------
           18600
             658    Remainder
        ---------
Proof     763258    Equal to the Dividend,
```

Another Example.

```
70993)  42009876  (591
             ...
       -----------
        354965
         651337
       -----------
         638937
          124006
       -----------
           70993
           53013    Remain
       -----------
        42009876
```

Exam-

Examples of the Second Way.

```
                 664  ┐
               17698  ├ Remainders.
               7447·  ┘
            ────────────
Divisor  5678) 2345678 (413   Quotient
                   •••
            ────────────
              22712··  ┐
               5678·   ├ Products
              17034    ┘
            ────────────
              2345014
Remaind. add      664
            ────────────
Proof         2345678   Equal to the Dividend.
```

Another Example.

```
                  473  ┐
                2215·  ┘ Remainders
            ──────────
Divisor   542) 76353 (140   Quotient
                  •••
            ──────────
               542··  ┐
               2168·  ┘ Products
            ──────────
               75880
Remaind. add     473
            ──────────
Proof          76353   Equal to the Dividend.
```

POST-

POST-SCRIPT

TO

DIVISION:

OR, A

RULE,

BY

Which you may certainly know what Figure to ſet in your Quotient; and never to take one too great, or too little, but that which will juſt ſerve:

AND

Alſo how to perform (with Eaſe and Certainty) the moſt difficult Sum that can be propoſed in *Diviſion*, without the Aſſiſtance of *Multiplication*; only by *Addition* and *Subſtraction*: Not burthening the Memory at all.

IN the Practice of Diviſion, there is nothing more difficult, than in large Sums (eſpecially if the firſt Figures of the Diviſor be either 1, 2, 3, or Cyphers, and the laſt Figures 7, 8, or 9) to know certainly what Figure to put in the Quotient, when you demand how often the Diviſor may be had in the

the Dividend; for the certain finding whereof (a little Pains being taken before you begin your Work) do thus;

Suppose you were to divide any Sum, as 1097909, by 309: First, set down the nine Digits, 1, 2, 3, &c. one under another; and against the Figure 1 set 309, your Divisor, which doubled is 618, which set against 2; these added together make 927, which stands against 3: Add the Divisor 309 to 927, it makes 1236, which is against 4; to this add the Divisor, and it makes 1545, which stands against 5. And thus to every last Number, still add the Divisor, till you have gone through all the nine Digits; then will they be as in the Margin.

1	309
2	618
3	927
4	1236
5	1545
6	1854
7	2163
8	2472
9	2781

Having prepared this Table, set your Dividend and Divisor down, as in the First Way of Division, pricking the Dividend, and drawing a Line under it, as is there directed, and as you see here done;

```
309) 1097909 (3553
     ....
     -------
     927
     1709
     -------
     1545
      1640
     -------
     1545
       959
     -------
       927
        32
```

Then laying your little Table before you, look in it for 1097, the four first Figures of the Dividend, which you cannot exactly find there, but the nearest Number less, (which you must always take when you cannot find the just Number you look for) is 927, against which stands 3; set 3 in the Quotient, and substract 927 out of 1097, and there will remain 170, to which bring down 9, the next Figure of your Dividend, and it is 1709; Look this Number in your Table, which you cannot find, but the next less is 1545, against which stands 5; set 5 in the Quotient, and substract 1545 out of 1709, and there will remain 164; to which bring down the next Figure of your Dividend (which here is 0) making it 1640; Look this 1640 in the Table, which you cannot

not find, but the next less is 1545, against which stands 5; set 5 in the Quotient, and substract 1545 out of 1640, and there will remain 95; to which bring down the last Figure in your Dividend, which is 9, making it 959: Look this Number in the Table, or the next less, which is 927, against which stands 3; set 3 in the Quotient, and substract 927 from 959, the Remainder is 32. So is your Division ended, and the Quotient is $3553\frac{32}{309}$. And with what Ease and Certainty this is effected, no Multiplication being used, I leave to the Reader to judge.

Compendiums in Division.

To divide any Number by 10, 100, *or* 1000, &c.

RULE.

If from the Number to be divided, you cut off so many Figures towards the Right-hand, *as there are Cyphers after the Unit in the* Divisor; *Then the Figures towards the* Left-hand *shall be the* Quotient, *and those cut off, the* Remainder: *So,*

6583	divided by	10	the Quotient is	658	Remains	3
40208		100		402		8
791634		1000		791		634

To divide any Number readily, by any of the nine Digits.

First, The Digit 1, neither multiplies nor divides any Number, but leaves it the same: But,

Secondly, To divide any Number by 2. It is but taking the half of the given Number: So 730 divided by 2, the Quotient will be 365; for the half of 730 is 365.

Thirdly, To divide any Number by 3; Take one third part of the Number for the Quotient: But if the Number given cannot be parted into 3 equally, the Digits remaining are *Thirds* of the *Unit*: So if 1095 were to be divided by 3, the Quotient would be 365; but if 1097 were to be divided by 3, the Quotient would be $365\frac{2}{3}$; for one third part of 1097, is 365, and there will 2 remain, which are two *Thirds*.

Fourthly, If you you would divide any Number by 4; One *fourth* part thereof is the Quotient, and the Remainder (if any be) are *Fourths*: So 1460 divided by 4, the Quotient is 365; but 1463 divided by 4, will give for the Quotient $365\frac{3}{4}$.

Fifthly, If you divide any Number by 5; Double the Number, cut off the Figure towards the *Right-hand*, and the Figures towards the *Left-hand* shall be the Quotient, and the Figure cut off the Remainder, or *Fifths*: So if you would divide 1825 by 5, the

the Quotient will be 365; for the double of 1825 is 3650, and the Cypher abated, the Quotient is 365.

Sixthly, If you would divide any Number by 6; Take half the given Number, and one third part thereof will be the Quotient: So if you would divide 2190 by 6; the half of 2190 is 1095, one third part thereof is 365, for the Quotient.

Seventhly, If you would divide any Number by 7; double the Number given, and cut off the last Figure towards the Right-hand; then take the seventh part of that Number, and double it, and substract that double from the former Number, the Remainder will be the Quotient: So if you would divide 2555 by 7, that Number doubled is 5110, from which cut off the Cypher towards the Right-hand, and it is 511, one seventh part whereof is 73, the double whereof is 146, and that substracted from 511, the Remainder is 365, and that will be the Quotient of 2555 divided by 7.

Eighthly, If you would divide any Number by 8; Take half the Number given successively three times, the third half shall be the Quotient: So if 2920 were to be divided by 8; First, the half of 2920 is 1460, the half of 1460 is 730, and the half of 730 is 365; and that is the Quotient of 2920 divided by 8.

Ninthly, To divide any Number by 9; Take the third part of the given Number twice successively, the second Remainder shall be the Quotient: So if you would divide 3285 by 9; One third part of 3285 is 1095, and one third part of 1095 is 365, which is the Quotient required.

Questions performed by Division only.

Question 1. *If a Piece of Land lying in a long Square or Parallelogram, contain 42952 square Perches, and one of the sides thereof be 236 Perches long, how long must the other side be?* Divide 42952 by 236, the Quotient will be 182, and so many Perches long must the other side be.

Question 2. *In a Year there are 8760 Hours, and in every natural Day there are 24 Hours, I demand how many Days be there in a Year?* Divide 8760 by 24, the Quotient will be 365 and so many Days be there in a year.

Question 3. *The distance from* London *to* Coventry is 133760 *Yards, and in one Mile there is contained* 1760 *Yards, now I would know how many Miles it is from* London *to* Coventry; Divide 133760 by 1760 the Quotient will be 76; and so many Miles it is from *London* to *Coventry*.

These Questions performed by *Division* only, are the converse of those that were performed by *Multiplication*, which I the rather

make choice of, that the Reader might see how *Multiplication* and *Division* prove each other.

There are one or two more kinds of *Division*, something like these last, but I shall forbear exemplifying them; for Variety helps to make a Book rather great than useful.

¶ Here is to be noted, That in the following Rules, where there is continual Use of Division, I sometimes make use of one Kind, and sometimes another: But the Practitioner may use which he is best skill'd in, for they all produce the same Effect.

REDUCTION

IS twofold; First, That which turns *Great* Denominations into *Smaller*, as *Pounds* into *Shillings* or *Pence*; this is done by *Multiplication*: As followeth.

Example I. Let it be asked, How many Pence are contained in 729 *l.* 11 *s.* 7 *d.*?

l.	
729	
20	*multiply*
14580	
11	*add*
14591	*Shillings*
12	*multiply*
29182	
14591	
175092	
7	*add*
175099	*Pence*

First, A Shilling is contained in a Pound 20 times; therefore multiply 729 by 20, or (which is the same but somewhat shorter) by 2, and put 0 to the Product, as in the Margin, this shews, that in 729 *l.* there are 14580 *Shillings*. To which add 11 *s.* it makes 14591 Shillings.

Again; Because one Peny is contained in one Shilling 12 times, multiply 14591 by 12, it produceth 175092, to which add the 7 pence: So the Sum will be 175099; and so many Pence are contained in 729 *l.* 11 *s.* 7 *d.*

Example II. Let it be asked, How many Pints are contained in 4 Tuns, 3 Hogsheads, and 27 Gallons?

First,

First, One Tun is equal to 4 Hogsheads, therefore 4 Tun is equal to 16 Hogsheads; to which add the 3 Hogsheads, so there is 19 entire Hogsheads.

Again, Because 1 Hogshead contains 63 Gallons, multiply 19 by 63, it produceth 1197 Gallons, to which add 27, it gives 1224 Gallons.

Lastly, because every Gallon contains 8 pints, multiply 1224 by 8, it produceth 9792, and so many pints are contained in 4 Tuns, 3 Hogsheads, and 27 Gallons.

After the same sort might dry Measures be reduced, as Quarters to Bushels, Pecks, or Gallons; and likewise Weights and Outlandish Coins, of which the proportion of the greater to the lesser is (before) known or given.

```
  63
  19 multiply
----
 567
 63
----
1197
  27 add
----
1224 Gallons
   8 multiply
----
9792
```

Secondly, it is often requisite to turn *Smaller* denominations to *Greater*: This is done by *Division*, as followeth.

Example, I. Let it be asked how many pounds are contained in 80976 shillings?

Divide 80976 by 20, the quotient is 4048*l.* and 16*s.* remaining, which is the true Answer.

```
20) 80976 (4048 Pounds
    ....
    ----
     97
     80
    ----
     176
     160
    ----
      16   Shillings
```

Example, II. Let it be asked how many pounds are in 109754*d.*?

Because One pound contains One shilling 20 times, and One shilling contains One peny 12 times, therefore if 102754 be divided first by 12, the quotient shall be 9146 shillings and 2 Pence over; then if 9146 be divided by 20, the quotient is 457 pounds and six shillings remaining; so that 109754 pence is equal to 457 *l.* 6 *s.* 2 *d.*

```
12) 109754 (9146 Shillings
     ....
    ------
    108
     17
    ----
     12
      55
     ---
      48
       74
      ---
       72
        2 d.
```

```
20) 9146(457 Pounds.
     ...
    ----
    80
    114
    ---
    100
     146
     ---
     140
       6 s.
```

Or if 109754 had been at first divided by 12 times 20, that is 240, (which is the number of pence contained in a pound) the quotient had been 457 pounds and 74 pence remaining, which is all one with the former; ſo 74 pence is equal to 6 ſhillings and 2 pence. More inſtances ſhall not need herein, becauſe the thing of it ſelf is very clear.

PRO-

PROGRESSION

IS alſo of two Sorts; the *firſt* of certain Numbers in *Arithmetical Proportion* from 1; that is ſuch as differ equally, as 1, 2, 3, 4, 5, 6, where the common Difference is 1 (as is eaſily ſeen) or 1, 3, 5, 7, 9, 11, where the common difference is 2; or any other, as 1, 8, 15, 22, 29, 36, where the common difference is 7, this is called *Arithmetical Progreſſion.*

Secondly, of certain Numbers in *Geometrical* Proportion from 1; that is, ſuch as increaſe by a common Multiplication, as 1, 2, 4, 8, 16, 32, where the common Multiplier is 2, that is the firſt multiplied by 2 produceth the ſecond, and the ſecond by 2 produceth the third, and ſo on.

Or as 3, 9, 27, 81, 243, where the common Multiplier is 3, this is called *Geometrical Progreſſion.*

Both the common Difference (in the firſt) and the common Multiplication (in the latter) ſhall for ſhortneſs hereafter be called *the Common Exceſs.*

Firſt, now of the firſt ſort, or *Arithmetical Progreſſion*, the prin-

Arithmetical Progreſſion.

cipal uſe of this is,

1. *If the number of Places and Common Exceſs be given, to find the Laſt number.*

2. *When the Number of Places and the Laſt Number is given, to find the Aggregate, or Total Sum of all the Numbers.*

3. *When the laſt Number, and the Total Sum is given, to find the Number of Places.*

4. *The Number of Places, and Total Sum being given, to find the laſt Number.*

5. *The laſt Number, and number of Places given, to find the common Exceſs.*

6. *The Laſt Number and Common Exceſs being given, to find the number of Places.*

I will inſtance in no more, few of theſe ever happening to be uſed.

For the firſt of theſe, let there be given,

The Number of Places ———————————— 100
The common Exceſs ———————————— 1
To find the laſt number alſo ———————————— 100

RULE.

RULE.

Multiply the Number of Places leſs by 1, *by the Common Exceſs, and to the Product add the firſt Number: the Sum is equal to the laſt Number.*

So here, multiply 99 by 1, the Product is 99, (for 1 neither Multiplies or Divides) to this add the firſt Number 1, it gives 100 for the laſt Number.

Or let the Numbers be 1, 7, 13, 19, 25, 31, where the Common Exceſs is 6, and the Number of places alſo 6.

Now if the Number of places leſs by 1, that is 5, be multiplied by the common exceſs, which is 6, the Product is 30, to which adding the firſt Number which is 1, the laſt Number 41, is thereby compoſed. This is ſo eaſie that it needs no proof.

2. For the ſecond, which is, *The laſt Number, and the Number of places given, to find the Total ſum of all the Numbers.*

RULE.

Add the firſt and laſt numbers together, and multiply the Sum by half the number of places, the Product is equal to the aggregate or Sum of all the numbers added together.

So if the firſt Number 1 be added to the laſt Number 100, it gives 101, which multiplyed by 50 (which is half the number of places) produceth 5050, which is equal to all the hundred numbers added together.

And hereby may that vulgar queſtion be anſwered which is,

If a man take up 100 *ſtones placed a yard one from another, all in a Right Line by one at a time, and bring them back one by one to his firſt ſtanding, how many yards doth he go backward and forward.*

It is ſhewed before that he goes forward 5050 Yards, and he muſt needs come back juſt as much, that is, in all 10100 yards, which is 5 Miles and 3 quarters; wanting 20 Yards.

Or ſecondly, ſuppoſe the numbers were 1, 9, 17, 25, 33, 41. Whereof the common exceſs is 8, the firſt and laſt added is 42, which multiplied by 3, (half the number of Places) the Product is 126, which is the ſum of them all.

3. For the third thing, that is, *By the laſt number and the Total to find the number of Places.*

RULE.

RULE.

Add the first and last Numbers, and by the sum of them divide the Total, the Quotient will be equal to half the Number of places.	42) 126 (3
This is so plain, it needs no clearing.	126

4. For the fourth, *If the Total and Number of places be given, to find the last number.*

RULE.

Divide the Total by half the Number of places, the Quotient is a Number, from which if 1 be taken, the rest is the last Number.

As let the numbers be 1, 3, 5, 7, 9, 11, 13, 15, or any other (in *Arithmetical proportion*) whatsoever.

The sum of these is to be 64 and the number of places is 8, the half of it 4. Now if 64 be divided by 4 the quotient is 16, from which if 1 be taken there remains 15 for the last number.	4) 64 (16 4 24 24

5. Now for the fifth variety, If the last number, and the number of places be giveu, to find the common excess

RULE.

From the last Number take 1, and the remain shall be the Dividend; then from the number of places also take 1, and make this latter remain the Divisor; then the quotient of this Division shall be the common excess.

Example. Let the numbers be 1, 4, 7, 10, 13, 16 from 16 take 1, remains 15 for the dividend, then from 6 (which is the Number of places) take also 1, remains 5 for the Divisor.

Now when 15 is divided by 5, the quotient is 3. And 3 is also the common excess, or difference between 1 and 4, or 4 and 7, &c.	5) 15 (3
6. *Lastly*, Let the last number, and the common excess be given to find the number of places.	15

RULE.

RULE.

From the last number take 1, and divide the remainder by the common excess; then to the quotient add 1, the sum is the number of places.

As, let the numbers be 1, 5, 9, 13, 17, 21, 25, 29, from 29 take 1, remains 28, which divided by 4 (which is the excess) the quotient is 7, to which add 1, the sum is 8, which is the number of the places, as the Reader may easily count.

4) 28 (7

28

7 & 1 is 8

Geometrical Progression.

I shall not be so large in this as in the former, because these things are of little use to the Arithmetician, except where a number is to be many times doubled, tripled, or the like, which cannot be so easily abridged here as in the other, because there the last number arising of many *Additions* of the excess to 1, was easily found by one Multiplication: but here the last number being made by many *Multiplications* of the excess, is therefore many times harder then the other.

The varieties here shall be but two

1. The common excess, and number of places being given, to find the last number.
2. The Excess, and last Number being given, to find the Total Sum.

The first of these may thus be found. Let the Numbers be 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, the excess is 2, the places 10, find out the fifth Number (which is easily done for any one may reckon so far by heart) that is here 16, and multiply 16 by 16, it produceth 256, which is the ninth Number: Lastly, multiply 256 by the excess 2, thence ariseth 512, the number desired.

So if the places had been more, as 72, having found the 9th Number 256, multiply it by 256 thence comes 65536 for the 17th Number, which multiplied by the excess 2, gives 131072 for the 18th place, which multiplied by 131072, gives 17179869184, for the 35th place; and that multiplied again by the excess 2 gives 34359738368, for the 36th place, that Multiplied by 34259738368 the product will be 1180591620717411327424, for the 71st place, which lastly multiplied by the excess, gives 2361183241434822-654848,

654848, for the 72 place, which is the laſt Number of the Progreſſion required to be found.

Perhaps this may ſeem ſomewhat tedious, but where things cannot be performed without Labour, the Reader muſt content himſelf with ſuch Rules as make it leſs; for it is certain, that this way is much ſhorter than to have Multiplied ſtill by the Exceſs 71 times, which elſe he muſt have done.

All this notwithſtanding, he is not bound to uſe the ſame Numbers, much leſs in other Queſtions where the Number of places is not the ſame: But whereas I began from the place 9, he may begin at 8, 10 or 12, or where he pleaſes, ſo as he remembers ſtill where he is; for this is general, *If the Number belonging to any place whatſoever be Multiplied by it ſelf, the Product ſhall be the Number belonging to twice ſo many places want one place.*

Now for the ſecond thing, which is, to find the Sum of all the Numbers,

RULE.

From the laſt Number take the firſt, and Divide the Remain by the Exceſs want 1, then Multiply the Quotient by the Exceſs, and to the Product add the firſt Number, the Sum of them is equal to the Sum of all the Numbers.

So if from the laſt Numbers, of 72 place, be taken 1, the Remain is 2361183241434822654847, which ſhould be Divided by 1, (that is the Exceſs want 1, for the Exceſs is but 2) but becauſe 1 neither Multiplies nor Divides, that Labour is ſaved: Now Multiply this Remain by the Exceſs, the Product is 4722366482869-645309694, to which adding the firſt Number 1, by making the Figure 7, next the Right-hand to be 6, you have the total Sum of all the 72 Numbers.

A Queſtion reſolved by Geometrical Progreſſion.

A Londoner *ſojourning in a Country Market Town in Winter, made himſelf a new Freez Suit and Coat, on which were ſet 6 dozen of Buttons of Silk and Silver; a Baker being in his Company liked it ſo well, he would buy it of him; the Citizen conſented to let him have it, paying for the firſt Button a ſingle Barly-corn, for the ſecond* 2, *for the third* 4, *and ſo on doubling to the laſt.*

The Bargain was liked on both parts for the preſent, but ſhortly after revoked, for it could not be performed, and no Man can be holden to an impoſſibility.

But why this could not be performed, may be judged: Firſt, by inquiring the worth of ſo much Barly in Mony: And ſecondly, the weight of it; and how it ſhould be removed.

1. For the firſt, allowing 10000 Corns to a Pint (which is more than enough) then 512000 Corns make a Quarter; and yet (for ſhortning the Diviſion) we will allow 10000000 Corns to a Quarter; by which Dividing the whole Number of Corns (which is done by cuting off the firſt 7 Figures towards the right hand) the Quotient will be 4722366482869 64, and ſo many whole Quarters there are, omitting the Remain, as in this caſe inconſiderable.

Now allowing Barly were to be ſold at 15*d.* the Buſhel (which is cheap) it is ſo many Angels; and therefore dividing by 2, it is 236118324143482 pounds Sterling; which is in words, Two hundred thirty ſix millions of millions, one hundred and eighteen thouſand three hundred twenty four millions, one hundred forty three thouſand four hundred eighty two Pounds, which I take to be too much for any Tradeſ-man to get or keep.

And reckoning Land for ever at twenty Years Purchaſe, if this Sum of Pounds be Divided by 20, the Quotient is the yearly Rent of 11805916207174 Pounds.

And this Divided again by 564 (the Number of Days in a Year) the Quotient is 32344975918, that is above Thirty two thouſands of Millions a Day for ever. So great a Vanity may be concluded on for want of a little premeditation.

2. Now ſecondly, for the weight of it, if we put 8 Buſhels to weigh 2 hundred weight, (for ſure it doth weigh more) then the whole Number of Quarters Multiplied by 2 gives the Weight of all to be 944473296573928 hundred weight, and if this be Divided by 20, (which is but cutting off one Figure towards the Right-hand, and Dividing the reſt by 2) or which is all one, cut off one figure from the Number of Quarters, the Quotient 47223664828696 is ſo many Tuns. And therefore it will require 427223664828 Ships of a 1000 Tuns a piece to carry it: And conſequently, if every Nation in the World had above 10000 ſuch Ships, yet there muſt be above four Millions of ſuch Nations; which I ſuppoſe are not to be found in this World.

And here I will leave this, having uſed this long Example, (which though it require more labour as all great Examples do, yet the ſame skill will do it, as if the places had been Fewer) that the Reader being throughly Exerciſed thereby, may the eaſier leap over others which are ſhorter.

THE

THE GOLDEN RULE, OR, Rule of Three Direct.

THis is the most *useful* and most *easie Rule* in *Arithmetick*, and deserves a *Golden Name*. It is, when there are three Numbers given or known, to find a fourth in proportion with them.

But 4 Numbers are in proportion, and called *Proportional*, when, as the first is to the third; so is the second to the fourth.

As if there were given 3, 4, and 6, to find a fourth, which may be to 4, as 6 to 3, that is double, and that fourth Number is 8; and this is called *Proportion Direct*; and the Rule whereby it is done, *The Direct Rule*.

There is also another Proportion called *Reciprocal*; which is, when, as the first is to the third, so is the fourth to the second: As 3, 4, 6, and 2, this is called *The Reverse Rule*.

In *Direct Proportion*, the Product of the two middle Numbers Multiplied together, is ever equal to the Product of the first and last Multiplied together, which serves not only for a *Proof*, but a ground of the *Rule*, which *Rule* shall here follow: The *Reverse Rule* being deferred till we have done with this: And for Working thereof, this is the

RULE.

Multiply the Second Term *(or Number) by the* Third, *and Divide the* Product *by the* First; *the* Quotient *shall be the* Fourth Number *desired*.

Example. Let the three Numbers given be 2, 6, 3, Multiply 6 by 3, the Product is 18; then Divide 18 by 2, the Quotient is 9, which is the fourth Number in Proportion with 2, 6, and 3.

For, as 2 to 3; so 3 times 2, which is 6, is to 3 times 3 which is 9.

And so the Product 18 Divided by 2, and the Quotient 9, causeth that the Product of 2 into 9 shall be also 18, and consequently

if 2 be the first of the 4 proportional Numbers, and 6 and 3 the two middlemost, then 9 is the last.

Otherwise.

Divide the second by the first, and Multiply the third by the Quotient, the Product shall be the fourth.

So if one Divide 6 by 2, the Quotient is 3, by which Multiply 3, the Product is 9, for the fourth Number, as before. Otherways this Rule might be expressed, but where the first Way is so short and clear, there many other ways would rather trouble than help the Person that should use them.

In the first Way (which here we mean to use, and no other) if the *first* Number be 1; then the Product of the *second* and *third* gives the *fourth*, without any *Division*: Or, if the *second* or *third* Number be 1, then there needs no *Multiplication*, but Dividing the greater of them by the first, the Quotient (in whole Numbers, for yet we speak of them) is the fourth Number which was sought. But,

Note I.

To know when to use the Direct, or the Reverse Rule. Consider, if *More*, require *More*; or if *Less*, require still *Less*, then use the *Direct Rule.* But if *More* require *Less*, or *Less*, *More*, then use the *Reverse Rule*; this will be easily understood when we come to Example.

Note II.

To know how to place the three Numbers when they are confusedly given. Remember that 2 of them are always of one Denomination, as both *Pounds*, or both *Sheep*, or both *Yards*, or *Acres*; and the other Number hath another Denomination: Now know, that this single Number is ever the second Number in order.

And one of the other two, namely, that which hath some relation to this *second* is the *first*; and the other is the *third* Number, whose relation is sought for in the *fourth*; whence it's plain, that the *second* and *fourth* are also of the same Denomination.

And having Premised these things, let us now exemplifie the Rule in some Questions.

Question I.

If three Yards of Cloth cost 4 *l.* what shall 21 Yards cost?

Set the Numbers in order, as in the Example. If 3 Yards cost 4 *l.* what 21 Yards? Here you see that the first Number and the

the third Number, are both of one Denomination, *viz.* both *Yards*, and the ſecond Number is of another Denomination, namely *Pounds*, wherefore the *fourth* Number which is ſought for, muſt be alſo *Pounds*; therefore multiplying (according to the Rule before given) the *ſecond* Number by the *third*, and Dividing the Product by the *firſt*, the Quotient ſhall anſwer the Queſtion.

Firſt, 21 Multiplied by 4, (which is the third Number Multiplied by the ſecond) produceth 84, which Divided by 3 the firſt Number, the Quotient is 28 *l.* and ſo much ſhall 21 Yards coſt: For 28 is to 4, as 21 to 3, ſeeing each contains other 7 times.

And the Work will ſtand thus.

Yards *l.* *Yards*
If 3 coſt 4 what 21?

4

3) 84 (28 *Pounds.*

6
24

Queſtion II.

If 4 Men eat 2 Pecks of Corn in a Week, how many Pecks ſhall ſerve 100 Men?

Place your Numbers as here you ſee, then Multiply 100 by 2, (that is the third Number by the ſecond) and the Product is 200, which Divided by 4, the Quotient is 50, for the Number of Pecks required.

Men *Pecks* *Men*
If 4 eat 2 what 100?

2

4) 200 (50

200

Queſtion III.

If 20 Sheep coſt 13 Pound 13 Shillings 4 Pence, what is that for every Sheep?

Turn

Turn the Shillings and Pounds into Pence; thus.

Multiply 13 *s.* by 12, the Product is ———————— 156
And 13 *l.* by 240 (because 240 pence is a Pound ———— 3120
To which add the 4 *d.* ———————————————— 4

It makes in all 3280

Then the Question will be, If 20 Sheep cost 3280 Pence, what shall one Sheep cost?

```
Sheep       d.       Sheep
If 20 cost 3280, what 1?
             1
        ---------
  20)  3280  (164 Pence.
        ...
        ---------
        20
        128        12)  164  (13 Shillings.
        ---------            ..
        120             ---------
         88              12
        -------          44
         80             -------
          8              36
                          8 d.
```

By the Rule before delivered, I should Multiply, the second Number by the third, but in this example, the third Number being 1, it doth not Multiply; I therefore Divide 3280 the second Number, by 20 the first Number, and the Quotient 164 is the Price of one Sheep in Pence, which Divided by 12, the Quotient is 13 *s.* and 8 *d.* remaining, the price of every Sheep therefore is 13 *s.* 8 *d.*

Question IV.

How many 10 inch Tiles will pave a Floor that contains 16 square Yards?

First, remember there are 36 Inches in one Yard in length; which Multiplied into 36, gives 1296, for the square Inches in one square Yard; Multiply 1296 therefore, by 16, thence comes 20736, the Sum of all the 16 Yards in Inches.

Secondly, seeing every Tile is 10 Inches in length, and 10 in breadth, Multiply 10 by 10, it produceth 100 for the square Inches in one Tile; then, by the Golden Rule say,

If

Inches		*Tile*		*Tiles*
If 100	require	1,	; what	20736?
				207.36

Here, Becauſe 1 doth neither *Multiply* nor *Divide* (as hath been ſeveral times intimated) therefore, Divide the *Third Number*, 20736, by the *Firſt* 100, (which is done by cutting off two Figures to the Right-hand) and the *Quotient* is 207 and 36 remaining,

So it appears, that 207 is too little, and 208 too much to do the Work: The juſt Number being 207 $\frac{12}{100}$, but we ſhall not trouble the Reader with this till he know ſomething of Fractions.

Queſtion V.

If 100 *l.* gives 6 *l.* Intereſt for a Year, how much ſhall 750 *l.* give?

Multiply 750 by 6, the Product is 4500, which Divided by 100, the Quotient is 45 *l.* for the thing required.

	l.		*l.*		*l.*
If	100	give	6,	what	750?
					6
					45.00 *Pounds.*

Queſtion VI.

If 750 *l.* gives 45 *l.* Intereſt for a Year, what ſhall 100 *l.* give?

Multiply 45 by 100, the Product is 4500, which Divided by 750, the Quotient is 6 *l.* for the Intereſt of a 100 *l.* for a Year.

	l.		*l.*		*l.*
If	750	give	45,	what	100?
					45
				750)	4500 (6 *Pounds.*
					45.00

Many other Queſtions might be added, but the Rule is ſo plain, that it needs them not; and ſo general, that he which can reſolve one, may as well reſolve any other: And for that reaſon, and becauſe in all the Rules which follow, this Rule will be conſtantly made uſe of, I will ſay no more of it here.

The

The Golden Rule *Reverſe.*

IF 12 Workmen do any piece of Work in 8 Months, how many Workmen ſhall do the ſame in 2 Months?

RULE.

Multiply the firſt *Term by the* ſecond; *and Divide the Product by the* third, *the Quotient is the Number deſired.*

Here 12 is not the firſt Number, though it be firſt named; but the three Numbers placed in order, ſtand thus, 8, 12, 2, for the middle Term muſt always be of the ſame Denomination with that which is required.

Now Multiply 12, by 8, the Product is 96, which Divided by 2, the Quotient is 48, which anſwers the Queſtion, As in this Example.

Months	*Men*	*Months*
8	12	2
	8	
	2) 96 (48	
	8	
	16	
	16	

For, If 8 Months require 12 Men; then (a fourth Part of 8) 2 Months, ſhall require four times 12, that is 48 Men.

For here *Leſs* requires *More*; that is, *Leſs time*, *More hands*; and therefore it is wrought by the *Reverſe Rule*.

Queſtion II.

How many Ells of Tapeſtry will ſerve to hang a Room 3 Yards high, 6 Yards long, and 5 Yards broad? Not regarding Doors, Windows or Chimney, but as if there were no ſuch.

Firſt, Multiply 6 by 3, the Product is 18, which doubled (becauſe there are 2 ſides called *Lengths*) is 36 Yards for all the Length.

Secondly,

Secondly, (for the ſame reaſon) Multiply 3 by twice 5, that is by 10, the Product is 30 Yards, for all the breadth; which added to 36, gives 66 Yards, equal to all the length and breadth in Yards.

But now becauſe Ells, that is, *Flemiſh* Ells (for ſuch Meaſure are Hangings ſold by) is equal to 3 Quarters of a Yard, that is, their Ells is to our Yard as 3 to 4. Say therefore, if 4 give 66, what 3? Multiply 66 by 4, it produceth 264; then Divide 264 by 3, the Quotient is 88. Again, Multiply 88 by 4, and Divide the Product (which is 352) by 3, the Quotient is 117, and 1 remaining, to which the Diviſor 3 being applyed; the Number juſtly anſwering the Queſtion is 117 Ells, and one third part of an Ell.

Note I. Becauſe here we had to deal with things which had equal length and breadth, that is ſquare Yards, and ſquare Ells, therefore one Multiplication and Diviſion was not ſufficient to proportion this: But if inſtead of working by 4 and 3, we had done it by their Squares which is 16 and 9, it might have been performed at once; thus Multiply 66 by 16, the Product is 1056, which Divide by 9, the Quotient is $117\frac{1}{3}$, as before, but I began not with this way, for I ſuppoſed my Reader ignorant of Squares.

Note II. It might alſo have been done, by reducing all the Terms into Quarters of a Yard at the firſt, and after the Number is found, reducing them again to Ells; but becauſe it is more proper to work thus, till Fractions have been taught: I leave that, and proceed to another Queſtion.

Queſtion III. If 1 Cloſe would graze 21 Horſes for 6 Weeks; (ſuppoſing no waſte to be made) how many Horſes would it feed for 7 Weeks?

Multiply 21 by 6, it produceth 126, which Divided by 7 the Quotient is 18. At that rate therefore it would keep 18 Horſes for 7 Weeks.

Queſtion IV. If 1 Cloſe will feed 18 Horſes for 7 Weeks, how long ſhall it feed 63 Horſes?

Multiply (according to the Rule) 18 by 7, the Product is 126, which Divide by 63, the Quotient is 2, therefore 2 Weeks it ſhall keep them.

The like way ſerves for Hay, Oats, or any other Proviſion for Man or Beaſt; which may be of uſe in *Garriſons*, and ſuch like caſes where ſcarcity may be feared, to proportion either the *Mouths* to the *Meat*, or *Meat* to the *Mouths*.

Before I leave this Rule, (becauſe it comes not ſo much in uſe and Practice as the *direct Rule* doth, and therefore may be more apt to be forgotten) I will, to exercise the Reader herein, propoſe the following *Queſtions*, giving the Anſwers of them, and leave the Practice to the Reader to find out of himſelf, the better to fix it, the Rule in his Memory.

Queſtion I. If 12 Men would raiſe a Frame in 10 days; in how many days would 8 Men raiſe the ſame?

Here, becauſe the fewer Men would require the longer time, though the Number be 12, 10, 8, yet you ſhall (by obſerving what hath been already delivered in this Rule) find the fourth proportional (which is the Number anſwering the Queſtion) to be 15, and ſo many Men will do the Work in 8 Days.

Queſtion II. If 60 Yard of Hangings of three Quarters broad would hang a Room; How many Yards of half a Yard in breadth would ſerve to Hang the ſame Room?

Anſwer Ninety Yards.

Queſtion III. If a Board being 12 Inches in breadth do require 12 Inches in length to make a foot Square; What number of inches in length will make a foot Square, when the breadth of the Board is 16 Inches?

Anſwer 9 Inches.

Queſtion IV. If the Baſe or end of any Solid (as a piece of Timber or Stone) being 144 Inches, do require 12 Inches in length of that piece to make a ſolid Foot: What Number of Inches in length will make a ſolid Foot, when the Square at the end is 216 Inches.

Anſwer, 8 Inches.

I will ſay no more of this Rule; Neither will I treat of the *Double Rule of Three*, as a Rule by it ſelf; but come to the *Rule of five Numbers*, which is an Abridgment of the other.

The

The Golden Rule *Compounded of five Numbers.*

Question I. IF a hundred pound weight (that is 112 pound weight) carried 126 Miles cost 14 *s.* how much shall three quarters of a hundred (that is 84 pound) cost; being carried 40 miles?

RULE.

Multiply the three last Numbers one into another, (*that is*) *the* third *by the* fourth, *and that Product by the* fifth; *the last Product shall be the* Dividend.

Again, *Multiply the* two first *Numbers together; the Product shall be the* Divisor. *This Division being made, the* Quotient *will be the Number of Shillings desired.*

Example of the former Question.

First, Place your Numbers according to the Tenor of the Question thus:

l.	*Miles*	*s.*	*l.*	*Miles*
112	120	14	84	40

112
120
—
2240
112
—
13440

14
12
—
28
14
—
168
84
—
672
1344
—
14112
40
—
13440) 564480 (42 *Pence.*
—
53760
26880
—
26880

Your Numbers being placed in order, reduce the 14 *s.* into Pence, and it is 168 *d.* then multiply 168 by 84, the Product is 14112, which Multiplied by 40, it produceth 564480 for the *Dividend.*

Then Multiply 112 by 120, it produceth 13440 for the *Diviſor.*

Divide 564480 by 13440, the Quotient will be 42 Pence which is 3 *s.* 6 *d.* anſwers the Queſtion.

In this Rule, the *firſt* Number and *fourth*, alſo the *ſecond* and and *fifth*; and the *third* and *ſixth*, are of like Denomination and Nature.

Queſtion II. If 10 *l.* for 6 Months Yield 3 *l.* Intereſt, what ſhall 625 *l*, yield for 36 Months?

Place them thus; 100, 6, 3, 625, 36.

Multiply the three laſt, as before is ſhewed, the latter Product is 67500 for the *Dividend*; and the 2 firſt Multiplied make 600 the *Diviſor*, then Divide 67500 by 600 the Diviſor (or 675 by 6, which is all one) the Quotient will be 112 whole pounds; and 300 (or 3) remaining, which becauſe it is half the Diviſor, ſignifies the half of a pound; that is 10 Shillings. So the anſwer to the Queſtion is 112 *l.* 10 *s.*

```
   l.    m.   l.            l.   m.
   100   6    3             625  36
     6                        3
  ─────                    ──────
   600                      1875
  ─────                       36
600) 67500 (112½           ──────
     ...                   11250
  ─────                    5625
   600                     ──────
    750                    67500
  ─────
   600
   1500
  ─────
   1200
    300
```

Which might have been given in one Denomination, namely 2250 Shillings, if before the work the Pounds had been turned into Shillings, Multiplying them by 20, as hath been ſhewed before.

But ſince moſt Queſtions, except ſuch as are ſtudied for the purpoſe, are apt to end in ſome Fraction, I ſhall next treat of Fractions.

Only

Only firſt, having ſpoken of the double Rule of Three, this may let you know, that all Queſtions which are wrought at once by the compound Rule of Five, may be done at twice by the ſingle Rule of Three; and the doing of them ſo by two Operations, is called, *The Double Rule.*

As in our laſt Queſtion, there are two things conſiderable, the difference of Money; and the difference of Time.

Firſt, for the Money.

Say, if 100 *l.* give 3 *l.* what 625 *l?* Anſwer 18 $\frac{75}{100}$ *l.*

Secondly, for the time.

Say, if 6 *mo.* give 18 $\frac{75}{100}$ *l.* what 36 *mo.* Anſwer 112 $\frac{50}{100}$.

But this will be better underſtood anon; and then the Reader may uſe that which he likes beſt.

OF

FRACTIONS.

THe word *Fraction* ſignifies a *breaking* or *breach* of any intire thing into parts; and when a Number is broken ſo, the parts (which muſt needs be every one leſs than the whole; and the whole is accounted but *One* or *Unity*) being leſs than Unity, are called Fractions (that is, fragments or pieces) of Unity. Now the Unite, or intire Number which is to be broken, may be any thing, as one *Pound*, in reſpect of which, *Shillings* and *Pence* and *Farthings* are *Fractions*; or, one *Shilling*, in reſpect of which, *Pence* and *Farthings* are *Fractions*; or, one *Peny*, in reſpect of which *Farthings* are *Fractions*; and the like of *Weights* and *Meaſures*, or any other thing to be broken into Parts.

In *Fractions*, we ſhall treat firſt of *Numeration*, then of *Multiplication* and *Diviſion*, then of *Reduction*; and laſtly, of *Addition* and *Subſtraction*.

The reaſon of this Order will ſoon be ſeen; for *Multiplication* and *Diviſion* are here much eaſier than *Addition*, &c. and therefore ought to be learned before them.

NUME-

NUMERATION.

NUmeration is nothing elſe but the way of writing *Fractions*; and that this may be done, we muſt conſider that any *Unity*, or *Number* repreſenting an *Unite*, may be broken into two parts equal; and then each of the Parts is called *one ſecond*, or *half*; or it may be parted into three equal parts, and then each part is called *one third*, and two of them are called *two thirds*, and the like may be underſtood if it were parted into 4, 5, 6, 7, 8, 9, 20, 50, or 100, or how many ſoever.

Now to write theſe; do thus:

Write	One half	Thus	$\frac{1}{2}$
	One third		$\frac{1}{3}$
	One fourth		$\frac{1}{4}$
	One fifth		$\frac{1}{5}$
	One ſixth		$\frac{1}{6}$
	One ſeventh		$\frac{1}{7}$
	One eighth		$\frac{1}{8}$
	One ninth		$\frac{1}{9}$
	One tenth		$\frac{1}{10}$

In every one of theſe 10 *Fractions*, the Number below the line is called the *Denominator*, and ſhews into how many parts the *Unite* is broken.

The Number above the line ſhews how many of thoſe parts are taken, or contained in the *Fraction*. and is therefore called the *Numerator*: So in the *Fraction* $\frac{3}{5}$, the *Denominator* 5 ſhews the *Unite* to be broken into 5 parts; and the *Numerator* 3 ſignifies 3 of ſuch parts to be contained in the *Fraction*; which Fraction therefore is called *Three Fifths*.

And here it is plain, that, As the *Numerator* is in proportion to the *Denominator*: ſo is the *Fraction* to 1, or Unity, for $\frac{3}{3}$ or $\frac{5}{5}$; or any the like, is equal to 1.

And therefore all *Fractions* are Quotients of leſſer Numbers divided by greater, as $\frac{4}{7}$ ſignifies 4 to be divided by 7, and as the Dividend 4, is to the Diviſor 7, ſo is the Quotient $\frac{4}{7}$ to Unity.

And therefore this line of ſeparation which is drawn between the *Dividend* and *Diviſor*, doth properly ſignifie *Diviſion*.

Hitherto

Hitherto we have ſpoken only of ſuch *Fractions* as are leſs then 1, and thoſe are called *Proper Fractions*; but there are alſo 2 $\frac{1}{2}$, 3 $\frac{3}{4}$, 5 $\frac{1}{7}$, 6 $\frac{3}{5}$, and the like mixed Numbers; which ſo ſignifie *two and an half*, 3 *and* 3 *quarters*, *five and a ſeventh*, 6 *and* 3 *fifths*. Theſe by the multiplying the whole Numbers, by the Denominator, and to the product adding the *Numerators* reſpectively, are turned to $\frac{5}{2}$, $\frac{15}{4}$, $\frac{36}{7}$, $\frac{33}{5}$, which are called *Improper Fractions*, becauſe every one of them contains more than Unity.

Theſe, neverthleſs may be *Multiplied*, *Divided*, *Added*, or *Subſtracted* in the ſame way as are proper *Fractions*. And this ſhall ſerve for *Numeration of Fractions*.

MULTIPLICATION.

RULE.

M*Ultiply all the* Numerators *together, the laſt* Product *ſhall be the* Numerator *of the* Product *required: Likewiſe Multiply all the* Denominators *together, the laſt* Product *ſhall be the* Denominator *of the* Product *ſought.*

Example I. If $\frac{3}{5}$ be to be Multiplied by $\frac{4}{9}$ Multiply the *Numerator* 3 by the *Numerator* 4, the Product is 12, for the *Numerator* of the new Product. Alſo Multiplying the *Denominator* 5, by the *Denominator* 9, they produce 45, for the *Denominator* of the deſired Product, ſo that Product which was required, is $\frac{12}{45}$.

Example II. If $\frac{1}{2}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{9}$, and $\frac{3}{11}$ were to be Multiplied all together, begin with the *Numerators*, ſaying, once 3 is 3, and 3 times 4 is 12, and 12 times 5 is 60, and 60 times 3 is 180, for the *Numerator*: Then Multiply the *Denominators*; ſaying, 2 times 4 is 8, and 8 times 5 is 40; and 40 times 9 is 360, and 360 times 11 is 3960, for the new *Denominator*. So that the product of all theſe is $\frac{180}{3960}$, that is 1644 to $\frac{1}{22}$, as ſhall be ſeen hereafter in *Reduction*.

And thus it appears, that proper *Fractions* being leſs than *One*, are ſtill made leſs by *Multiplying*: As here the product $\frac{1}{22}$ is much leſs then $\frac{3}{11}$, which is the leaſt Multiplier; and the reaſon hereof is plain, for ſeeing *Multiplication* is but the taking of a Number, a certain Number of times, if that Number of times be more than 1, then the Number to be taken is increaſed by being taken more than once; but if the Number of times be 1, it is not increaſed

nor

nor diminished, but is still the same; Lastly, if that Number of times be less than 1, as $\frac{1}{2}$, the Number not being taken once, but half of once, produceth a Number less by half; that is, the half of the Number to be taken; and the like reason is of all others.

Example III. Multiply the mixt Numbers, $3\frac{1}{2}$, $4\frac{1}{3}$, and $5\frac{3}{4}$: First (as hath been shewn already) turn them to improper *Fractions*; thus, first say, 2 times 3 is 6, and 1 is 7. So the first is $\frac{7}{2}$. Secondly, 3 times 4 is 12, and 1 is 13: So the Second is $\frac{13}{3}$. Lastly, 4 times 5 is 20, and 3 is 23: So the last is $\frac{23}{4}$. Now the Fractions to be Multiplied are $\frac{7}{2}$, $\frac{13}{3}$, and $\frac{23}{4}$; First, for a new *Numerator*, say, 7 times 13 is 91, and 91 times 23 is 2093, for a new *Numerator*.

Then say, 2 times 3 is 6, and 6 times 4 is 24. So the new *Denominator* is 24.

And the product of all these *Fractions* is $\frac{2093}{24}$, that is, if real Division be made, $87\frac{5}{24}$.

DIVISION.

Division, to Divide one Fraction by another, is but the cross Multiplication of them; that is, the *Numerator* of the one, by the *Denominator* of the other, and hereby the proportion of one Fraction to another is seen.

$$\begin{array}{ccc} & 24 & \\ \frac{3}{4} & \times & \frac{6}{8} \\ & 24 & \end{array}$$

Example I. Divide $\frac{3}{4}$ by $\frac{6}{8}$, to do it, set them thus: And Multiply as the Cross leads; Saying, 3 times 8 is 24, which set over the Cross for a new *Numerator*, and 6 times 4 is also 24: Which set under the Cross for a new *Denominator*; so the Quotient is $\frac{24}{24}$, that is 1, which shews the Fractions to be equal one to another.

$$\begin{array}{ccc} & 27 & \\ \frac{3}{5} & \times & \frac{4}{9} \\ & 20 & \end{array}$$

Example II. Divide $\frac{3}{5}$ by $\frac{4}{9}$. First, set them thus: And say, 3 times 9 is 27, for a *Numerator*, and 5 times 4 is 20, for the *Denominator*: So the Quotient is $\frac{27}{20}$, and so many times is $\frac{4}{9}$, contained in $\frac{3}{5}$ that is, as 27 is to 20, so is $\frac{3}{5}$ to $\frac{4}{9}$, and so is $\frac{27}{20}$ to 1.

In *Division* it is to be remembred, that the *Numerator* of the *Quotient* ever ariseth of the *Numerator* of the *Dividend*: And the *Denominator* of the *Quotient* comes of the *Denominator* of the *Dividend*,

vidend, each being cross Multiplied as before. And also remember always to set the Dividend on the left hand of the Cross.

If a Fraction be to be Divided by a whole Number; Multiply the *Denominator* by that Number, the Product gives the new *Denominator*, and the *Numerator* remains the same. So if $\frac{1}{4}$ be divided by 9, say 9 times 4 is 36. So the Quotient is $\frac{1}{36}$.

Or if $\frac{1}{4}$ were to be Multiplied by 9, the Product (by Multiplying the Numerator by 9,) will be $\frac{9}{4}$, that is, $2\frac{1}{4}$.

Example III. Divide $\frac{320}{8}$ by $\frac{45}{9}$, thus: Say 320 times 9 is 2880, for a *Numerator*: And 8 times 45 is 360 for a Denominator. So the Quotient is $\frac{2880}{360}$ or $\frac{8}{1}$.

$$\begin{array}{ccc} & 2880 & \\ \frac{320}{8} & \times & \frac{45}{9} \\ & 360 & \end{array}$$

For $\frac{320}{8}$ is equal to 40, and $\frac{45}{9}$ equal to 5, but 40 contains 5 eight times.

And so in the second Example, it may be proved, that as 27 to 20, so is $\frac{3}{5}$ to $\frac{4}{9}$. For first, Multiply the two middle most, than 20 times $\frac{3}{5}$ is $\frac{60}{5}$, that is 12.

Secondly, multiply the first and last, and then 27 times $\frac{4}{9}$ is $\frac{108}{9}$: that is also 12.

Wherefore by that which hath been said in the *Golden Rule*, the four Numbers 27, 20 $\frac{3}{5}$, $\frac{4}{9}$, are proportional.

REDUCTION.

Reduction of Fractions is threefold.

1. *To reduce one Fraction* (which is not already in the least) *to its least Terms.*

2. *To reduce many Fractions of divers Denominations, to one Denomination.*

3. *To reduce any Fraction from one Denomination* (as near as may be) *to any other Denomination desired.*

I. For the first of these, *To reduce a* Fraction *to its least Terms.* Divide both the *Numerator* and the *Denominator* by the greatest *Common Divisor* that you can think of; the two *Quotients* being placed respectively in a Fraction, that Fraction shall be equal to the former Fraction, and in lesser Terms.

So (in the 3 Examples of *Division*) to reduce $\frac{2880}{360}$, to $\frac{8}{1}$, Divide 1880 by 360, the *Quotient* is 8: Then Divide 360 by 360, the *Quotient* is 1, and the new Fraction $\frac{8}{1}$ is equal to the former Fraction $\frac{2880}{360}$, and in leſs Terms, as you may ſee. But to find the greateſt common Diviſor, this is

The RULE.

Divide the greater Term *by the* leſſer (I mean by Terms, the *Numeraror* and *Denominator) and by the remainder* (if any be) *divide the* Diviſor, *and if any thing ſtill remains, by that divide the laſt* Diviſor, *continuing this courſe till nothing remain greater than* Unity *that* Diviſor *which is leaſt of all, is the greateſt* Common Meaſure *of both Terms, by which both being divided, and the* Quotient *placed like a* Fraction, *that* Fraction *ſhall be equal to the former* Fraction, *and in the* leaſt Terms.

Example. Reduce $\frac{148}{16}$ to the leaſt Terms; firſt divide 148 by 16, the Quotient is 9, and 4 remains: Again, divide 16 by 4, the Quotient is 4, and nothing remains; wherefore taking 4, (the laſt Diviſor) for the greateſt common Diviſor, by it divide 148, the Quotient is 37, and by it Divide 16, the Quotient is 4. Theſe two laſt Quotients placed orderly in a Fraction, make $\frac{37}{4}$, which is equal to $\frac{148}{16}$, and in the leaſt terms, for no Number greater then 1, will divide evenly both 37 and 4.

Other ways there are of leſſening Fractions, as Dividing the Terms (if they be even Numbers) by 2, and the Quotients (if even) again by 2, or elſe by 3, or any other Number that will divide them both evenly, that is, leave nothing remaining, but the former Rule being general and eaſie ſhall ſerve for all.

II. Now ſecondly, *To reduce many* Denominations *to one* common Denominator. Let the Fractions be $\frac{1}{2}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{7}{8}$, $\frac{9}{10}$, to be reduced all to one Denomination.

RULE.

Multiply all the Denominators together, and the laſt Product ſhall be the common Denominator to all the Fractions ——— *Then Multiply every particular Numerator into all the* Denominators *except his own, and the laſt* Product *ſhall be* Numerator *to that Fraction.*

Thus to reduce the forementioned Fractions $\frac{1}{2}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{7}{8}$, $\frac{9}{10}$, into one Denomination: Say, 2 times 4 is 8, and 8 times 5 is 40, and 40 times 8 is 320, and 320 times 10 is 3200, this laſt Product 3200 ſhall be the common *Denominator*. Then to get Numerators

merators for every one of them: As first, for the first, say 1 times 4 is 4, and 4 times 5 is 20, and 20 times 8 is 160, and 160 times 10 is 1600. For the first *Numerator*; so the first Fraction reduced is $\frac{1600}{3200}$. Then for the second *Numerator*: Say, 3 times 2 is 6, and 6 times 5 is 30, and 30 times 8 is 240, and 240 times 10 is 2400. So the second Fraction reduced, is $\frac{2400}{3200}$. After the same manner may the other three be reduced to $\frac{2560}{3200}$ for the third: $\frac{2800}{3200}$ for the fourth: and $\frac{2880}{3200}$ for the last: These are severally equal to the other, the first to the first, &c. as may be proved thus.

Let the Unity be a pound *Sterling*, then

	s.	
The $\frac{1}{2}$ of it is	10	
and $\frac{3}{4}$ is	15	
and $\frac{4}{5}$ is	16	d.
and $\frac{7}{8}$ is	17	6.
and $\frac{9}{10}$ is	18	
In all	76s. 6d.	

That is 3 whole Unites, and 16 s. 6 d. over; Turn 16 s. 6 d. all to six pences, it is 33, and because 6 d. is the fortieth part of a Pound, therefore all the Fractions are equal to 3 $\frac{33}{40}$.

Now add the new Fractions (which being all of one Denomination) may be added like whole Numbers: Thus,

	1600
	2400
	2560
	2800
	2880
In all	12240

Which divided by the Denominator 3200, the Quotient is 3 $\frac{2640}{3200}$. Now $\frac{2640}{3200}$, reduced to the least Terms, as hath been shewed how it may, will be $\frac{33}{40}$, so the Sum of these also is 3 $\frac{33}{40}$, which is equal to the Sum of the Fractions given to be reduced, and therefore they are equal in Sum, and might be thus proved equal severally, that is, the first of them propounded to the first reduced. Divide the Numerator 1600 by the Numerator 1, the Quotient is 1600. Also divide the Denominator 3200, by the Denominator 2, the Quotient is also 1600: and so may any of the rest be proved equal by the Equality of Quotients. But I leave it as plain enough already.

III. *Thirdly*, Any Fraction being given, to change the *Denomination* to any other more requisite, retaining still (as near as may be) the same Value.

RULE.

Multiply the Numerator *given, by the* Denominator *required, and divide the* Product *by the* Denominator *given; the* Quotient *shall be the* Numerator *required.*

Example. Let the Fraction given be $\frac{7}{13}$ of a pound *Sterling*, what is that in the twentieth Parts or Shillings? Multiply 7 by 20, the Product is 140, which divided by 13, the Quotient 10 $\frac{10}{13}$, that is, 10 *s.* and $\frac{10}{13}$ of a Shilling; which may be brought to pence thus, Multiply 10 by 12, Product is 120, which divided by 13 Quotient is 9 $\frac{3}{13}$ *d.* And again, Multiply 3 by 4, the Product is 12, which Divided by 13, Quotient is $\frac{12}{13}$ of a Farthing, so seven thirteenths of a Pound is 10 *s.* 9 *d.* and almost a Farthing.

But he which is resolved to have it in the smallest Coin, do it at first Work; for seeing a Farthing is the 960 part of a Pound, Multiply 7 by 960, they produce 6720, which Divided by 13. the Quotient is 516 farthings, and $\frac{12}{13}$ of a farthing: These farthings: may be turned to Shillings, Dividing by 48, or to pence by 4, as in *Reduction*.

This Rule though it be brief and plain is of great use in *Arithmetick*; either for turning natural and surd Fractions into Decimals; or any other desired *Denomination*, with such facility and speed as may be wished.

V. *Fraction of Fractions.*

In Reduction of Fractions, some make another, or more parts as *Fraction of Fractions* for one: That is, when there is part of a *Fraction*; or a part of a part of a *Fraction*, &c. to be valued in one *Fraction*.

RULE.

Multiply all the Numerators *together, the last* Product *shall be the* Numerator *desired: Then Multiply all the* Denominators *together, and this last Product shall be the* Denominator *sought.*

Example. Let the *Fractions of Fractions* propounded, be $\frac{4}{5}$ of $\frac{3}{4}$ of $\frac{1}{2}$, for so they are usually written; and let the *Numerators* be

be Multiplied: Saying, 4 times 3 is 12, and 12 times 1 is 12, the *Numerator* therefore required is 12: Then for the *Denominator* say, 5 times 4 is 20, and 20 times 2 is 40, for the *Denominator* required; and $\frac{12}{40}$ is equal to $\frac{4}{5}$ of $\frac{3}{4}$ of $\frac{1}{2}$.

The Proof.

Let the Unite be 40 *s.* one fifth of 40 is 8, and therefore $\frac{4}{5}$ is 32, of which one fourth is 8, and $\frac{3}{4}$ is 24, of which one half is 12, and therefore $\frac{12}{40}$ is the just Sum of all the Fractions: This needs no farther exemplifying.

ADDITION.

To add many Fractions into one Sum, consider whether they be of one Denomination or divers: If of one, Then add all the *Numerators* together into one Sum, that Sum is the new Numerator, and the *Denominator*, in this case is not altered.

Let the Fractions to be added be $\frac{2}{4}$, $\frac{4}{4}$, $\frac{5}{4}$, $\frac{1}{4}$. Add the Numerators: Saying 2 and 4 is 6, and 5 is 11, and 1 is 12. So the Sum of them all is $\frac{12}{4}$, that is 3 Unites.

As, let the Unite be 20 *s.* one fourth is 5 *s.* and $\frac{2}{4}$ is 10 *s.* and $\frac{4}{4}$ 20 *s.* which added to 10 *s.* is 30 *s.* then $\frac{5}{4}$ is 25 *s.* which added to thirty Shillings gives 55 *s.* And lastly, $\frac{1}{4}$ is 5 *s.* which added to 55 *s.* makes 60 *s.* that is 3 times 20 *s.* that is 3 *l.* or 3 Unites.

But if the Fractions to be added, be of divers Denominations; as let them be $\frac{2}{3}$, $\frac{1}{4}$, $\frac{4}{5}$, $\frac{7}{8}$, then (by the *Reduction* afore-going) they must be turned all into one Denomination, and then they will be $\frac{320}{480}$, $\frac{160}{480}$, $\frac{384}{480}$, and $\frac{420}{480}$, and may be added like those before: Thus,

	320
	160
	384
	420
In all	1284

So

So the Sum of all is $\frac{1484}{480}$, or $\frac{371}{120}$, or that is $3\frac{11}{120}$, which if it be in Money, and the Unite 1 *l.* it is then 3 *l.* 1 *s.* and 10 *d.* as may be tryed thus, First $\frac{2}{3}$ of a pound, is 13 *s.* and 4 *d.* and $\frac{3}{4}$ is 15 *s.* and $\frac{4}{5}$ is 16 *s.* Lastly, $\frac{7}{8}$ is 17 *s.* 6 *d.* These all added together, the Sum is 3 *l.* 1 *s.* 10 *d.*

SUBSTRACTION.

IN *Substraction* of one Fraction from another, if they be both of one Denomination: It is done by taking the *Numerator* of one from the *Numerator* of the other, the remain is the new *Numerator*, and the *Denominator* the same as before.

So if $\frac{3}{9}$ be substracted from $\frac{8}{9}$, the remain is $\frac{5}{9}$, the like of all other.

But if they be not of one Denomination, they must first be reduced to be so; then that which is said before is sufficient.

Concerning the Golden Rule *in* Fractions.

THe *Golden Rule in Fractions* is the same as in whole Numbers, I will give you but one instance.

If $\frac{3}{4}$ of a Yard of Tape cost $\frac{1}{2}$ of a Penny, what shall one Inch, that is, $\frac{1}{36}$ of a Yard cost?

Multiply the second by the third, the Product is $\frac{1}{72}$, which divided by $\frac{3}{4}$, the Quotient is $\frac{4}{216}$ of a Penny, for the Price of $\frac{1}{36}$ of a Yard.

Otherwise. Seeing $\frac{3}{4}$ of a Yard may be turned to 27 Inches: Say if 27 cost $\frac{1}{2}$, what 1? Divide $\frac{1}{2}$ by 27, it makes $\frac{1}{54}$ for the Answer: Which is equal to $\frac{4}{216}$, and in the least Terms.

And wheresoever this may be done, to have the first and third Numbers of Fractions of one Denomination, the best way is to work with their Numerators, not regarding their Denominators at all; As, If $\frac{2}{3}$ cost $\frac{3}{4}$, what $\frac{7}{3}$? Instead thereof write. If 2 cost $\frac{3}{4}$, what

$\frac{3}{4}$, what 7? Multiply $\frac{3}{4}$ by 7, it produceth $\frac{21}{4}$, which divided by 2, the Quotient is $\frac{21}{8}$, and that is the anſwer in the leaſt Terms.

And all this while it ſhould have been noted that the Fractions are ever written in a ſmaller figure then the whole Numbers.

The Rule of Practice.

IN the *Golden Rule*, or *Rule of Three Direct*, I intimated; that if the firſt of the three Proportional Numbers given were *One*, that then the Product of the ſecond and third Numbers gives the fourth Proportional Number ſought without uſing of any *Diviſion*; ——Alſo, that if the ſecond or third of the Proportionals given were *One*, then there was no need of *Multiplication*; but dividing the greater of them by the firſt, the Quotient ſhall be the fourth Proportional ſought for.

And from hence is framed this Rule of *Practice*, (by ſome called the *Merchants Rule*) which always hath *One*, an ingredient in the Queſtion, and it is no other but an *Abridgement* or *Compendium* of the *Rule of Three*, when *One* is one of the three Proportionals given.

And that ſuch Queſtions that are to be reſolved by this Rule may be the more readily and eaſily anſwered (Money commonly being one of the three Terms) it is expedient that he which intendeth to make much uſe of this Rule; ſhould have readily in his Mind the *Even* or *Aliquot* parts of a *Pound*, of a *Shilling*, and of a *Penny*. And alſo to have in *Memory* the ſeveral *Products* of 12 (the Number of *Pence* in one *Shilling*) Multiplied into 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12. All which are ſet down in the ſmall *Tables* following, which ought firſt perfectly to be learned by heart, before farther Progreſs be made into this Rule.

TABLE I. *The Aliquot or Even parts of a Pound or 20 Shillings.*

ſh.	*d.*				
1	0	is the	$\frac{1}{20}$	One Twentieth	of a Pound or 20*s.*
2	0	is the	$\frac{1}{10}$	One Tenth	of a Pound or 20*s.*
2	6	is the	$\frac{1}{8}$	One Eighth	of a Pound or 20*s.*
3	4	is the	$\frac{1}{6}$	One Sixth	of a Pound or 20*s.*
4	0	is the	$\frac{1}{5}$	One Fifth	of a Pound or 20*s.*
5	0	is the	$\frac{1}{4}$	One Fourth	of a Pound or 20*s.*
6	8	is the	$\frac{1}{3}$	One Third	of a Pound or 20*s.*
10	0	is the	$\frac{1}{2}$	One Half	of a Pound or 20*s.*

TABLE

TABLE II. *The Aliquot or Even parts of a Shilling.*

d.	q			
1	0	is the	$\frac{1}{12}$ One Twelfth	of a Shilling.
1	2		$\frac{1}{8}$ One Eighth	
2	0		$\frac{1}{6}$ One Sixth	
3	0		$\frac{1}{4}$ One Fourth	
4	0		$\frac{1}{3}$ One Third	
6	0		$\frac{1}{2}$ One Half	

TABLE III. *The several Pence in a Shilling Multiplyed by* 12.

2	Pence Multiplied by 12 produceth	24
3		36
4		48
5		60
6		72
7		84
8		96
9		108
10		120
11		132
12		144

For the working of the *Rule of Practice*, when the Price given is of equal Parts of a *Shilling* this is,

The RULE.

Knowing by your Table what part of a Shilling it is, (whether $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, *&c.*) *divide the Sum propounded by it, and the Quotient shall be the number of Shillings answering your Question.*

Example. At 6 d. the Ounce, what 7625 *Ounces?* Six Pence is (by your Table) $\frac{1}{2}$ of a Shilling, wherefore take one half of 7625, and it is 3812 *s* and 1 remaining, which 1 is 6 *d*. So that 7625 Ounces, will cost 3812 *s*. 6 *d*. which is reduced into Pounds, by cutting off the last Figure towards the Right-hand of 3812, and taking the half of the other Figures, which will be Pounds, and if one remain, in taking of the half it is 10 *s*. ——So the Figure 2 being cut off from 3812, the half of 381 is 190 and 1 remaining, which is 190 *l*. 12 *s*. So the price of 7625 Ounces will be 190 *l*. 12 *s*. 6 *d*. And so must you do for all others. As if the price be $\frac{1}{3}$, take $\frac{1}{3}$, if $\frac{1}{4}$ take $\frac{1}{4}$, as by the Examples following.

(1) At

(1) At 6 *d.* the Ounce, what 7625 Ounces?

$\frac{1}{2}$ 3812|12——6 *d.*

190 12——6 *d.*

(2) At 4 *d.* the yard, what 3621 yards?

$\frac{1}{3}$ 120|7

60 *li.* 7 *s.* 0 *d.*

(3) At 3 *d.* the Gallon, what 989 Gallons?

$\frac{1}{4}$ 24|7 3 *d.*

12 *li.* 7 *s* 3 *d.*

(4) At 2*d.* the Pound, what 6760 Pounds?

$\frac{1}{6}$ 112|6 8 *d,*

56 *li.* 6 *s.* 8 *d.*

(5) At 1 *d.* 2 *q.* the Ell, what 9623 Ells?

$\frac{1}{8}$ 120|2 10 *d.* 2 *q.*

60 *l.* 2 *s.* 10 *d.* 2 *q.*

(6) At 1 *d.* the Ounce, what 672 Ounces?

$\frac{1}{12}$ 5|6

2 *li.* 16 *s.*

Thus have you *Examples* when the price is *even parts* of a *Shilling*, But when they are *uneven parts* of a *Shilling*, as 5 *d.* 7 *d.* or the like, then you must do the work at two or three Operations, tho' in the same manner, as Pence.

	Pence		*d.* *d.*
If the Price be	5	take for it	3 and 2
	7		4 and 3
	8		4 and 4
	9		6 and 3
	10		6 and 4
	11		6 and 3 and 2

Examples of these uneven Parts of a Shilling.

(1) At 5 *d.* the Gallon, what 6254 Gallons?

$\frac{1}{4}$ is 3*d.*	1563	6 *d.*
$\frac{1}{6}$ is 2*d.*	1042	4 *d.*
5	260|5	10 *d.*
	130 *li.* 5 *s.*	10 *d.*

(2) At 7 *d.* the Ounce, what 9271 Ounces?

$\frac{1}{3}$ is 4 *d.*	3090		4 *d.*
$\frac{1}{4}$ is 3 *d.*	2317		9 *d.*
7	540|8		1 *d.*
	270 *li.*	8 *s.*	1 *d.*

(3) At 8 *d.* the Yard, what 7952 Yards?

$\frac{1}{3}$ is 4 *d.*	2651		8 *d.*
$\frac{1}{3}$ is 4 *d.*	2650		8 *d.*
8	530|1		4 *d.*
	265 *li.*	1 *s.*	4 *d.*

(4) At 9 *d.* the Ell, what 3769 Ells?

$\frac{1}{2}$ is 6 *d.*	1884		6 *d.*
$\frac{1}{4}$ is 3 *d.*	942		3 *d.*
9	282|6		9 *d.*
	141 *li.*	6 *s.*	9 *d.*

(5) At 10 *d.* the Dozen, what 625 Dozen?

$\frac{1}{2}$ is 6 *d.*	312		6 *d.*
$\frac{1}{3}$ is 4 *d.*	208		4 *d.*
10	52|0		10 *d.*
	26 *li.*	0 *s.*	10 *d.*

(6) At 11 *d.* the Pound, what 6952 Pound?

$\frac{1}{2}$ is 6 *d.*	3476		
$\frac{1}{4}$ is 3 *d.*	1738		
$\frac{1}{6}$ is 2 *d.*	1158		8 *d.*
11	637|2		8 *d.*
	318 *li.*	12 *s.*	8 *d.*

(7) At 12 *d.* or 1 *s.* the Ounce, what 9871 Ounces?
$\frac{1}{20}$ of 20 *s.* therefore $\frac{1}{2}$ of 987|2 is
(493 *li.* 12 *s.*

If the price of the Commodity is in Farthings, or Halfpence, bring the Sum into Pence, and work as in the preceeding Questions, and according to the following Examples.

(1) At

(1) At 1 *q.* the Pound, what 6392 Pound?

$\frac{1}{4}$	1598			
$\frac{1}{12}$	13|3			2 *d.*
	6 *li.*	13 *s.*		2 *d.*

(2) At 2 *q.* the Ell, what 3625 Ells?

$\frac{1}{2}$	1812			2 *q.*
$\frac{1}{12}$	15|1			
	7 *li.*	11 *s.*	0 *d.*	2 *q.*

(3) At 3 *q.* the Ounce, what 7321 Ounces?

$\frac{1}{2}$	3660			3 *q.*
$\frac{1}{12}$	305			
$\frac{1}{24}$	152			6 *d.*
	45|7			
	22 *li.*	17 *s.*	6 *d.*	3 *q.*

This is the manner of working for the even parts of a Peny, but if they be uneven parts; As two pence 3 farthings, five pence 1 farthing or the like, Work, first for the even part of a *Shilling*, and then for the *Farthings*, which added the Work is done. As in these Examples.

(1) At 3 *d.* 3 *q.* the Ell, what 817 Ells?

$\frac{1}{4}$	204			3 *q.*
$\frac{1}{4}$	51			$\frac{3}{4}$
	25|5			
	12 *li.*	15 *s.*	8 *d.*	$3\frac{3}{4}$ *q.*

(2) At 4 *d.* 1 *q.* the Pound, what 7138 Pound?

$\frac{1}{6}$	1189		8
again	1189		8
$\frac{1}{8}$	148		$8\frac{1}{2}$
	252|6	2	$0\frac{1}{2}$
	126 *li*	8 *s.*	0 *d* $\frac{1}{2}$

For the even parts of a *Pound*, you must take the parts as you find them expressed in the Table; as for 10 *s.* the $\frac{1}{2}$, for 4 *s.* the $\frac{1}{5}$; as in Example,

(1) At 2 *s.* 6 *d.* the Ell, what 6294. Ells?
$\frac{1}{8}$ 786 *li.* 15 *s.*

(2) At 4 *s.* the Ream, what 735 Reams?
$\frac{1}{5}$ 147 *l.*

If (in this Rule) at any time the Question consists of the part of an *Ell*, *Yard*, *Pound*, *Ounce*, *Gross*, or the like; you must deal with the whole *Ells*, *Yards*, *Ounces*, &c. first, and afterwards add the price of the $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$, or what other Part soever it be. And thus much shall suffice for this *Rule* of *Practice*.

Of Tare, Trett, *&c.*

IN *Merchandize* there is an *Allowance*, made by the Merchant to the Buyer, for the Weight of the *Cask*, *Bag*, *Chest*, *Freal*, *&c.* in which any Goods are put: And this Allowance is called *TARE*, which being deducted from the *Gross Weight* (which is the *Commodity* and *Cask*, &c. together) the Remainder is the Weight of the Commodity only; and is called *NETT-WEIGHT.*

There is also an Allowance made by Merchants to the Buyer, for *Refuse* or *Waste* that may be mixed with the Commodity; as *Dust Moats*, &c. (as in Commodities *Garblable*, as *Spices*, &c.) and this Allowance is called *TRETT*; which is always 4 *lib.* in the *Hundred Weight*: But the Allowance for *Tare* is various.

In such Commodities where *Trett* is allowed, the Remainder after such Allowance, is called *SUTTLE-WEIGHT*, and out of that the Allowance for *Trett* is made; and when that is deducted, the Remainder is called *NETT-WEIGHT.*

Exemplary Questions will make these RULES plain.

Question I. There are 4 *Chests* of *Sugar*, the *Gross Weight* of all which is 44 *C.* 1 *qr.* 13 *lib.* And the *Tare* allowed for each Chest 37 *lib.* What is the *Nett-Weight* of *Sugar* in all the 4 Chests?

	C.	qr.	lib.	
From	44	1	13	Total *Gross Weight*,
Substr.	1	1	8	Total *Tare*,
Remain	43	0	5	Total *Nett-Weight*.

Question

Question II. If from 990 *C.* 3 *qr.* 21 *lib.* *Gross Weight*, the *Tare* is to be substracted, after the Rate of 14 *lib.* *per C.* (or 112 *lib.*) of *Gross Weight*; How many *C.* *Nett-Weight* will remain?

1. The *Gross-Weight* 990 *C.* 3 *qr.* 21 *lib.* reduced into *Pounds*, will be 110985 *lib.*

2. Say by the *Rule of Three Direct*,

As 112 *lib.* : to 14 *lib.* :: So 110985 : to $13873\frac{1}{8}$.

Wherefore,

3.	From	110985	*Gross-Weight*,
	Substr.	$13873\frac{1}{8}$	*Total Tare*,
	Rests *Nett*	$97111\frac{7}{8}$.	

Which reduced is equal to 867 *C.* $7\frac{7}{8}$ *lib.*

Note, That when the Number of *Pounds* to be abated *per C.* for *Tare*, are an *Aliquot* part of 112 *lib.* as in the former Example, where it is 14 *lib.* which is equal to $\frac{1}{8}$ of 112: Then the Proportion may be,

As 8 : to 1 :: So 110985 : to $13873\frac{1}{8}$.

Or,

	C.		C.		C.	qr.	lib.		C.	qr.	lib.
As	1	: to	$\frac{1}{8}$	:: So	990	3	21	: to	123	3	$13\frac{1}{8}$.

For $\frac{1}{8}$ of	990 *C.*	is equal to	123	3	00
	3 *qr.*		0	0	$10\frac{4}{8}$
	21 *lib.*		0	0	$2\frac{5}{8}$
		Total Tare	123	3	$13\frac{1}{8}$

Question III. A Merchant buys 1175 *lib.* Weight of a Commodity, (as *Cloves*, *Nutmegs*, or the like) for which he is to be allowed for *Trett*, 4 *lib.* in the *Hundred* Weight; How many *Pounds* Weight ought he to receive?

Then say by the *Rule of Three Direct*,

	lib.		lib.		lib.		lib.
As	100	: is to	104	:: So is	1175	: to	1222.

Question IV. A Merchant hath 1222 *lib.* Weight of a Commodity, part whereof he bought at a certain Rate *per lib.* and the rest was allowed him as an Overplus; after the Rate of 4 *lib.* Weight in the 100 *lib.* Weight, which he bought: I demand how many Pounds *Nett-Weight* did he buy?

Say

Say by the *Rule of Three Direct*;

lib.		*lib.*		*lib.*		*lib.*
As 104	: is to	100	:: So is	1222	: to	1175.

This Question is but the Reverse of the former; and shews the way whereby to make abatement for *Trett.*

Question V. If from 55 *C.* 1 *q.* of Gross Weight, Tare is to be Substracted after the rate of 16 *per Cent.* and from the remaining *Trett*, is to be abated after the rate of 4 *l. per* 104 *l.* The Question is, what the *Nett Weight* is worth in Money, after the rate of 8 *l.* 8 *s.* for every *C.* (or 112 *lib.*)?

1. The Gross Weight in Pounds is, 6188.

Then,

lib.		*lib.*		*lib.*		*lib.*
As 112	: is to	16	:: So is	6188	: to	884,

Then,

From	6188	the Grosse Weight,
Substr.	184	the Tare
Remain	5304	the Trett.

Then,

lib.		*lib.*		*lib.*		*lib.*
2. As 104	: to	100	:: So	5304	: to	5100.

And then,

As 112 *lib.* Weight;
Is to 8 *l.* 8 *s.*
So is 5100 *lib.*
To 382 *l.* 10 *s.*

Which is the worth of the Commodity,

THE

THE RULE OF *FELLOWSHIP*.

THis Rule is useful for *Merchants*, and all such as Trade in *Companies*, with a Joynt stock; and must share a proportional part of the gains, or loss; every one according to his stock which he laid in.

The Rule is two-fold, with *equal* time; or with *unequal* time.

That which is with *equal time*, is commonly called, the *Rule of Fellowship without Time*.

Of this we will first speak.

THE RULE.

As the whole Joint Stock is to all the gain or loss: So is each mans particular Stock, to his part of the gain, or loss.

Example I. Two Purchasers *A.* and *B.* buy 700 *l.* a year Land for ever, (when money is at 8 *per Cent.*) for 14000 *l.* of which *A.* paid 8000 *l.* and *B.* 6000 *l.* after 5 years (money being fallen to 6 *per Cent.*) they sell it for 18700 *l.* so there is gained 4700 *l.* how much of this must *A.* have,

First for *A.*

Say, if 14000 gain 4700, what 8000? Answer, 2685 $\frac{10000}{14000}$.

Then for *B.*

If 14000 gain 4700, what 6000? Answer, 2014 $\frac{4000}{14000}$. As by the following Operation doth appear.

(I) For *A.*

l. *l.* *l.*
If 14000 gain 4700, what 8000.

```
                        8000
                        4700
                       ------
       14000)  37600000  (2685 10/14
                   ....
               ------
               28000
                96000
               ------
                84000
                120000
               ------
                112000
                 80000
               ------
                 70000
                 10000
```

(II) For *B.*

l. *l.* *l.*
If 14000 gain 4700, what 6000?

```
                        6000
                        4700
                       ------
       14000)  28200000  (2014 4/14
                   ....
               ------
               28000
                20000
               ------
                14000
                60000
               ------
                56000
                 4000 Remainder.
```

Here note, That this Work might have been much abreviated, if from each of the three Numbers you had cut off two Cyphers towards the right Hand, as hath been formerly shewed in the Compendiums of *Multiplication* and *Division.*

Now

Now for the Proof hereof,
If you add — $2685\frac{10000}{14000}$
which is the sum that *A.* gained;
To — $2014\frac{4000}{14000}$
The sum which *B.* gained; the sum of them is — 4700
Which is equal to the total Gain.

And according to the proportion of these two Numbers: That is, as 8 to 6, or 4 to 3. So they ought to have parted the yearly Rent also, all the time they received it: That is, A, ought to have 400 *l.* Yearly; and *B.* 300 *l.*

Example II. *A. B.* and *C.* joyn their moneys to make a stock of 25000 *l.* of which *A.* laid in 10000 *l. B.* 8000 *l.* and *C.* put in 7000 *l.* with this (after a certain time in trading) they gained 7500 *l.* how must this be parted?

First for *A.*

Say, if 25000 gain 7500, what 10000?

Or shorter, if 25 get $7\frac{1}{2}$, what 10? Multiply $7\frac{1}{2}$ by 10, it produceth 75, which divided by 25, the quotient is 3, that is, (restoring the three Cyphers) 3000 *l.* for *A.*

Then for *B.*

Say, if 25000, gain 7500 what 8000?

Or shorter, if 250 get 75; what 80?

Multiply and divide as the *Golden Rule* requires, to the quotient restore the two Cyphers, then it will be 2400 *l.* for *B.*

Lastly, for *C.*

Say, if 250 give 75, what 70? Answer 21, to which put the two Cyphers, it makes 2100 for *C.*

And these three 3000, 2400, and 2100, being added together, make 7500: And have that proportion as the particular stocks had: And therefore the Work is right.

(I) for *A.*

If 25 gain $7\frac{1}{2}$, what 10

$$\begin{array}{ccc} & 150 & \\ 7\frac{1}{2}\ \frac{15}{2} & \times & \frac{10}{1} \\ & 2 & \end{array}$$

2) 150 (75

14
10 (75

10

25) 75 (3000 *l.*

75

(II) for *B.*

If 250 gain 75, what 80?

75

---

250) 6000 (24...

500
1000

1000
2400 *l.* for *B.*

(III) for *C.*

If 250 gain 75 what 70?

70

---

250) 5250 (21...

500
250

250
2100 *l.* for *C.*

And

And if inſtead of gaining 7500 *l.* whereby every one is ſuppoſed to have his Stock, and part of the gains; they had loſt 7500 *l.* then their particular Stocks had not been due to them, but ſo much as would be left after their proportional Parts of the loſs were abated.

Example III. *A. B.* and *C.* with a joynt Stock of 25000 *l.* gain 7500: of which *A.* gets 3000, *B.* 2400, *C.* 2100; what was their Stock?

This is but the Reverſe of the former, therefore ſay, if 7500 require 25000, what doth 3000 require? 10000 for *A*, and ſo work for the other two.

Many Examples are of little uſe (except to load the Readers memory) where the Rule is ſo ſhort and plain; I will therefore add no more to this part of the Rule but immediately come to the Rule of Fellowſhip with Time.

THE RULE OF *FELLOWSHIP* With Time.

THis Rule is to be uſed when the *Times* of the continuance of the particular *Stocks* are unequal, and differ; ſo that here the *difference of Time*, and alſo the *difference of Stock* being both to be conſidered; it can be done no better way than by taking the *Power* of them both to be the *particular Stock*; and all thoſe *Powers* added, to be the *whole Stock*, that which I call the *Power*, is the *Product* of the *Money* of every one, multiplied by his *Time*; And then,

THE RULE.

As the ſum of thoſe Products, *is to the whole* Gain; *ſo is each particular* Product, *to its part of the* Gain.

Question I. Three Merchants *A. B. C.* make a Stock of 10000 *l.* of which *A.* layes in 4000 for 3 Months, *B.* 3000 *l.* for 6 Months; and *C,* 3000 *l.* for 8 Months, with this they gain 2000 *l.* what is each Mans ſhare?

Firſt, for *A.* multiply 4000 by 3, it makes 12000, let that be accounted his *particular Stock.*

Secondly, For *B.* multiply 3000 by 6, it makes 18000, his *particular Stock.*

Laſtly, for *C.* multiply 3000 by 8, it produceth 24000, for his *Stock.* Add theſe, they make 54000 *l.* for the *general Stock*; then ſay,

For *A.*

If 54000 give 2000, what 12000? Anſwer, 444 $\frac{24000}{54000}$.

Then for *B.*

If 54000 give 2000, what 18000? Anſwer, 666 $\frac{36000}{54000}$.

Laſtly, for *C.*

If 54000 give 2000, what 24000? Anſwer, 888 $\frac{48000}{54000}$.

The three Fractions may be reduced (by dividing each Numerator, and Denominator by 6000) and then the three Shares will be 444 $\frac{4}{9}$, 666 $\frac{6}{9}$, and 888 $\frac{8}{9}$, which altogether make 2000, as they ought.

Question II. Three Farmers, *A. B.* and *C.* lay out 1000 *l.* to Stock their Grounds with Cattel, of which *A.* put in 200 *l.* for 6 Years; *B.* had 300 *l.* going for 4 Years; and *C.* 500 *l.* for 2 Years; at the end (by unſeaſonable Times) there was loſt 200 *l.* which made the remain of their Stock but 800 *l.* what had each Man loſt?

Multiply 200 by 6, it gives 1200: Likewiſe, 300 by 4, it gives 1200. Laſtly, 500 by 2, the product is 1000: All theſe are 3400 for the joynt Stock.

Then firſt for *A.*

Say, if 3400 loſe 200, what 1200? Anſwer, 70 $\frac{2000}{3400}$ for *A.* to which *B.* is equal, becauſe the Power of his Stock is ſo.

Therefore for *C.*

Say, if 3400 loſe 200, what 1000? Anſwer, 58 $\frac{2800}{3400}$. So the 3 ſhares are 70 $\frac{20}{34}$, 70 $\frac{20}{34}$, and 58 $\frac{28}{34}$, equal to 200 $\frac{24}{34}$.

Now becauſe *A.* put in 200 *l.* and loſt 70 $\frac{20}{34}$, Subſtract the loſs from the Sock, remains 129 $\frac{14}{34}$.

And ſo doing for *B.* his remain will be 229 $\frac{14}{34}$.

And for *C.* his remain is 441 $\frac{6}{34}$. Now theſe three Remains, 129 $\frac{14}{34}$, 229 $\frac{14}{34}$, and 441 $\frac{6}{34}$, make up 800 *l.* which was the whole Remain.

Question

Question III. *A.* rents a Close for a Year, to pay 80 *l.* he puts into it 200 Sheep: 2 Months after *B.* puts 40 Sheep in; and 5 Months after that *C.* puts in 100 Sheep, how much must every one pay of the Rent?

Multiply 200 by 12, it produceth	2400
And 40 by 10, produceth	400
Lastly, 100 by 5, (which is *C.* time) produceth	500
In all	3300

Then for *A.*

If 3300 pay 80, what 2400? Answer, $58\frac{600}{3300}$.

Then for *B.*

If 3300 pay 80, what 400? Answer, $9\frac{2300}{3300}$.

And for *C.*

If 3300 pay 80, what 500? Answer, $12\frac{400}{3300}$.

The whole Numbers make 79, and the broken Numbers make 1. In all 80.

Note, Whereas, hitherto we have considered only difference of *Time* and *Money*; it may be noted, that there may be difference of other kinds, as *Persons* or *Place*; but whatsoever they are, the Power of all is found like these by multiplication; and are to be wrought like these, with so many Uses of the *Golden Rule*, as the Question requires. I will therefore add but one Question more, which is this:

Question. One leaves a Legacy of 900 *l.* among four Kinsfolk, *A. B. C. D*: So as *B.* may have twice as much as *A.* and *C.* thrice as much as *B*; and *D.* as much and half as much as *C*; what is every one to have?

Say, If *A.* be 1. *B.* is 2, *C.* 6, and *D.* 9, add these Numbers: 1, 2, 6, 9, together, they give 18, then say, If 18 require 900, what 1? Answer is 50. So *A.* is to have 50 *l.* *B.* 100 *l.* *C.* 300 *l.* and *D.* 450 *l.* which are their just Parts; and altogether are equal to 900 *l.* and the work is right.

BAR-

BARTER.

TO Barter is to exchange one Commodity for another, the Nature whereof will beft appear by the refolving of fome Queftions.

Queftion I. Two Merchants Barter, One hath Sugar at 4 *l.* the C. ready Money, but in Barter he will have 4 *l.* 13 *s.* 4 *d.* The other hath *French* Wine at 13 *l.* the Hogfhead ready Money; at what price muft he rate his Wine, to equalize the others advance of his Sugar in Barter?

Say, by the Rule of Three direct,

If 4 *l.* in Barter, require 13 *s.* 4 *d.* advance, what fhall 13 *l.* in Barter require?

```
     l.        s.        d.      l.
If   4 ——————— 13 ——————— 4 what 13 ?
               12                  4)  2080  (520 d.
             ————                      ...
               30                     ———————
               13
             ————                 12)  520   43 s.
              160  Pence.               ..
               13                     ———————
             ————                       48
              480                       40
              160                     ———————
             ————                       36
             2080                     ———————
                                      (4 Pence.)
```

That is 43 *s.* 4 *d.*, or 2 *l.* 3 *s.* 4 *d.* the Anfwer.

Queftion II. Two Barter, one hath $3\,C\frac{1}{2}$ of Ginger at 13 *d.* $\frac{1}{2}$ *per* Pound. The other hath Sugar at 15 *d.* $\frac{1}{4}$ *per* Pound. How much Sugar muft be delivered for the $3\,C\frac{1}{2}$ of Ginger.

Firft, by the Rule of Three (or Practice) find what the $3\,C\frac{1}{2}$ of Ginger comes to at 13 *d.* $\frac{1}{2}$ *per* pound, which will be found to be 22 *l.* 1 s. For,

If

If 1 *lib.* coft 13 *d.* $\frac{1}{2}$ what 3 C $\frac{1}{2}$ coft?

Anfwer, 22 *l.* 1 *s.*

Secondly, fay, If 15 *d.* $\frac{1}{4}$ buy 1 *lib.* of Sugar, what fhall 22 *l.* 1 *s.* buy?

Anfwer, 347 $\frac{7}{61}$.

Queftion III. Two Barter, One hath Tobacco at 14 *d. per* pound, which he will Barter for Sugar at 10 *d. per l.* how much Tobacco muft be given for 8900 *lib.* of Sugar?

Firft, the 8900 *l.* of Sugar at 10 *d. per* pound, comes to 370 *l.* 16 *s.* 8 *d.*

Then, If 14 *d.* buy 1 *lib.* of Tobacco, what Number of pounds will 370 *l.* 16 *s.* 1 *d.* buy?

Anfwer, 6357 Pound, and fo many pounds of Tobacco at 14 *d.* muft be given for 8900 pound of Sugar at 10 *d.*

Queftion IV. Two Barter, One hath broad Cloth at 15 *s.* the Yard ready Money, for which in Barter he will have 16 *s.* 3 *d.* The other hath Wool at 2 *s.* 10 *d. per* pound ready Money? What price muft his Wool be fet at in Barter to equalize the advance which he puts upon his Cloth.

Say by the Rule of Three direct.

If 15 *s.* ready Money require 1 *s.* 3 *d.* in Barter; what fhall 2 *s.* 10 *d.* ready money require?

Anfwer, 2 *d.*——3 *q.* $\frac{1}{3}$

So that he muft rate his Wool at 3 *s.* 3 *q.* $\frac{1}{3}$ of a farthing *per* pound.

OF
INTEREST
Simple *and* Compound.

IN the *Second Part* of this *Book*, I have *Tables of Compound Intereft*, *Rebate* or *Difcount* of *Money*, *Purchace* of *Leafes* and *Annuities*, whofe *Conftruction* and *Ufe* are There *Exemplified* by Refolving of *Queftions* fuitable to each *Table*, as by having recourfe thither will appear. As for Simple Intereft, any Queftion there-

thereunto relating, ſuch may be Reſolved by the *Single* or *Compound Rules* of *Proportion* before taught in this Book; of which alſo an *Example* or two I will here inſert: And omit ſaying any thing of the other, till I come to ſpeak thereof in the *Second Part*.

Queſtion I. If 100 *l.* in 12 Months gain 6 *l.* what ſhall 625 *l.* gain in 3 Years or 36 Months?

The Proportion is,

As 100 *l.* is to 6 *l.* in a Year,

So is 625 *l.* to 112 *l.* 10 *s.* in a Year.

Wherefore multiply 625 *l.* by 6 *l.* and divide the Product by 100, (by cutting off two figures) the Quotient will be 37 $\frac{50}{100}$ that is, 37 *l.* 10 *s.* and this being Multiplied by 3, giveth 112 *l.* 10 *s.* as by the Work appears.

```
 l.        l.        l.
100———————6————————625
                     6
              ———————————
                 37|50
                   |3
   l.   s.    ———————————
Or 112——10      112/50
```

Queſtion II. If 100 *l.* in 12 Months gain 6 *l.* what will 236 *l.* 10 *s.* 5 *d.* gain in 16 Months?

The Proportion.

As 100 *l.* is to 6 *l.* in a Year,

So is 236 *l.* 10 *s.* 5 *d.*

To 14 *l.* 3 *s.* 9 *d.* 3 *q.* in a Year.

So is 236 *l.* 10 *s.* 5 *d.* to 14 *l.* 3 *s.* 9 *d.* 3 *q.* in a Year.

```
            l.    s.      d.
Multiply 236——10————————5 by 6,
The Product is 1419—— 2————————6,
```

This divide by 100, which is done by cutting off two figures of the Integer, leaving 14 *l.* on the Left-hand of the Line. The figures on the Right-hand multiplied by 20, and the figures (or remains) again by 12; and laſtly by 4; ſhall in all give 14 *l.* 3 *s.* 9 *d.* 3 *q.*

Which divide by 3, and add that third part to 14 *l.* 3 *s.* 9 *d.* 3 *q.* the ſum will be 18 *l.* 18 *s.* 5 *d.* 0 *q.* as by the Work appeareth.

l. l. l. s. d.
100—6—236—10—5
6

l. 14|19—2—6
|20

s. 3|82
|12

d. {1|70
9 {8|2

|20
|4

q. 3|60

	l.	s.	d.	q.
In a Year	14	3	9	3
In $\frac{1}{3}$ of a Year	4	14	7	1
In 16 Months	18	18	5	0

By this manner of Work, If 417 *l.* 11 *s.* 8 *d.* be put out at Intereſt for 2 Years at 6 *l. per cent.* it will amount unto (I mean the Intereſt) 50 *l.* 2 *s.* 2 *d.* As by the Work appears.

li. li. li. s. d.
100—6—417—11—08
6

25|05—10—00
|20

|10
|12

1|20
|4

|80

	li.	s.	d.
In one Year	25	01	01
			02
In two Years	50	02	02

Thus theſe Queſtions are wrought by the *Single Rule of Three*, but they may be otherwiſe wrought by the *Golden Rule Compound of 5 Numbers.* Of which in that *Rule* you have an Example.

ALLIGATION.

THis *Rule* taketh its *Name* from *Binding*, *Tying* or *Uniting*, many *Particulars* in one *Mass* or *Sum*: And it is either *Medial* or *Alternate*. Examples in both which follow:

I. *Of Alligation Medial.*

Alligation Medial is; When having the several *Quantities* and *Rates* of diverse *Simples* propounded, we do discover the *Mean Rate* of a *Mixture* Compounded of those *Simples*.

As, having 10 *Bushels* of *Wheat* at 4 Shillings (or 48 Pence) the *Bushel*, 40 *Bushels* of *Rye*, at 3 Shillings (or 36 Pence the *Bushel*:) 50 *Bushels* of *Barley*, at 2 Shillings (or 24 Pence) the *Bushel*: And 20 *Bushels* of *Oats* at 12 Pence the *Bushel*: This *Rule* of *Alligation Medial*, will tell you the *Mean Price* of that *Mistling*. And for the performance thereof this is

The RULE.

First, Sum up the given Quantities: *Then find the* Total Value *of all the* Simples: *Which done,*

The Proportion will be,

As the Sum of the *Quantities*,
Is to the *Total Value* of the *Simples*,
So is any part of the *Mixture* propounded,
To the required *Mean Rate*, or *Price*, of that Part.

Example, In the fore-mention'd *Grains*, I demand how much one *Bushel* of that *Mistling* is worth?

Now the Sum of the given *Quantities* of *Bushels*, (*viz.* 10, 40, 50, 20) is 120 *Bushels*: And

					d.		
10	Bushels of	Wheat	at		48	comes to	480
40		Rye			36		1440
50		Barley			24		1200
20		Oats			12		240
120	In all				Pence in all		3360

Then

Then say by the *Golden Rule Direct*

If 120 *Bushels*, give 3360 *Pence*: What 1?

```
120) 3360 (28 Pence.
      ..
     ----
     240        or
      960       s. d.
     ----       2  4
      960
```

In like manner; if it had been demanded what 8 *Bushels*, or One *Quarter* of that *Mistling* is worth; the *Answer* would have been, 224 *Pence*, which is 18 *s.* 8 *d.* for the *Price* of the *Quarter*.

```
120---3360----8
        8
     -----
120) 26880 (224
      ...
     -----
     240
      288        12) 224 (18 s.
     ----             .
     240             ---
      480             12
     ----             104
      480            ----
                       96
                      (8 d.
```

The Proof of this Rule.

The tryal of the Work *is, by comparing the* Total Value *of the several* Simples; *with the* Value *of the* Whole Mixture: *For, if these Sums* accord, *the* Operation *is perfect* ——— So in the preceding Example,

	Bushels				s.		l.		
Value of The	10	Bushels of	Wheat	at	4	is	2	0	0
	40		Rye		3		6	0	0
	50		Barley		2		5	0	0
	20		Oats		1		1	0	0
						The Total	14	0	0

All which amounts to 14 *l.* which is likewise the Value of 120 *Bushels* at 2 *s.* 4 *d.* the *Bushel*, for that also *Amounts* to 3360 *s.* or 14 *l.*

Another Example.

A Goldsmith *hath melted* 12 lib. *of* Gold Bullion *of* 18 Carracts fine, *with* 4 lib. *of* 21 Carracts Tine: *How many* Carracts Tine *is* 1 lib. *of this* Mass *worth?*

The Answer is $18\frac{12}{16}$, or $18\frac{3}{4}$ *Carracts fine.* See the Work.

12 *lib.*	4 *lib.*		12 *lib.*
18 *Car.*	21 *Car.*		4 *lib.*
96	84		16 Divisor.
12		16)	300 ($18\frac{12}{16}$ or $\frac{3}{4}$
216			16
84			140
300 Dividend			128
			12

II. *Of Alligation Alternate.*

Alligation Alternate is, When having the several *Rates* of diverse *Simples* given; to discover such *Quantities* of them, as are necessary to make a *Mixture*, which may bear a certain *Rate* proposed.

For the Solution of *Questions* belonging to *Alternate Alligation*; Observe these

RULES.

I. *You must* Rank *the* Terms *in such sort, That the* Given Rate *of the* Mixture, *may represent the* Root; *and the several* Rates *of the* Simples, *may stand as* Branches *issuing from that* Root.

II. *Having* Ranked *the* Terms *in their due Order*; Link *the* Branches *together* (Two *and* Two) *in such sort*; *that* One *that is* Greater *than the* Root *(or Rate) may always be coupled with another that is* Less *than the same* Root *or* Rate.

III. *Having* Alligated *the* Branches, *and found the* Differences *between them and the* Root; *write the* Difference *of each* Branch, *just against his correspondent yoak follow.*

The Nature of this *Rule* will be understood, in working some *Questions* for *Examples.*

Question

Question I. A Corn Master would Mix Four sorts of Grains together, *viz.* Wheat at 4 *s.* 6 *d.* the Bushel; Wheat at 4 *s.* the Bushel; Rie at 3 *s.* the Bushel; and Barley at 2 *s.* 8 *d.* the Bushel ——— So as to make 15 Quarters in all, to be sold at 3 *s.* 6 *d.* the Bushel: How much must he take of each sort of Grain.

I. Reduce the 15 *Quarters* into *Bushels*, and they make 120 *Bushels*.

II. Turn all your *Rates* of *Grain* into *Pence*: So will the first Wheat be 54 *d.* the second 48 *d.* the Rie at 36 *d.* and the Barley at 32 *d.* the *Bushel*.

III. Reduce the *Rate* of the required *Bushel*, *viz.* 3 *s.* 6 *d.* into *Pence* and they will be 42 *d.* And that is the *Root*, being thus prepared; Set the Number down as in the *Margine*, and *Link* them so, that a *Greater* and a *Lesser* may still be together; as 54 and 32, and 48 with 36. Then place the *Price* of the *Bushel* Required 3 *s.* 6 *d.* or 42 *d.* by it self on the *Left-hand*, and take the *Difference* between that, and the *Price* of a *Bushel* of every one particular *Grain*:

d.		*Differences.*
42	54	10
R	48	6
	36	6
	32	12
The Sum		34

As, the Difference between 54 *d.* and 42 *d.* that is 12, which must not be set against 54, but against that number which is Linked with 54, that is, against 32. Also, the *Difference* between 42 and 36, which is 6, must not be set against 36, but against 48, which is *Linked* with 36: And so must all the *Differences* be Ordered; as is easie to be seen in the *Margine*. Then proceed by this

RULE.

Multiply the Whole Mass *to be made, by any particular* Difference; *and Divide that* Product *by the* Sum *of all the* Differences; *the* Quotient *shall give the just* Quantity *of that Particular kind, whose* Price *standeth against the* Difference *you wrought with:* So,

Multiply 120, (the whole *Mass* to be made) by 10, (the *Difference* standing against 54) the *Product* will be 1200; Which *Divided* by 34 (the *Sum* of all the *Differences*) the *Quotient* will be $35\frac{5}{17}$ *Bushels*. And so much must be taken of that *Wheat*, whose *Price* is 4 *s.* 6 *d.* or 54 *d.* And working so for all the rest, you shall find that there must be taken of that *Grain* whose Price,

	s.	d.			
Is	4	6	the Bushel	$35\frac{5}{17}$	Bushels
	4	0		$21\frac{6}{34}$	
	3	0		$21\frac{6}{34}$	
	2	8		$42\frac{12}{34}$	

For,

For,

$$\begin{array}{llllllll}
\text{As } 34 & \text{is to} & 120 & :: & \text{So is } 10 & : & \text{to } 35\frac{5}{17} \\
34 & : & 120 & :: & 6 & : & 21\frac{3}{17} \\
34 & : & 120 & :: & 6 & : & 21\frac{3}{17} \\
34 & : & 120 & :: & 12 & : & 42\frac{6}{17} \\
\hline
 & & & & & \text{In all} & 119\frac{17}{17}
\end{array}$$

Now to *Prove* this Right:

Multiply 120 *Bushels* (the *whole Mass*) by the desired *Price* 42 *d.* the *Product* will be 5040 *d.* the *Price* of the whole *Mass* in *Pence.*

Then for the *Price* of the *Bushels* of every sort of *Grain*, say by the *Golden Rule Direct.*

As One *Bushel* of *Grain*,
Is to the *Price* of One *Bushel* of that *Grain.*
So is the Number of *Bushels* to be taken of that *Grain*,
To the *Price* of those *Bushels.*

So the *Price* of the First sort of *Wheat* was 54 *d.* and there was to be taken of that sort $35\frac{5}{17}$ *Bushels:* Therefore, Multiply $35\frac{5}{17}$ by 54 the Product will be $1890\frac{270}{17}$ and so much will the Quantity of that sort of *Wheat* amount unto.

And working with all the rest in the same manner, you will find the *Rates* of the several *Quantities* to be as followeth, *viz.*

$$\text{The}\left\{\begin{array}{l} \overset{\text{Bushels}}{35\frac{5}{17}} \\ 21\frac{3}{17} \\ 21\frac{3}{17} \\ 42\frac{6}{17} \end{array}\right\}\begin{array}{c}\text{Bushels}\\ \text{at}\end{array}\left\{\begin{array}{l} \overset{\text{Pence}}{54} \\ 48 \\ 36 \\ 32 \end{array}\right\}\begin{array}{c}\text{comes}\\ \text{to}\end{array}\left\{\begin{array}{l} 1890\frac{270}{17} \\ 1008\frac{144}{17} \\ 756\frac{108}{17} \\ 1344\frac{192}{17} \end{array}\right.$$

$$\text{Which added together do make } 4998\frac{714}{17}$$

And the *Numerator* of the *Improper Fractions* being divided by the *Denominator*, the *Quotient* will be 42, and that added to 4998 makes it 5040 *Pence* equal to the *Price* of the *Whole Mistlin* at 3 *s.* 6 *d.* the *Bushel.*

Question II. One hath 6 sorts of *Fruits* at several prices; *Dates* at 2 *s.* *Almonds* at 1 *s.* 4 *d.* *Currants* at 10 *d.* *Raisins* at 5 *d.* *Prunes* at 4 *d.* and *Figs* at 4 *d.* the Pound; and would take of every sort some, to make a *mixed quantity* of 30 *l.* weight, to sell one with another for 9 *d.* the pound, how much must he take of each?

Having

Having placed the Numbers and their differences, and the ſum of thoſe differences diſtinctly as hath been ſhewed before, and may be ſeen by the Figure in the Margine; the Work is evermore like that in the former Queſtion. So 38 is the firſt number in the *Golden Rule*; 30, the ſecond (which that it may not be forgotten, may be ſet at the Right-ſide of the figure) and every particular difference, as 6, 5, 4, 3, *&c.* is the third in the Rule, to be repeated till all the differences have been employ'd.

	24	6	
	16	5	
	10	4	
9			(30
	5	1	
	4	7	
	3	15	
	Sum	38	

So 30 multiplied by 6, produceth 80, which divided by 38, the Quotient is $4\frac{28}{38}$ of a pound weight, and ſo much muſt be taken of Dates, at 24 *d.*

Secondly, 5 times 30 is 150, which divided by 38, the Quotient is $3\frac{36}{38}$ for Almonds. And working after the ſame manner with 4, 1, 7, 15, their reſpective Quantities will be found to be theſe:

	pounds	*38th parts*
Dates	4,	28
Almonds	3,	36
Currants	3,	6
Raiſins	0,	30
Prunes	5,	20
Figgs	11,	32
In all	26	152

That is $26\frac{152}{38}$ and the reduction of the Fraction will make it 30, as it ought to be, and by comparing the prices of theſe particulars added, with the price of 30 *lib.* weight, at 9 *d. per lib.* weight, which makes 470 *d.* This may be be proved like the former.

But that the Reader may be perfect in it, I will do it here alſo as followeth:

Say

Say first, 24 times 4 is 96, and 24 times 28 is 672: for the first,

Set them thus,	96,	672
Secondly, 16 times 3 is 48 and 16 times 36 is 576	48,	576
Thirdly, 10 times 3 is 30, and 10 times 6 is 60,	30,	60
And 5 times 30 is 150	00,	150
Fourthly, 4 times 5 is 20, and 4 times 20 is 80.	20,	80
Lastly, 3 times 11 is 33, and 3 times 33 is 96	33,	96
In all	227,	1634

Now this 1634 being the ſum of the Numerators of Fractions, whoſe common Denominator is 38, muſt be divided by 38, and the Quotient will be 43, which added to the whole number 227; the ſum is 270. And ſo much is 30 multiplied by 9, which ſhews the work to be right.

The Combination or linking of Numbers may be varied at pleaſure, as whereas above I linked 24 and 3, alſo 16 with 4, and 10 with 5; it might have been 24 with 5, and 16 with 4, and 10 with 3. Or 24 with 4, and 16 with 3, and 10 with 5, of which diverſity of linking would follow diverſity of ſolutions, but all true, as the Reader may eaſily prove by himſelf.

Likewiſe, if the Numbers to be linked were 3, 5, 7, or any odd Numbers, one of them may be linked to two ſeverally, to make the work even.

Example III. If the Numbers were 12, 10, 8, 6, and 4, and the *mean* or *common Price* required were 9, you might firſt link them as you ſee here, taking 12 twice, or elſe you might take any other twice as you ſhall think fit, and ſo the work will be every way right, though not the ſame; if the differences be rightly ſet off, and orderly uſed, as is taught before in the firſt Queſtion.

	12	5
	12	3
	10	1
9		
	8	1
	6	3
	4	4

Queſtion

Question III. A Goldsmith would mix 3 sorts of Silver, *A. B. C. A.* is 10*d.* weight better, *B.* 7*d.* weight better; and *C.* 4*d.* weight better, to make an Ingot of 50*l.* weight, which should be in fineness 8*d.* weight better: How much must be taken of each?

Set them, their differences, and the sum of their differences, as in the Margin. Then,

9	10	4	50
	10	1	
	7	2	
	4	2	
		9	

First, 50 Multiplyed by 4, is 200, and divided by 9, the Quotient is	$\frac{200}{9}$
Secondly, 50 Multiplyed by 1, is 50, and divided by 9, the Quotient is	$\frac{50}{9}$
Thirdly, 50 Multiplyed by 2, is 100, and divided by 9, the Quotient is	$\frac{100}{9}$
Fourthly, The same again	$\frac{100}{9}$
In all	$\frac{450}{9}$

Which is equal to 50, the Quantity required.

Now the first Fraction multiplied by 10, (omitting the Denominator) is 2000
The second also by 10 gives 500
The third by 7 makes 700
The last by 4 makes 400

In all 3600

That is, $\frac{3600}{9}$, which is equal to 400, and if the whole Ingot 50, be multiplied by the betterness required, namely, by 8, they shall produce 400 also: So this is proved.

In every Alligation, or linking of two Numbers, this is evident, that if the sum of the Numbers linked be greater than the mean Number required, taken so many times as there are Numbers to be linked, the question would be absurd; and the resolution thereof impossible. And this shall serve for the Rule of *Alligation*.

THE

THE RULE OF FALSE POSITION.

THIS Rule serves to resolve Questions, which are not presently fit for the *Golden Rule*; and therefore instead of the true Number which is sought: Suppose any Number *Great* or *Small*, and make trial of it, whether it resolve the *Question* without any Error; if so, it is the *True Number*: If not, Note what *Error* and whether it be *Too Much*, or *Too Little*; if *Too Much* mark it thus +, but if *Too Little*, thus —.

Then suppose, again, another Number, (it imports not whether it be nearer or farther off) and try as before, and mark that *Error* also with + or —, according as you find it to be, either *More* or *less*, And then Work according to this following

RULE.

Multiply the first Position, *by the second* Error, *and the second* Position *by the first* Error, *and* (if the Errors be both + or both —) Substract *the* Lesser *Product from the* Greater, *and keep the* Remain *for a* Dividend; *and the* Difference *of the* Errors *for the* Divisor; *the* Quotient *of that Division is the* True Number *required*.

But if the Errors *be one* +, *the other* —, *the* Sum *of the* Products *added together must be the* Dividend: *And the* Sum *of the* Errors, *the* Divisor; the rest of the Work is the same as before.

Question I. A Man is to drive 48 young *Turkies* 40 Miles, and for every *Turkey* which comes alive to the end of the Journey, he is to receive 3 *d.* but for every one which dies by the way, he is to pay 6 *d.* At the end he received 72 *d.* How many died by the way?

Let

Let the First Supposition be; That, by the way, there Dyed Twenty:

	d.
For them he was to pay Twenty Six-Pences, or	120
And for 28 which Lived he was to Receive 28 Three-Pences, or	84
So he paid more than he receiv'd	36
But he should have gotten	72
Add:	108

Wherefore the First Error is ——— 108

Let the Second Supposition be; That, by the way, there Dyed Ten:

	d.
For them he was to pay Ten Six Pences; Or	60
And for the 38 which Lived, he was to Receive Nine Shillings Six Pence, Or	114
The Difference is	54
But it should be	72

So the Second Error is ——— 18

Now,

$$\left.\begin{matrix}20\\10\end{matrix}\right\}\text{Multiplied by}\left\{\begin{matrix}18\\108\end{matrix}\right\}\text{Produceth}\left\{\begin{matrix}360\\1080\end{matrix}\right.$$

The Difference ——— 720 Dividend.

Also

The Difference of the Errors 108 and 18, is 90, Divisor. And 720, divided by 90, gives in the Quotient 8, which is the True Number which Dyed by the Way. As here appears:

	d.
Eight that Dyed, he Paid 4 *s.* or	48
Forty that Lived, he Received 10 *s.*	120
The Difference	72

Question II. If it were required to make up a pound *Sterling* of Shillings and Groats only; and so as the Number of Groats may be to the Number of Shillings, as 7 to 1: How many Shillings must there be?

First, suppose the Shillings to be 4 *Shillings*
then the Groats must be equal to 16 *s. viz.* 48 *Groats*
but the Shillings taken 7 times
are 28, to which 48 should be equal, but is + 20

Secondly, suppose the Shillings 2
then the Groats (making 18 *s.*) are 54
which should be equal to 7 times 2, but is + 40
Multiply 4 by 40, Product is 160, then
Multiply 2 by 20, the Product is 40, which taken from 160,
rests for the *Dividend* 120
And the difference of Errors is 20

Lastly, 120 divided by 20, the Quotient is 6
The Number of Shillings therefore is 6
And the Number of Groats is 42
For as 7 to 1, so is 6 times 7 which is 42, to 6 times 1, which is 6: So the Work is done.

Question III. If there be 4 several weights, *A. B. C. D.* of which *D.* is 24 Ounces, and *C.* is double to *B.* and triple to *A.* and *D.* with twice *A.* is double to *C.* and quadruple to *B.* How much doth every one of these Weights weigh?

Oun.
First, suppose *A.* to be 8
then *D.* with twice *A.* is 24, and 16, that is 40,
of which *C.* being the half is 20, and *B.* 10.
Now thrice *A.* is 24, to which *C.* should be equal, but is — 4

Secondly, let *A.* be supposed 4
then *D.* more, twice *A.* is 32 and *C.* 16
and *B.* is 8, but thrice *A.* is 12, to which 16
should be equal, but is + 4

Then

Then 8 multiplied by 4, gives 32, and 4 by 4, produceth 16: Both these Products give 48 for the Dividend: And the sum of the Errors (because the first is —, the other +) gives 8 for the Divisor, and the quotient will be 6, to which *A.* is equal, and twice *A.* more *D.* is 36, of which *C.* being half is 18, and *B.* is 9, and thrice *A.* is equal to *C.* namely 18, and all right.

Whereas the first Error is equal here to the second, it follows that the Positions were equally False; and therefore their difference which is 4, being parted into two equal Parts, 2 and 2, if 2 be taken from 8, the remain is the true Number 6, or if 2 be added to 4, (which was the second position) the sum will be also 6.

And further, whensoever the *Errors* be one +, the other —, though they be not Equal; yet then, if the *Difference* between the Positions be parted into two Parts, which are in Proportion one to another, as the two Errors are one to another respectively: Then if the *first Part* be taken from the *first Position* (if that be the *Greater*) or added to it (if it be the *Less*) the same Number required is thereby had.

As, let the last Question be resumed,	
And let the First Position for *A.* be	15
Then the first Error will be	— 18
Then let the Second Position be	3
And so the second Error will be	+ 6
And the difference of Position is	12

Which divided into two Parts 9 and 3, which have that Proportion one to another as have the Errors 18 and 6, then if the first part 9, be taken from the first Position 15, there remains the true Number 6. Or else if the second part 3, added to the second Position 3: Thereby also is made the true Number 6.

The way of parting 12 (or any other) into two parts proportional with the Errors, is easily done by the *Golden Rule*, thus:

As the sum of the Errors 24,
is to the difference of Position 12;
So is the greater Error 18,
to the greater Part required, namely 9.

Many

Many other Queſtions are in the other Books exemplified and wrought by this Rule; but ſeeing I intend not to write a great Book; and alſo becauſe ſome of thoſe Queſtions may be reſolved without this *Rule*, I will add no more: Only mention one of thoſe Queſtions.

If there be a Ciſtern with 4 Cocks, which holds 8 Barrels of Water, and the firſt Cock will run it all out in 6 hours, the ſecond in 4, the third in 3, and the laſt in 2 hours: In what time ſhall all of them run it out?

If the firſt in 6 hours runs	8
the ſecond in the ſame time would run	12
the third	16
the laſt	24
In all	60

Then ſay, if 60 require 6, what 8?

The anſwer $\frac{48}{60}$, that is $\frac{4}{5}$ of an hour; in which time all the 4 Cocks together would run out all the 8 Barrels of Water.

THE

THE RULE OF CERES and VIRGINUM.

THIS is the most uncertain, and unnecessary Rule in *Arithmetick*; being seldom used except in *Sporting Questions* to puzzel young beginners, with easie Problems: Such as follow.

Question I. A Caterer bought 8 Birds of two sorts, as Geese and Hens for 20 *s.* the Geese cost 4 *s.* a Piece, the Hens 2 *s.* a piece; How many did he buy of each sort?

This may be done by the *Rule of False*; and also thus: Multiply the whole number 8, into the least price 2, it produceth 16, which taken from the whole price 20, rests 4 for a *Dividend*; which divided by 2, which is the difference of the particular prices the Quotient is 2, for the Number of *Geese*; and 6 must be for the *Hens*: The Proof is easie.

			s.		*s.*
For	2 Geese	at	4	gives———	8
	6 Hens	at	2	gives———	12
	8				20

Question II. If 21 Persons, *Men*, *Women* and *Children* spend 26 Shillings; so that every *Man* pays 2 *s.* every *Woman* 1 *s.* every *Child* 6 *d.* How many is there of each sort?

THE RULE.

Multiply the Number of Persons *by the* least Expence, *and take the Product of it from the* whole Expence, *the rest shall be the* Dividend; *which divided by the* difference *betwixt the* greatest *and*

and leaſt *particular* Expences : *The Quotient is a* Number, *which the Number of Men (or they which ſpend moſt) comes near to; but cannot exceed: Or if the ſaid* Dividend *be divided by the ſum of the* greateſt *and* leaſt *Expences, the Quotient is a Number, than which the Number of Men (or thoſe which ſpend moſt) cannot be much leſs.*

So here 21 Multiplyed by 6 *d.* that is, by $\frac{1}{2}$, the Product is 10 $\frac{1}{2}$, which taken from 26, reſts 15 $\frac{1}{2}$, for the Dividend: And then taking $\frac{1}{2}$ from 2, reſts 1 $\frac{1}{2}$ for the Diviſor, and the Quotient is $\frac{31}{3}$, which is ſomething more than 10; the Number of Men therefore muſt be but 9.

Then turn the Dividend, and the Diviſor both into whole Numbers, by multiplying them by the Common Denominator 2, ſo they reduced will be 31 and 3, as before is to be ſeen in the Quotient.

Multiply the Diviſor 3, by 9, (which is the Number of *Men*) the Product is 27, which taken from 31, (which is the reduced Dividend) the remain is 4, for the Number of *Women* ; and the *Children* muſt be 8,

Example I.

9 Men at 2 *s.* each	18 *s.*
4 Women at 1 *s.* each	4
8 Children at 6 *d.* each	4
In all 21	26

But the number of Men may be alſo 8, which multiplied by the reduced Diviſor 3; the product is 24, which taken from 31, the remain is 7 for the Women; and then the Children muſt be 6.

Example

Example II.

8 Men at 2 *s.* each	16
7 Women at 1 *s.* each	7
6 Children at 6 *d.* each	3
In all 21	In all 26

Or the number of Men may be 7, which multiplied by 3, produceth 21, which taken from 31, remains 10 for the Women, and 4 Children.

Example III.

7 Men at 2 *s.* each	14 *s.*
10 Women at 1 *s.* each	10
4 Children at 6 *d.* each	2
In all 21	In all 26

So there are already ſeen 3 various ſolutions of this Queſtion, which make this Rule the leſs to be regarded. But further, the number of Men may be 10, and not more, for if you put them 11, that multiplied by 3, produceth 33, which is greater than 31, from which it ſhould be taken, but I ſay it may be 10, and then there is only one Woman, and ten Children: this confirms the former part of the Rule.

Now for the latter part, if the Dividend 31 be divided by the ſum of the two extream expences (reduced by doubling as the Dividend is) 4, the Quotient will be $7\frac{3}{4}$. And the Men may be 7, as hath been ſhewed; but they may be alſo but 6, and fewer they cannot be: As 6 Men, 13 Women, and 2 Children; for if you put them 5, that multiplied by 3, produceth 15, which taken from 31, there remains 16 for the Women, and ſo there ſhould be no Children which is contrary to the Suppoſition.

And further, becauſe the Quotient was $7\frac{3}{4}$, the Number of Men might be ſo, if pure Arithmetical Diviſion be only regarded: And then the Women alſo are in Number $7\frac{3}{4}$, and the Children $5\frac{1}{2}$, as may eaſily be tryed; I need not exemplifie it.

Queſtion III. If there be an Exhibition of 900 *l. per Annum* to 30 Perſons: Some *Clerks*, ſome *Meſſengers*, and ſome *Doorkeepers*, at 60 *l.* each *Clerk*, 40 *l.* each *Meſſenger*, and 20 *l.* each *Doorkeeper*; how many muſt there be of each ſort?

Multiply (according to the Rule) 30 by 20, the Product is 600, which taken from 900, remains 300 for the Dividend; and 60 want 20, that is 40, for the Divisor: And the Quotient is $7\frac{1}{2}$, and more the *Clerks* cannot be; Also divide by 60 more 20, that is 80, Quotient is $3\frac{3}{4}$, and much fewer the *Clerks* cannot be.

Not to stand upon the Fractions (in this case of dividing Men) the Clerks may be 7, 6, 4, 3: And the Messengers 1, 3, 5, 7, or 9, and the Doorkeepers 22, 21, 20, 19, or 18, that the Clerks cannot (in whole Numbers) be more than 7, or less 3, may thus be proved; First, let them be 8, than 8 times 40 is 320, which is more than 300, out of which it should be taken: Secondly, let them be 2, then 2 times 40 is 80, out of 300 remains 220, which divided by 20, gives the Quotient 11, for the Messengers; so the Clerks and Messengers being 13, the Remain thereof to 30, namely 17, must be Doorkeepers.

But,

2	Clerks at 60 *l.* each	120 *l.*
11	Messengers at 40 *l.* each	440
17	Doorkeepers at 20 *l.* each	360
In all 30		In all 920

Which is 20 *l.* too much, therefore the Clerks cannot be two.

Note,

It may be asked, why the Remain 220 should be divided by 20: Whereas the like Remain in the former Example, namely, 16, was taken (without any Division) absolutely for the Number of Women, or Middle Number? I answer, although the greatest or first Number being found, (as here to be 2) the residue of 2 to 30, might be rightly parted into two fit Parts in the same manner as the first Question of this Rule was resolved, or else by the *Rule of False:* Yet to give further satisfaction, the cause of this is, the difference betwixt the two lesser Expences, was there $\frac{1}{2}$, which (before the Division was reduced to 1, which neither multiplies nor divides any Number, but leaves it the same, Whereas, in this last, the middle Expence (or Exhibition) being 40, and the least 20, the difference of them was 20, by which dividing the Remain of

of the laſt Subſtraction: The Quotient is ever the Number of the middle Perſons. Which may ſerve as an addition to the Rule, where the ſorts of things are but three.

Queſtion IV.

If there be 10 Perſons of four ſeveral Countries, *Engliſh French, Dutch* and *Spaniſh*, to pay a Debt of 1000 *l.* So that every *Engliſh-Man* pays 50 *l.* every *French-Man* 70 *l.* every *Dutch-Man* 130 *l.* and every *Spaniard* 150 *l.* How many is there of each?

The Dividend (according to the former Rule) is 500.

Now to make the Diviſor, take his ſum that pays leaſt (namely 50) out of each of the other three 150, 130, and 70, and the Remains will be 100, 80 and 20.

Add the firſt and leaſt for the Diviſor, it is 120.

And the Quotient will be $4\frac{2}{12}$, and the *Spaniards* cannot be more.

Secondly, add the firſt and ſecond together for the Diviſor, it is 180, and the Quotient is $2\frac{14}{18}$, and the *Spaniards* cannot be leſs.

I mean, they cannot be much more than 4, or leſs than 2: And therefore, ſeeing any one Solution will ſerve, let them be 3, and by that multiply 100, and take the Product out of 500, there remains 200 for a ſecond Dividend, which divided by (the ſecond Remain) 80, the Quotient is $2\frac{1}{2}$: Therefore the *Dutchmen* are 2, which multiplied by 80, make 160; take that out of 200, there remains 40 for a third Dividend: Which divided by (the Third Remain) 30, the Quotient is 2 for the *Frenchmen* alſo; and conſequently the *Engliſh* muſt be 3, becauſe all of them are 10: But the *Spaniards* may be alſo 4 or 2.

Example.

4	Spaniards at 150 *l.* each	600
1	Dutch-man at 130 *l.*	130
1	French-man at 70 *l.*	70
4	Engliſh at 50 *l.* each	200
10	In all	1000

P 2 2 Spaniards

2 Spaniards at 150 *l.* each		300
3 Dutch at 130 *l.* each		390
3 French at 70 *l.* each		210
2 English at 50 *l.* each		100
10	In all	1000.

The reaſon why the *Spaniards* and *Engliſh*, as alſo the *Dutch* and *French* are equal in Number, is becauſe their Payments differ equally from 100, which is the Mean Sum with which 10 Men ſhould pay 1000 *l.* and making it ſo, this Queſtion, and many other of this Nature, may be anſwered by the Rule of *Alligation*: Thus,

$$100 \left[\begin{array}{l} 50 \\ 30 \\ 30 \\ 50 \end{array}\right] 10 \qquad \overline{160}$$

If 160 give 10, what 50? Anſwer is $3\frac{30}{190}$, (that is in this caſe 3) for the *Spaniards*, and as many for the *Engliſh*, becauſe their reſpective differences from 100, the one 50 more; the *Engliſh* 50 leſs, are equal.

And alſo, becauſe the other two differences 30 and 30 are equal; the Number of the *French* is equal to the Number of the *Dutch*.

But both thoſe Numbers together are 4, becauſe 3 *Spaniards*, and 3 *Engliſh*, taken out of 10, the Remain muſt be 4.

Wherefore the Number of the *French* is 2, and the *Dutch* alſo are 2.

Or thus;

$$100 \left[\begin{array}{l} 30 \\ 50 \\ 50 \\ 30 \end{array}\right] 10 \qquad \overline{160}$$

Accounting the Men in the ſame order, as before.

If 160 require 10, what 30?

Anſwer,

Anſwer is $1\frac{160}{160}$, (that is, in this caſe 2) for the *Spaniards*, and conſequently 2 *Engliſh*; and therefore the *French* and *Dutch* each 3.

But where any one of the particular ſums is equal to the Mean Sum, there this cannot ſo well be done by *Alligation.*

Example.

If one ſhould buy 12 Loaves of Bread for 12 *pence* ſo that ſome might be *two-penny*, ſome *penny*, ſome *half-penny*, and ſome *farthing* Loaves: And it be required to know how many he muſt buy of each?

Then becauſe of 12 Loaves for 12 *pence*, the Mean Price is 1, but one of the particulars being alſo 1, there ſhould be no penny Loaves, becauſe there is no difference between the Mean *Price*, and a *Penny*.

But it may be found by the Rule of *Ceres* and *Virginum*, to be either.

4 Two-penny Loaves	8 Pence
2 Penny Loaves	2 Pence
2 Half-penny Loaves	1 Penny
4 Farthing Loaves	1 Penny
In all 12 Loaves.	In all 12 Pence.

Or elſe,

3 Two-penny Loaves	6 *d.*
4 Penny Loaves	4
3 Half-penny Loaves	$1\frac{1}{2}$.
2 Farthing Loaves	$0\frac{1}{2}$.
In all 12	In all 12

To

POSTSCRIPT.

To *this Rule of* Ceres *and* Virginum, *(to supply a Vacancy) I have added by way of* Postscript; *some few* Enigmatical Questions *with their* Answers *affixed: Leaving the manner how to resolve them, to the Ingenuity of the Learner.*

Question I. There are four several Measures, as *A*, *B*, *C*, *D*: Of which, *D* holds 24 Pints, *C* holds as much again as *B*; and 3 times as much as *A*: And *D*, with twice *A*, will hold twice as much as *C*; and 4 times as much as *B*. How many Pints doth each of these Measures hold severally?

Answer,	*A* holds	6	Pints.
	D	24	
	C	18	
	B	9	

So that *C* holds as much again as *B*, and 3 times as much as *A*; and *D* with twice *A*, holds as much as *C*; and four times as much as *B*.

Question II. One took a sum of Money with him, and went to a Gaming-House; where, at his first Game he doubled the Money he brought with him: At the second Game he lost 120 *l.* At the third Game he doubled the Money he had Remaining: at the fourth Game he lost 120 *l.* and had no Money left. What was the Sum of Money that he took with him?

Answer, 90 *l.* Forasmuch as he lost 120 *l.* at the fourth Game's End, and had then no Money left: It is evident that he had but 120 *l.* at the end of the third Game: Which was the double of the Money which he had at the end of the second Game, namely, 60 *l.* which with 120 *l.* that he lost at the end of the second Game makes 180 *l.* which was the double of the Money that he brought with him, namely, 90 *l.* which may be thus Proved.

	l.
The Money brought with him	90
The first Game doubled it	180
The second he lost 120 *l.*	120
Then he had left	60
At the third he doubled it	120
At the fourth he lost	120
There Remains	000

Question III. Two Persons *James* and *Paul*, had between them a certain Number of Sheep in two Droves; *James* said to *Paul*, if you put 70 of your Sheep into my Drove, I shall have three times as many

ny Sheep as you have: But *Paul* said to *James*, if you put 70 of your Sheep into my Drove, I shall have 5 times as many as you. How many Sheep had each of them?

Answer, *James* had 110, and *Paul* had 130 —— For, if you take 70 from *Paul*, and add them to *James*; then *James* will have 180, and *Paul* but 60, which is but one third part of what *James* has —— But if you take 70 from *James* and give them to *Paul*; then *Paul* will have 200, and *James* but 40, which is but one fifth Part of what *Paul* has.

Question IV. A Father gave to his Eldest Son 252 Crowns, and to his Youngest he gave but 28 Crowns: And to every Son successively, from the Youngest, he gave 28 Crowns more than to the preceding. How many Sons had the Father? And how many do the Crowns amount unto?

Answer, He had 9 Sons. And the Number of Crowns 1260.

Question V. A Drover driving of Sheep before him: One meets him, and says, good speed Friend with thy 20 Sheep. Nay says the Drover, I have not 20 Sheep; but if I had as many more; and half so many more; and 2 Sheep; and half a Sheep; then I should have 20 Sheep. How many Sheep had he?

Answer, 7 Sheep: For 7 and 7 is 14, and half 7 is 3 and a half, that is 17 and a half, and 2 and a half is 20.

Question VI. There is 273 *l.* to be divided amongst four Persons: *Andrew*, *Bennet*, *Christopher* and *Daniel*: Of which, *Andrew* is to have a part unknown; *Bennet* is to have twice so much as *Andrew*, and 30 *l.* more; *Christopher* is to have 3 times as much as *Andrew*, wanting 52 *l.* And *Daniel* is to have 5 times as much as *Andrew*, and 20 *l.* more. How much of the 273 *l.* must each Person have.

				l.
Answer,	*Andrew*	must have		25
	Bennet			80
	Christopher			23
	Daniel			145
			In all	273

Question VII. There was a May-pole, which in a windy Night was broken, so that the top thereof lit upon the Ground, at 30 Foot distance from the bottom thereof; and the Piece broken off was 50 Foot long: I demand how long the May-pole was in all; and how long was the standing Part?

Answer,

Anſwer,	Foot
The whole Length was	90
The Piece ſtanding	40

Theſe few Queſtions ſhall ſuffice in this place; ſuch as are delighted in this kind of Diverſion may peruſe my Recreations.

The End of the First PART.

Decimal Arithmetick.

The Second PART.

CONTAINING
The Grounds and Reaſon thereof: And applyed to Practice;

I. In all the Rules of Vulgar Arithmetick.
II. In Intereſt, Simple and Compound.
III. In diſcount and Rebate of Money.
IV. In Equation of Payments.
With Tables of all theſe.

ALSO,
In the Extraction of the Square and Cube Roots.
In the Menſuration of Superficies and Solids.
And in the Works of ſeveral Artificers: As,

Joyners,	*Carpenters,*
Painters,	*Brick-Layers,*
Plaiſterers,	and
Glaſiers,	*Maſons.*

Whereby, this Decimal Arithmetick, will be as ſerviceable to all of ſuch Profeſſions; As the foregoing Part was for Merchants and other Tradeſmen.

By *William Leybourn*, *Philomath*.

LONDON:
Printed for *Awnſham* and *John Churchill* at the *Black-Swan* in *Pater-Noſter-Row*, 1700.

Decimal Arithmetick.

The Second PART.

CONTAINING

The [illegible] and Reason thereof [illegible] and applyed [illegible] Practice:

I. [illegible] the Rules [illegible] Multiplication [illegible]
II. [illegible] and [illegible]
III. [illegible] and [illegible]
IV. [illegible] of [illegible]
With [illegible] of all [illegible].

[illegible]

[illegible] Square and [illegible] Roots [illegible]

[illegible] and Arit[illegible]

[illegible]

[illegible]

and

[illegible]

[illegible] Decimal [illegible] to [illegible] was [illegible]

[illegible]

LONDON,

Printed for [illegible] and [illegible] at the [illegible]

[illegible]

Decimal Arithmetick.

PART II.

INTRODUCTION.

THIS *Second Part*, which Treateth of *DECIMAL ARITHMETICK*: I ſhall divide into Four *Sections*. In the *Firſt*, ſhall be Taught how to *Reduce Vulgar Fractions* into *Decimal Parts*, or *Fractions*; and thereby to make *Tables* (if you pleaſe) to expreſs the ſeveral *Denominations* of the *Coins*, *Weights* and *Meaſures* of your own or other *Countries*, in *Decimal Parts* or *Fractions*: And how to make uſe of ſuch *Tables* upon all occaſions.

The *Second Section*, contains *Notation*; and how to Work all the *Rules* of *Vulgar Arithmetick*, (treated of in the Firſt Part) *Decimally*: And to Extract the Square and Cube Roots.

The *Third Section* treateth of *Simple* and *Compound Intereſt*— *Diſcount* and Rebate of *Money*— *Æquation* of *Payments*, &c. *Purchaſe* of *Annuities*, *Valuation* of *Leaſes*, and the like, with *Tables* of them ready Computed.

The *Fourth Section*, Teacheth how to *Meaſure Superficies* and *Solids*; As *Board*, *Glaſs*, *Land*, *Pavement*, &c. And *Solids*, as *Stone*, *Timber*, *Spheres*, *Bullets*, *Columns*, &c. And alſo of the *Works* of the ſeveral *Artificers* relating to *Building*; as *Bricklayers*, *Carpenters*, *Maſons*, *Joyners*, *Painters*, *Glaſiers*, &c.

SECT. I.

Of the Nature of Decimal Arithmetick.

I will not in this Place inſiſt upon the *Excellencies* or *Antiquity* of this kind of *Arithmetick*; but fall immediately upon the *Practice* of it: Which I ſhall do by the Solution of ſeveral *Problems*.

PROBL. I.

A *Vulgar Fraction* being given; how to Reduce the same into a *Decimal Part* or *Fraction*.

RULE.

To the Numerator *of the Fraction given, add what Number of* Cyphers *you please; then divide the* Numerator *by the* Denominator, *the* Quotient *shall be the* Decimal Fraction *requir'd.*

Example I. Let it be required to reduce $\frac{4}{17}$ into a *Decimal.*
First, to the Numerator 4, add five Cyphers, so will it be 400000, divide this number by the Denominator 17, and the Quotient will be 23529, which is the Decimal required.

¶ And here Note, that *Decimal Fractions* are not written in a smaller Figure with a Line between them, as *Vulgar Fractions* are, but of the same Figure, only there must be a Comma or Point put between the whole Number and the Fraction, and that is the distinction: And so the former Decimal must be Written thus, .23529.

Example II. If you would express $235\frac{4}{17}$ in a Decimal way, it must be Written as followeth.

By the last Example you find that $\frac{4}{17}$ reduced to a Decimal, was .23529, therefore $235\frac{4}{17}$ must be Written thus: 235, 23529. In *Decimal Fractions* the *Numerator* is only express'd, and the *Denominator* only intimated; for this *Rule* is general, *Of how many Figures soever the* Numerator *of a Decimal Fraction doth consist, of so many* Cyphers *with a* Unite *before them, doth the* Denominator *of the same Fraction consist.* So this Decimal 12, 625, if it were Written in a Vulgar way, would be $12\frac{625}{1000}$ but in a Decimal, only 12, 625, the Comma or Point between 12 and 625 distinguisheth the *whole Number* from the *Fraction*, and the *Fraction* 625 consisting of three Figures, intimates that the *Denominator* thereof must consist of three Cyphers and an Unite before them; so the Decimal before express'd, 235, 23529, if it were Written in the Vulgar way, would be $235\frac{23529}{100000}$.

But it sufficeth to Express in Decimals, the *Numerators* only, and omit the *Denominators*, the *Denominators* of all *Decimal Fractions* being either 10, 100, 1000, 10000, 100000, &c. according to the Number of figures contain'd in the *Numerators.*

Accor-

According to this Rule, you shall find that

$\frac{4}{5}$	will be in Decimals by adding 5 Cyphers.	5. 80000 or only 8 thus, . 8
$12\frac{3}{7}$		12.42857.
$132\frac{9}{17}$		132.52941.

And by this means, all manner of *Fractions* of *Coins*, *Weights* and *Measures*, may be reduced from *Vulgar Fractions*, to *Decimal Fractions*; as by the next Problem will appear.

PROBL. II.

How to express English-Coin *in Decimal Numbers.*

Let it be required to express 9 Shillings (which is $\frac{9}{20}$ of a pound *Sterling*) in a Decimal; To the Numerator 9 add two Cyphers, making it 900, which divide by twenty, the Quotient is 45, for the Decimal of 9 *s.* So the Decimal of 13 *s.* will be 65, and so for any Number of Shillings.

¶ Here Note, that in the Reduction of Vulgar Fractions into Decimals, that many times the first, second or third Places of the Decimal Fractions are Cyphers, as in the following Table, the Decimal of one Farthing is .00104167, and the reason is, because if you reduce $\frac{1}{960}$ into a Decimal (for one farthing is the 960th part of a pound *Sterling*) you shall by adding of six Cyphers to the Numerator find the Quotient to be 104167, but two Cyphers must be placed before it; because dividing 1000000 by 960, the place of Unites in the Divisor at the first demand extendeth unto the third Cypher in the Dividend for in reducing of Vulgar Fractions to Decimals, this is

A general RULE.

That if the place of Unites *in the* Divisor, *at the first Demand, extend but unto the first of the* Cyphers *annexed to the* Numerator *of the Fraction, there must be no* Cypher *put before in the Quotient, but if the place of* Unites *extend unto the second* Cypher *added, then one* Cypher *must be placed before in the Quotient, if unto the third* Cypher, *then two* Cyphers *must be placed before in the Quotient*, &c.

According to which Rule, if you make tryal you shall find that the Decimal of 7 *s.* will be 35, the Decimal of 5 *d.* will be .0208333, the Decimal of two Farthings will be .00208333, as in the Table.

By these Rules last delivered are the ensuing Tables of *English Money*, *Weight* and *Measure* compos'd, and the like may be done for a Foreign *Coin*, &c. according as every Mans occasion shall require.

The

THE TABLE OF *English Coins* in Decimals.

English Coin.

Sh.	19	.95
	18	.9
	17	.85
	16	.8
	15	.75
	14	.7
	13	.65
	12	.6
	11	.55
	10	.5
	9	.45
	8	.4
	7	.35
	6	.3
	5	.25
	4	.2
	3	.15
	2	.1
	1	.05
D.	11	.04583333
	10	.04166667
	9	.0375
	8	.03333333
	7	.02916667
	6	.025
	5	.02083333
D.	4	.01666667
	3	.0125
	2	.0833333
	1	.04166667
F.	3	.003125
	2	.00208333
	1	.00104167

Troy Weight in Decimals.

O.	11	.91666667
	10	.83333333
	9	.75
	8	.66666667
	7	.58333333
	6	.5
	5	.41066667
	4	.33333333
	3	.25
	2	.16666667
	1	.08333333
P.W	19	.07916667
	18	.075
	17	.07083333
	16	.06666667
	15	.0625
	14	.05833333

P. W.

P. W.	13	.05416667
	12	.05
	11	.04583333
	10	.04166667
	9	.0375
	8	.03333333
	7	.02916667
	6	.025
	5	.02083333
	4	.01666667
	3	.0125
	2	.00833333
	1	.00416667

Gr.	23	.00399395
	22	.00381944
	21	.00364583
	20	.00347222
	19	.00329861
	18	.003125
	17	.00295139
	16	.00277778
	15	.00260417
	14	.00243056
	13	.00225694
	12	.00208333
	11	.00190972
	10	.00173611
	9	.0015625
	8	.00138889
	7	.00121528
	6	.00104106
	5	.00086805
	4	.00069444
	3	.00052083
	2	.00044722
	1	.00017311

Averdupois great Weight in Decimals.

3 *qu.*		.75
2 *qu.*		.5
1 *qu.*		.25
Pounds	27	.24107142
	26	.23214285
	25	.22321428
	24	.21428571
	23	.20535714
	22	.19642857
	21	.1875
	20	.17857143
	19	.16964286
	18	.16071428
	17	.15178571
	16	.14285714
	15	.13392857
	14	.125
	13	.11607143
	12	.10714286
	11	.09821428
	10	.08928571
	9	.08035714
	8	.07141857
	7	.0625
	6	.05357143
	5	.04464286
	4	.03571428
	3	.02678571
	2	.01785714
	1	.00892857

Oun.	15	.00837053
	14	.0078125
	13	.00725446
	12	.00669643
	11	.00613839
	10	.00558035
	9	.00502232
	8	.00446429
	7	.00390625
	6	.00334821
	5	.00279018
	4	.00223214
	3	.00167411
	2	.00111607
	1	.00055804

3 *qu.*	*Ounce.*	.00041853
half	*Ounce.*	.00027902
1 *qu.*	*Ounce.*	.00013951

Averdupois

Averdupois little Weight in Decimals.

Ounces	15	.937
	14	.875
	13	.8125
	12	.75
	11	.6875
	10	.625
	9	.5625
	8	.5
	7	.4375
	6	.375
	5	.3125
	4	.25
	3	.1875
	2	.125
	1	.0625
Drams.	15	.05859375
	14	.0546875
	13	.05078115
	12	.046875
	11	.04296875
	10	.0390625
	9	.03515265
	8	.03125
	7	.02734375
	6	.0234375
	5	.01953125
	4	.015625
	3	.01171875
	2	.0078125
	1	.00390625
3 qu.		.00292969
half.		.00195312
1 qu.		.00097656

the decimal

Liquid Measures in Decimals.

Pints.	7	.875
	6	.75
	5	.625
	4	.5
	3	.375
	2	.25
	1	.125
3 qu.		.09375
half.		.0625
1 qu.		.03125

Dry Measures in Decimals.

Bushels.	7	.875
	6	.75
	5	.625
	4	.5
	3	.375
	2	.25
	1	.125
Pecks.	3	.09375
	2	.0625
	1	.03225
3 qu.		.0234375
half.		.015625
1 qu.		.0078125
Pints.	3	.0058594
	2	.0039063
	1	.0019531

Long Measures, the Integers being Yards and Ells in Decimals.

Qua.	3	.75
	2	.5
	1	.25
Nail.	3	.1875
	2	.125
	1	.0625

3 qu.

3 *qu.*	.046875
half	.03125
1 *qu.*	.015625

Time in Decimals.

Mo.	11	.916667
	10	.833333
	9	.75
	8	.666667
	7	.833333
	6	.5
	5	.416667
	4	.333333
	3	.25
	2	.166667
	1	.083333

Da.	30	.082193
	29	.097454
	28	.076714
	27	.073973
	26	.071233
	25	.068495
	24	.065755
	23	.063016
	22	.060274
	21	.057536
	20	.054795
	19	.052055
	18	.049316
	17	.046577
	16	.043837
	15	.041097
	14	.038357
	13	.035617
	12	.032877
	11	.030137
	10	.027397
	9	.024657
	8	.021918
	7	.029178
	6	.016438
	5	.013698
	4	.010959
	3	.0082192
	2	.0054795
	1	.0027397

Dozens in Decimals.

Do.	11	.9166667
	10	.8333333
	9	.75
	8	.6666667
	7	.5833333
	6	.5
	5	.4166667
	4	.3333333
	3	.25
	2	.1666667
	1	.0833333

Pa.	11	.076388
	10	.0694444
	9	.0625
	8	.0555555
	7	.0486111
	6	.0416667
	5	.0347222
	4	.0277778
	3	.0208333
	2	.0138889
	1	.0069444

The use of the foregoing TABLES.

THe *Tables* preceding are in Number nine; The first being of *English-Coin*; The second of *Troy Weight*; The third of *Averdupois great Weight*; The fourth of *Averdupois little Weight*; The fifth of *Liquid Measures*; The sixth of *Dry Measures*; The seventh of *Long Measures*; The eighth of *Time*, and the ninth of *Dozens:* These several Tables are made by the Rules immediately going before them, and their use is to express in *Decimal Numbers* either *Money*, *Weight*, or *Measure*, as by the following *Propositions* will appear.

PROBL. I.

How by the Table *to express* English-Coin *in* Decimals.

The first of the nine Tables is for this purpose; therefore if you would express either *Shillings*, *Pence* or *Farthings* in *Decimal Numbers*, you must repair to the first Table, which is of *English-Coin*, and there against 13 *Shillings* you shall find .65, which is the Decimal of 13 shillings, also against seven pence you shall find .02916667, which is the Decimal representing 7 pence: Also against 2 farthings you shall find .00208333, which is the Decimal answering 2 Farthings, and the like is to be done for any other Number of Shillings, Pence or Farthings.

But if it be required to find the Decimal of divers Denominations of Coin in one Sum, as of Shillings, Pence and Farthings together, you must add the Decimals of all the particulars together, and the sum of them shall be the Decimal sought.

Examples.

If you would know the Decimal of 13 *s.* — 7 *d.* — 2 *q.* in one Number: In the Table you shall find that,

$$\text{The Decimal of} \left\{\begin{array}{r} 13\,s. \\ 7\,d. \\ 2\,q. \end{array}\right\} \text{is} \left\{\begin{array}{l} .65 \\ .02916667 \\ .00208333 \end{array}\right.$$

The Decimal of 13 *s.* 7 *d.* 2 *q.* .68125000

PROBL.

PROBL. II.

How by the Table *to expreſs* Troy Weight *in* Decimals.

The ſecond Table is of *Troy Weight*, the ſeveral Denominations whereof are *Ounces*, *Penny-weights*, and *Grains*: So that by the Table you ſhall find that the Decimal belonging to five Ounces is .41666667, the Decimal belonging to 17 Penny-weight is .07083333, and the Decimal belonging to 13 Grains is .00225694, and ſo of any other number of Ounces, Penny-weights and Grains ſeverally.

But if it were required to expreſs theſe (or any other) ſeveral Denominations in one Decimal Fraction, then you muſt (as before you did for Money) take out of the Table the ſeveral Decimals belonging to the reſpective Quantities, and add them together, ſo ſhall the Sum of that Addition be the Decimal ſought.

Example. If it were required to find a Decimal which ſhould repreſent 5 Ounces, 17 Penny-weight, 13 Grains.

$$\text{The Decimal of} \left\{ \begin{array}{l} 5\ \textit{Ou.} \\ 17\ \textit{P. w.} \\ 13\ \textit{Gr.} \end{array} \right\} \text{is} \left\{ \begin{array}{l} .41666667 \\ .07083333 \\ .00225694 \end{array} \right.$$

$$\text{Decimal of } 5\ \textit{Ou.}\ 17\ \textit{P. w.}\ 13\ \textit{Gr.} \qquad .48975694$$

PROBL. III.

How by the Table *to expreſs* Averdupois great Weight *in* Decimals.

The third Table is of *Averdupois great Weight*, the ſeveral Denominations whereof are *Quarters* of *Hundreds*, *Pounds*, *Ounces* and *Quarters* of *Ounces*; thus you ſhall find in the Table, that the Decimal of 3 Quarters of a Hundred is .75, the Decimal of 22 pounds is .19642857, the Decimal of 7 Ounces is .00390625, and the Decimal of 3 Quarters of an Ounce is .00041853, and in this manner you may find the correſpondent Decimal belonging to any number of Quarters, Pounds, Ounces, and parts of Ounces ſeverally.

But if it be required to find one Decimal Number which ſhall repreſent divers Denominations, you muſt firſt find the Decimal belonging to the ſeveral particulars, and add them together, the ſum whereof ſhall be the entire Decimal required.

Example. Let it be required to find a Decimal which ſhall repreſent 3 Quarters, 22 Pounds, 7 Ounces $\frac{3}{4}$ of an Ounce.

$$\text{The Decimal of}\left\{\begin{array}{r}3\ Q.\\22\ P.\\7\ Ou.\\3\ Qu.\end{array}\right\}\text{is}\left\{\begin{array}{l}.75\\.19642857\\.00390625\\.00041853\end{array}\right.$$

$$\text{Decimal of } 3\ Q.\ 22\ l.\ 7\tfrac{3}{4}\ Ou. \qquad .95075335$$

PROBL. IV.

How by the Table *to expreſs* Averdupois little Weight *in* Decimals.

The fourth Table is of *Averdupois little Weight*, the Denominations whereof are *Ounces*, *Drams* and *Quarters* of *Drams*, ſo that the Decimal of 11 Ounces is .6875, the Decimal of five Drams is .01953125, and the Decimal of one Quarter of a Dram is .00097656.

But if it be required to find one Decimal Number, which ſhall repreſent 11 Ounces, 5 Drams and a half.

$$\text{The Decimal of}\left\{\begin{array}{r}11\ Ou.\\5\ Dr.\\1\ Qu.\end{array}\right\}\text{is}\left\{\begin{array}{l}.6875\\.01953125\\.00097656\end{array}\right.$$

$$\text{Decimal of } 11\ Oun.\ 5\tfrac{1}{4}\ Dr. \qquad .70800781$$

PROBL. V.

How by the Table *to expreſs* Liquid Meaſures *in* Decimals.

Becauſe there is ſo great variety of Liquid Meaſures that hardly any two commodities are ſold by the ſame, the difference of the Gallon continually making alteration, we have therefore in this fifth Table made the greateſt Denomination to be one Gallon, the next leſs Denomination being Pints and quarters of Pints, ſo that in the Table you ſhall find the Decimal belonging to three Pints to be .375, and the Decimal belonging to two Quarters, or half a Pint, to be .0625, and ſo for any other.

But for to expreſs Pints and parts of Pints in one entire Decimal Number, you muſt add the Decimals of the ſeveral Denominations together, and their Sum ſhall be the entire Decimal.

So

So if you were to express 3 Pints and a half.

$$\text{The Decimal of} \left\{\begin{array}{l}\text{3 Pints}\\ \text{half a Pint.}\end{array}\right\} \text{is} \left\{\begin{array}{l}.375\\ .0625\end{array}\right.$$

The Decimal of $3\frac{1}{2}$ Pints .4375

PROBL. VI.

How by the Table *to express* Dry measures *in* Decimals.

The sixth Table is of *Dry Measures*, the several Denominations whereof are *Bushels*, *Pecks*, *quarters of Pecks* and *Pints*, so may you find the Decimal of five Bushels to be .625 the Decimal of two pecks to be .0625, the Decimal of three quarters of a peck to be .023437, the Decimal of two Pints to be .0039063. Thus are the correspondent Decimals belonging to the several Denominations found.

But if you would have one number to express 5 Bushels, 2 Pecks, three quarters of a Peck, and 2 Pints,

$$\text{The Decimal of} \left\{\begin{array}{l}\textit{5 Bush.}\\ \textit{2 Pecks}\\ \textit{3 Quar.}\\ \textit{2 Pints.}\end{array}\right\} \text{is} \left\{\begin{array}{l}.625\\ .0625\\ .0234375\\ .0039063\end{array}\right.$$

Decimal of 5 *Bushels* $2\frac{3}{4}$ *Pecks* 2 *Pints*. .7148438

PROBL. VII.

How by the Table *to express* Long measures *in* Decimals.

The seventh Table is of *Long Measures*, the Integers being *Yards* and *Ells*: and the lesser Denominations are *Quarters* of *Yards* or *Ells*, *Nails*, and quarters of *Nails*. So you may find in the Table that the Decimal of three quarters of a Yard, or an Ell, is .75, the Decimal of two Nails, is .125, and the Decimal of one quarter of a Nail is .015695.

But if you would have one number to express 3 quarters of a Yard, or an Ell, two Nails and one quarter of a Nail;

$$\text{The Decimal of} \left\{\begin{array}{l}\textit{3 Quar.}\\ \textit{2 Nails.}\\ \textit{1 q. of a Na.}\end{array}\right\} \text{is} \left\{\begin{array}{l}.75\\ .125\\ .015625\end{array}\right.$$

Decimal of 3 *Qu.* 2 *N.* 1 *Qu.* .890625

PROBL.

PROBL. VIII.

How by the Table *to express the parts of* Time *in* Decimals.

Time is usually divided into *Years*, *Months* and *Days*: So the eighth Table which is of *Time*, consisteth of these two Denominations, *Months* and *Days*, you may find that the Decimal of 5 Months is .41667, the Decimal of 26 days is .071233. These are the principal Decimals, but the compound Decimal Number representing 5 Months, 26 days, is .487900, as you shall find, if you add .071233, which is the Decimal of 26 days, to .416667, which is the Decimal of five Months.

PROBL. IX.

How by the Table *to express* Dozens *in* Decimals.

The last Table is of *Dozens*, the Integer being a *Grosse*, and the small Denominations are Dozens, and parts of Dozens, so may you find the Decimal of seven Dozens to be .5833333, and the Decimal of five parts of a dozen to be .0347222, and these two Numbers added together, make .6220555, which is the Number which representeth 7 dozen, and $\frac{5}{12}$ parts of a dozen.

In the setting down of Decimal Fractions, to add them together, you must always observe to set *Primes* under *Primes*, *Seconds* under *Seconds*, &c. which the points before the several Fractions will direct you to do.

Hitherto we have shewed the use of the foregoing Tables in expressing of Fractions in decimal Numbers. It resteth now to shew the use of them in finding what Fraction either of Money, Weight or Measure, any decimal Number given doth represent, and that shall be made evident by the ensuing Proposition.

PROBL. X.

A Decimal Number being given, how to find what Fraction it doth represent.

Let .02916667 be a decimal Number, representing some Fraction-part of *English Coin*: Because it is required to find the value of this Fraction in *English Coin*, you must therefore repair to the Table of *English Coin*; in the second Column of which Table seek for the Number given (*viz.* .02916667) which you shall find to stand

ſtand againſt 7 pence, and ſo much is the value of the decimal Fraction .02916667, in *Engliſh Coin*.

Alſo if the decimal Fraction .75 were given, you ſhall find the Value thereof to be 15 Shillings, and the value of .003125 to be three farthings.

Likewiſe in in the Table of *Troy Weight*, if .41666667 were given, it would ſignifie five ounces, and .05416667 would expreſs 13 Penny-weight and .00173611 will expreſs ten Grains, &c.

After this manner may you find the value of any decimal Number given, either in *Money*, *Weight* or *Meaſures*, when the Number given may be exactly found in the Table: But if the Number given cannot be found exactly in the Table unto which it is directed at one entrance, Then you muſt find in the ſame Table, the neareſt Number you can, leſs than the given Number, and take the Number that anſwers unto it in the firſt Column, which will be the greateſt Fraction of the Number required: Then ſubſtracting the Decimal thus found, out of the Decimal given, you ſhall have a Remainder, which Remainder ſeek alſo in the ſecond Column of the Table, if it may be found, if not, ſeek the neareſt leſs and the Number anſwering thereto, in the firſt Column, ſhall be the next greateſt Fraction; then Subſtracting this Decimal found out of the former Remainder, there will be another Remainder, which alſo ſeek in the Table, and proceed as in the former: An Example or two will make all plain.

Example I. Let .68125000 be a Decimal given, repreſenting ſome part of *Engliſh Coin:* If you look in the Table of *Engliſh Coin* for .68125000, you cannot find it, but the neareſt Number in the Table leſs than it, is .65, againſt which I find .13 *s.* ſo that 13 *s.* is the greateſt Fraction-part of *Engliſh Coin* agreeing to this Number.

This done, Subſtract .65 out of .68125000, and there will Remain .03125000, which Number alſo you muſt ſeek in the Table of *Engliſh Coin*, but being you cannot find it there, you muſt take the neareſt Number leſs than it, which is .02916667, againſt which I find 7 pence, which is the next greateſt Fraction-part of *Engliſh Coin* agreeing to this Number.

Again, ſubſtract .02916667, out of .03125000, and there will Remain .00208333, which Number ſeek in the Table, and you ſhall find it to ſtand againſt 2 Farthings, and ſo much doth this laſt Remainder ſignifie in *Engliſh Coin*, and the whole given Number .68125000 doth repreſent in *Engliſh Coin* thirteen Shillings ſeven Pence two Farthings, as by the Operation following doth appear.

.68125000

.68125000 Number given.
.65 the next lesser Number in the Table, representing 13 *s.*

.03125000 first Remainder.
.02916667 the next lesser Number in the Table, representing 7 *d.*

.00208333 second Remainder, which represents two Farthings.
So doth the whole Number represent 13 *s.* 7 *d.* 2 *q.*

Example II. Let the Decimal .87426934 representing some Fraction of a pound Sterling, be given. If you look in the Table of *English Coin* for .87426934 you cannot find it; but the nearest Number in the Table less than it, is .85, against which I find 17 Shillings, so that 17 Shillings is the greatest Fraction-part of *English Coin*, agreeing to this Number.

Then substracting .85 out of .87426934 there will Remain .02426934, which number also you must seek in the Table of *English Coin*, but seeing you cannot find it there, you must take the nearest Number less than it, which is .02083333, against which I find five Pence, which is the next greatest Fraction-part of *English Coin*.

Lastly, substract .02083333, out of .02426934, and there will Remain .00343601, which Number you must also seek in the Table of *English Coin*; but not finding it exactly there, you must take the nearest Number less, which is .003125, against which you shall find 3 Farthings, which is the next greatest Fraction-part of *English Coin*, and the Decimal .87426934, doth in value signifie 17 Shillings 5 Pence 3 Farthings, and something more, for .003125 is the Decimal of 3 Farthings; and the Number you are to look for in the Table is .00343601, greater than the Decimal of 3 Farthings; wherefore, if you substract .003125 out of .00343601, there will Remain .31101, which is the $\frac{31101}{100000}$ part of a Farthing, which is inconsiderable. See the following Operation.

.87426934 Decimal given
.85 Decimal of———————17 *s.*

.02426934 First Remainder.
.02083333 Decimal of———————5 *d.*

.00343601 Second Remainder.
.003125 Decimal of———————3 *q.*

.00031101 Decimal part of a Farthing.

¶ And

¶ And here Note, that whatsoever hath been here said concerning the uses of the Table of *English Coin*, the same order is to be observed in the use of the other Tables of *Weight*, *Measure*, *Time*, &c. as by the following Examples (if you make trial) will appear.

Examples, I. If this Decimal .48975694, were given to know the value thereof in *Troy Weight*, you shall find it to contain 15 Ounces, 17 Penny-weights and 13 Grains.

II. Also if .9505335 were a Decimal given, and it were required to find the value thereof in *Averdupois great Weight*, you shall find it to contain 3 quarters of an Ounce.

III. Likewise, if .70800781 were a decimal Fraction given, you shall find the value thereof in *Averdupois little Weight* to be 11 Ounces, 5 Drams, and one quarter of a Dram.

IV. If .4375 were a Decimal, whose value is required in *Liquid Measures* you shall find it to contain 3 pints and an half.

V. Let .7148438 be a Decimal given, whose value is required in *Dry Measure*, you shall find it to contain 5 Bushels, 2 Pecks, 3 Quarters of a Peck, and 2 Pints.

Thus have I shewed you the use of these decimal Tables in expressing of the Fraction-parts of *Money*, *Weight*, *Measure*, &c. But because these Tables may not be always at hand, when there is need of Them, I will here shew you how the value of any Decimal given, may be known by Multiplication only; and this is

THE RULE.

Multiply the Decimal *given, by the Number of known parts of the next inferiour* Denomination, *which are equal to the* Integer, *the* Product *is the value of the* Decimal *proposed in that inferiour* Denomination; *and if there happen to be any* Decimal *in the* Product, *you may in like manner find the value thereof in the next inferiour* Denomination, *and so proceed till you come to the least known parts of the Integer.*

Example. Let .67395834 be a Decimal given, representing the Fraction of a *Pound Sterling*. First multiply .67395834 by 20 (the Number of Shillings in a Pound Sterling) and the Product will be 1347916680, from which cutting off the last eight Figures with a point, or dash of the Pen (because there were eight Figures in the given Fraction) there will stand before the point (towards the left hand) 13, which are Shillings, and the Remainder .47916680, standing behind the point, will be the Fraction-part of one Shilling Sterling, which Number .47916680, you must multiply by 12 the Number of Pence in one Shilling) and the Product

will be 575000160, from which Number cut off the last eight Figures as before, and there will be 5 left to the left Hand, which are 5 Pence, and the Figures on the right Hand of the Point, *viz.* 75000160 are the Fraction-part of one Penny Sterling, which therefore multiply by 4 (the number of Farthings in one Penny) and the Product of that Multiplication will be 300000640, from which cut off the last eight Figures to the right Hand, and there will be left 3 towards the left Hand, which representeth 3 Farthings, and the remaining Figures towards the right Hand are but the Fraction-part of a Farthing, which we therefore reject. And thus you may find by *Multiplication* only, that this Fraction .67395834 doth represent in the known parts of *English Coin*, 13 Shillings, 5 Pence, 3 Farthings, as by the following operation appeareth.

```
             .67395834
                    20
           ------------
Shillings  13,47916680
                    12
           ------------
             95833360
            47916680
           ------------
Pence       5,75000160
                     4
           ------------
Farthings   3,00000640
```

In like manner, if this Fraction .94809028 were given, representing some Fraction-part of *Troy Weight*; you shall find the value thereof to be 11 Ounces, 7 Penny Weight, 13 Grains, as by the operation following appeareth.

```
                  .94809028
                         12
                 ------------
                  189618056
                  94809028
                 ------------
Ounces.         11,37708336
                         20
                 ------------
Penny Weights.   7,54166720
                         24
                 ------------
                  216666880
                 208333440
                 ------------
Grains.        13|00001280
```

In

In this manner may any Decimal given be reduced into the known parts of the Integer by *Multiplication* only. And

¶ Here note, that whereas in the preceding Tables the Decimal Fractions consist of *seven* or *eight* Figures, we shall in the prosecution of our Work make use only of *four* or *five* of the first of them, which will be sufficient in ordinary practice, and come near enough to the truth in any ordinary question whatsoever.

So if instead of .02916667, which is the Fraction-part of 7 Pence, you take out only .02916. it will be sufficient.

$$\text{Also for}\left\{\begin{array}{l}.05833333\\ .0058594\\ .58333333\end{array}\right\}\text{take}\left\{\begin{array}{l}.05833\\ .0058\\ .5833\end{array}\right\}\begin{array}{l}\text{In } \textit{Troy Weight.}\\ \text{In } \textit{Dry Measure.}\\ \text{In } \textit{Time.}\end{array}$$

Thus much concerning the construction and use of the decimal *Tables*, we shall now come to the Practice of *Decimal Arithmetick*.

The End of the First Section.

SECT. II.

Of the Nature of Decimals, *and how by them to perform the Works of the several Rules in* Arithmetick.

I. *Of Notation of* Decimals.

NOTATION of Decimals is contrary to that of whole Numbers: For whereas in whole Numbers the value of Figures are increased ten-fold by continual addition of Cyphers towards the right Hand: So on the contrary, the values of the places of Decimals do decrease in the same proportion.

And whereas in whole Numbers, Cyphers in the first place towards the left Hand are unnecessary, yet in Decimals, they are absolutely necessary to discover the true Denominator. Also Cyphers at the end (or towards the right Hand) of decimal Numbers, are of no value, for one single Figure in decimals signifies as much

as the same Figure would do, if there were Cyphers placed behind it, so 7 is equivalent unto 70, 700, or 7000, &c. For the Denominators of decimal Fractions are always Cyphers with an Unite towards the left Hand, as hath been already intimated. So $\frac{70}{100}$ being reduced to its least Terms will be $\frac{7}{10}$ and $\frac{7000}{10000}$ will be reduced to $\frac{7}{10}$ also, and so of any other, as by the Table following doth evidently appear.

	987654321	123456789	
	100000000	.000000001	
	10000000	.00000001	
	1000000	.0000001	
	100000	.000001	
	10000	.0001	
a thousand	1000	.001	or $\frac{1}{1000}$
a hundred	100	.01	or $\frac{1}{100}$
Ten	10	.1	or $\frac{1}{10}$

II. *Of Addition of* Decimals.

IN Addition of Decimals, the same order is to be observed as in Addition of Numbers of one Denomination before taught in the first Part, in which there is no difficulty: But in Decimal Numbers the chief care to be taken is in placing your whole Numbers and Fractions in their due order, which you shall easily and certainly do, if you observe this general Rule, *viz.* to *place your whole Numbers and Fractions one under another, so that the Points of Seperati n which* (in decimal Numbers) *distinguish the whole Numbers from the Fractions, stand directly one under the other*, then are you to proceed in the addition of them in all respects, as you did in whole Numbers.

Example I. Let it be required to add together in one sum these several sums following, in a decimal Way, *viz.* 36 *li.* 2 *s.* 8 *d.* 29 *li.* 0 *s.* 2 *d.* 31 *li.* 16 *s.* 9 *d.* and 6 *li.* 2 *s.* 5 *d.*

First, set down 36 *li.* and a Point or Comma after it, then for the Fraction-part of 2 *s.* 8 *d.* look in your Table of *English Coin*, where you shall find the decimal Fraction of 2 *s.* 8 *d.* to be, .1333 therefore for 36 *li.* 2 *s.* 8 *d.* set down 36. 1333.

Secondly, for your 29 *li.* 0 *s.* 2 *d.* set down 29. 0083.

Thirdly, for your 31 *li.* 16 *s.* 9 *d.* set down 31. 8375.

Lastly, for your 6 *li.* 2 *s.* 5 *d.* set down 6. 1208 as you see done in the operation following.

For

	l.	*s.*	*d.*		
For	36	02	8	set down	36, 1333
	29	00	2		29, 0083
	31	16	9		31, 8375
	6	02	5		6, 1208
	103	02	0		103, 9999

Your decimal Numbers being thus placed in due order one under another, proceed to the adding of them together, as if they were whole Numbers, and you shall find the sum or total of them to be 103. 0999.

Now the 103 which stand towards the left Hand, are 103 pounds; and the .0999 which stands towards the right Hand of the Comma, is the Fraction-part of one pound Sterling, the value whereof you may find (by the Proposition beforegoing) to be two shillings *fere*, which should be two shillings exact, but it wanteth somewhat, *viz.* the $\frac{2}{1000}$ part of a Farthing, which is insensible; for if by the fore-mention'd Rule you seek the value of the decimal Fraction, .0999, you shall find it to be 1 shilling, 11 pence, 3 farthings, and the $\frac{2}{1000}$ part of a Farthing, which you may call in all 2 shillings, for decimal Numbers will seldom happen to give the exact value of Fractions, but will be either greater or lesser than they ought to be; but in such a sum as this is, the thousandth part of a Farthing is not to be regarded.

Example II. Let it be required to add together in a decimal way these sums following, *viz.* 29 *l.* 18 *s.* 7 *d.* 3 *q.* 63 *l.* 11 *s.* 2 *d.* 1 *q.* 129 *l.* 4 *s.* 0 *d.* 2 *q.* and 3 *l.* 7 *s.* 10 *d.* 1 *q.*

First, for 29 *l.* 18 *s.* 7 *d.* 3 *q.* set down 29. 73229.
Secondly, For 63 *l.* 11 *s.* 2 *d.* 3 *q.* set down 63. 55937.
Thirdly, for 29 *l.* 4 *s.* 2 *q.* set down 129. 20208.
Lastly, for 3 *l.* 7 *s.* 10 *d.* 1 *q.* set down 3. 39271 as you see here set down in the Margine.

Your decimal Numbers thus placed in order, add them together, as if they were whole Numbers, and you shall find the sum of them to contain 226. 08645.

29. 93229
63. 55937
129. 20208
3. 39271
226. 08645

Now the 226, which stand towards the left Hand of the Comma, are 226 pounds, and the other Figures towards the right Hand, *viz.* 08645 are the Fraction-parts of a Pound Sterling, which if you reduce by the fore-mentioned Proposition, you shall find the Value thereof to be 1 Shilling, 8 Pence, 3 Farthings, so the whole sum is 226 *l.* 1 *s.* 8 *d.* 3 *q.*

And

And here note, that what hath been said, as concerning *Money*, the same is also to be understood of *Weight*, *Measure*, *Time*, &c. as by the following Examples will appear.

Other Examples for Practice.

Example I.	*Example* II.
In *Money*.	In *Troy Weight*.
135. 8833	7. 97413
95. 5583	6. 65330
3. 2875	3. 62187
234. 7291	18. 24930
234 *l.* 14 *s.* 7 *d.*	18 *lib.* 2 *Ou.* 19 *P.W.* 21 *Gr.*

Example III.	*Example* IV.
In *Averdupois little Weight.*	In *Averdupois great Weight.*
12. 7227	37. 9442
76. 3594	9. 3053
32. 625	33. 6786
91. 4883	10. 0000
32. 1398	12. 8142
246. 0398	103. 7423
246 *lib.* 00 *Ou.* 9 *dr.*	103 *C.* 2 *q.* 27 *lib.* 3 *Ou.*

III. *Of Substraction of* Decimals.

THe Substraction of Decimals differeth nothing from the Substracting of one whole Number from another, and the decimal Numbers to be Substracted one from another, must be placed in the same order, as in *Addition* of decimal Numbers, the Practice of Substraction shall be seen in the following Examples.

Example I. Let it be required to Substract 31 *l.* 16 *s.* 9 *d.* out of 36 *l.* 2 *s.* 8 *d.*

First, for your 36 *l.* 2 *s.* 8 *d.* set down the Decimal thereof, which is 36. 1333.

Secondly, for your 31 *l.* 16 *s.* 9 *d.* set down the Decimal thereof 31, 8375.

This

This done draw a Line under them, and Subſtracting the Leſſer from the Greater you ſhall find the Remainder to be 4.2958 the 4 on the left ſide of the Comma are four Pounds, and the .2958 which ſtandeth towards the right Hand, is the Fraction-part of a Pound, the Value whereof being ſought, will be found to be 15 *s.* 11 *d.* So that if you Subſtract 31 *l.* 16 *s.* 9 *d.* there will Remain 4 *l.* 5 *s.* 11 *d.*

36, 1333
31, 8375
4. 2958

But if divers Sums be to be Subſtracted out of one greater Sum then you muſt firſt add all the ſeveral ſmaller Sums together, and Subſtract the Sum of them from the Greater given Sum, ſo ſhall the reſidue be the Sum deſired.

Examples for Practice.

Example I.

In *Money.*

Lent	2784. 8375
Paid at ſeveral times.	36. 1333
	29. 0083
	31. 8375
	6. 1208
paid in all	103. 0999
reſts to	2681. 7376
pay	2681 *li.* 14 *s.* 0 *d.*

Example II.

In *Averdupois great Weight.*

Bought	103. 7423
Sold	37. 9442
Unſold	65. 9442
65 *C.* 3 *q.* 5 *lib.* 7 *ou.*	

Example III.

In *Troy Weight.*

Delivered to a Goldſmith of old Plate	7. 97413
Receiv'd of new Plate	5. 59670
Reſts in the Goldſmiths Hands	2. 7743
	2 *lib.* 4 *oun.* 10 *p. w.* 14 *gr.*

IV. *Of Multiplication of* Decimals.

MULTIPLICATION of *Decimals* differeth nothing at all from the *Multiplication* of whole Numbers, for making the greater Number the *Multiplicand*, and the leſſer Number the *Multiplier*, the Number iſſuing from that Multiplication ſhall be called the *Product*.

Now

Now in the Multiplication of decimal Numbers one by another, if there be any Fraction either in the *Multiplicand* or *Multiplier*, or Fraction in both: So many Figures as the Fractions contain, so many Figures must be cut off from the *Product* towards the right Hand, which shall be the Fraction of the *Product*, and the Figures towards the left Hand of the Comma in the Product, shall be the Integers of the *Product*.

Example I. Let it be required to multiply 34 pounds, five shillings, three pence, by 16 pounds, six shillings, six pence.

First, seek the Decimal of 34 *l.* 5 *s.* 3 *d.* which you shall find to be 34. 2625, make this your Multiplicand, then seek the Decimal of 16 *l.* 6 *s.* 6 *d.* which you shall find to be 16. 325, make this Decimal Number your Multiplier; then draw a Line, and Multiply these two Numbers together, as if they were whole Numbers, and you shall find the Product of them to be 559.3353125. Now because there are four figures in the Multiplicand which are Fractions, namely, those four towards the right Hand, *viz.* 2625, and there are also three figures in the Multiplier, which are Fractions, namely, these three towards the right Hand, *viz.* 325. that is in all seven figures, representing Fractions, I therefore cut off from the Product the seven figures towards the right Hand, by making of a Comma there, to distinguish the whole Number from the Fraction: So is 559 the integer or whole Number, and .3353125 the Fraction of this Multiplication.

Multiplicand	34.2625
Multiplier	16.325
	1713125
	685260
	1027875
	255750
	342625
	379.3353125

Example II. If there be Fractions in the Multiplicand, and none in the Multiplier, yet the work is still the same, for you must cut off only so many Figures from the Product, as there are Fractions either in Multiplicand, Multiplier, or both; So if it were required to multiply 5767 Yards, and 3 quarters of a Yard, by 235 Yards, you must first set down 5767. 75 for your 5767 Yards, and three quarters, which Number must be your *Multiplicand*: And also set down 235 yards for your *Multiplier*, then multiplying them together, as if they were whole Numbers, you shall find the *Product* to be 1355421. 25, and because there are only two Fraction figures, both which are in the Multiplicand, namely, the two last thereof

5767.75
235
2883875
1730325
1153550
1355421.25

·75, and none in the Multiplier. I therefore cut off only two figures of the Product, Namely, the two laſt, which are .25, ſo is the Product of this Multiplication 1355421. 25 which is 1355421 ſquare yards, and one quarter of a Yard. And ſo if a Garden or other piece of Land, lying ſquare, ſhould contain in length 5767 Yards and three Quarters, and in breadth 235 Yards, the whole piece would contain 1355421 ſquare Yards, and one quarter of a Yard.

Example III. If decimal Fractions be to be multiplied by decimal Fractions, you muſt then (as before) multiply them as whole Numbers, and from the Product cut off ſo many Figures towards the right Hand, as there are Figures in the Multiplicand and the Multiplier. So if it were required to multiply .953 by .782, you ſhall find their Product to be .745246, which being but ſix figures in all, I cut them off and that Fraction .745246 is the Product of the Multiplication of the two given Fractions.

.953
.782
1906
7624
6671
745246

Example IV. If any two decimal Fractions being multiplied together, the Product thereof doth not conſiſt of ſo many places as are required (by the former rules) to be cut off, you muſt then ſupply that defect by prefixing a Cypher, or Cyphers before the product towards the left Hand: So if theſe decimal Fractions .063 and .0752 were to be multiplied, their Product would be 47376. Now (by the former Rules) you ſhould cut off ſeven Figures of the Product towards the right Hand, but this product 47376 conſiſteth but of five Figures; wherefore to make it ſeven Figures, I prefix two Cyphers before the product on the left Hand, making it .0047376, and that is the true Product produced by this Multiplication.

.0752
.063
02256
004512
.0047376

Example V. If you would multiply any Decimal (either Fraction only, or whole Number and Fraction together) by 10, 100, 1000, &c. You muſt add ſo many Cyphers to the Multiplicand, as there are Cyphers in the Multiplier, and cut off ſo many Figures as there are Fractions in the Multiplicand, and the Number ſhall be the Product required: So if 7,856025 were a Decimal given to be multiplied by 100, add two Cyphers to the Number given, making it 7.85602500, then becauſe there were ſix Figures of this Number towards the right Hand, it will be 785.602500, which is the true Product required.

Examples for Practice.

Example I.

```
      7.432
       2.61
   --------
       7432
     44592
    14864
   --------
    1939752
```

Example II.

```
    22.358
        32
  --------
    44716
   67074
  --------
   715.456
```

Example III.

```
     .352
      .24
   ------
     1408
     704
   ------
    .8448
```

Example IV.

```
     375.6218
          100
  -----------
   37562.1800
```

A Compendious way for the Multiplication of Decimal mixt Numbers.

IT hath been much objected against *Decimal Multiplication*, for that, when the *Mixt Numbers* to be Multiplied together, do consist of many *Decimal Parts*; the *Product* increasing to 8, 9, 10 or 12 Places of *Parts*; whereas 2, 3 or 4 at the most, will be sufficient. It is true, but that *Objection* may be removed by this following *Compendium:* For the manner of working whereof this is the

RULE.

Set down the biggest of the two Mixt Numbers; *and (although there be several Places of* Parts, *both in the* Multiplicand *and* Multiplier) *set the place of* Unity *of the* Integer *of the* Multiplier, *under that Figure of* Parts *in the* Multiplicand, *whose Number of* Parts *you would have to be in the* Product: *Then set all the* Figures *of the* Multiplier, *the contrary way to what they were given: And by each Figure of the* Multiplier

tiplier, Multiply *the Figure over it in the* Multiplicand (*having regard to the Figure going before*) *and ſet down the Product*, &c.

This will beſt appear by *Examples.*

Example I. Let it be required to Multiply 34.2625, by 16.325, ſo that there may be but two Places of Parts in the *Product.*

I. Set down the bigger *Number* 34.2625 for the *Multiplicand*; and (becauſe you would have but two Places of *Parts* in your *Product*) ſet the *Unites Place* of the *Integer* in the *Multiplier*, which is 6, under the ſecond place of *Parts* in the *Multiplicand*, which is 6 alſo; and tranſpoſe all the other *Figures* of the *Multiplier*, as you ſee them ſet down in the *Margine*: For the *Multiplier* given, was 16.325, and here it is made 523.61, and 6 the Place of *Unity* in the *Multiplier*, ſtands under 6 (the ſecond place of *Parts*) in the *Multiplicand.*

II. The *Multiplicand* and *Multiplier* being placed in this order, you are to begin your Work at the Right Hand, as followeth: Saying, once 5 is 5, for which (becauſe it is half, keep 1 in mind) but ſet it not down; but ſay again, once 2 is 2 and 1 in mind is 3; ſet 3 under 1, and go on with that Line, ſaying once 6 is 6, once 2 is 2, once 4 is 4, and once 3 is 3, ſetting them down; ſo ſhall the firſt Line, or *Product* be 34263.

Multiplicand	34.2625
Multiplier	523.61
	34263
	20557
	1028
	68
	17
Product	559.33

III. Go to the next Figure of the *Multiplier*, 6, and ſay 6 times 2 is 12, for which bear 1 in mind, and ſay again 6 times 6 is 36 and 1 in mind is 37; ſet 7 under 3, and go on, ſaying 6 times 2 is 12 and 3 in mind is 15; ſet down 5 and bear 1; and ſay, 6 times 4 is 24 and 1 is 25, ſet down 5 and bear 2; then 6 times 3 is 18 and 2 is 20, which, being the laſt, ſet down; ſo ſhall your ſecond Line or Product be 20557.

IV. Then go to the third Figure of the *Multiplier* 3; and ſay, 3 times 6 is 18, for which (it being above 15) bear 2 in mind; and ſay, 3 times 2 is 6 and 2 in mind is 8, which ſet under 7, and ſay, 3 times 4 is 12, ſet down 2 and bear 1, then 3 times 3 is 9 and 1 is 10, which being the laſt ſet down, and ſo the third Line or Product is 1028.

V. Then go to the fourth Figure 2, ſaying 2 times 2 is 4, which being leſs than 5 bear none in mind but go to the next Figure, ſaying, 2 times 4 is 8, ſet 8 under 8, and ſay 3 times 2 is 6, which ſet down alſo; ſo is the fourth Line or Product 68.

VI. For the laſt Figure of the *Multiplier* 5, ſay 5 times 4 is 20, for which bear 2 in mind, and ſay 5 times 3 is 15 and 2 in mind is 17, which is the laſt Line or Product, and your *Multiplication* is ended.

VII. Draw a Line under the ſeveral *Products*, and add them together, in the ſame order they ſtand, and the *General Product* will be 55933; from which ſeperate *two Places* to the *right Hand*, and the *Product* will be 559.33. And this may you do, and have any Number of *Parts* in the *General Product*.

Example II. Let it be required to multiply the ſame *Mixt Numbers* 34.2625, and 16.325, ſo that there may be no *Places* of *Parts* in the *General Product*.

Multiplicand	34.2626
Multiplier	523.61
	343
	205
	10
	1
General Product	559.

In this Caſe, ſet the *Unites Place* of the *Multiplier* under the *Unites Place* in the *Multiplicand* (and tranſpoſe all the *Figures* of the *Multiplier*, as before) and as you ſee them placed in the *Margine*. Then,

I. Begin with the firſt Figure of the *Multiplier* 1, and ſay, once 6 is 6, (which being above 5) bear 1 in mind, and ſay, once 2 is 2 and 1 in mind is 3; ſet 3 under 1; then ſay, once 4 is 4; and once 3 is 3; ſet them both down; and ſo your *Firſt Product* will be 343.

II. Go to the ſecond *Figure* 6, and ſay, 6 times 2 is 12; for which bear 1 in mind, and ſay, 6 times 4 is 24, and 1 in mind is 25, ſet 5 under 3 and bear 2 in mind: Then ſay, 6 times 3 is 18, and 2 in mind is 20; which ſet down; ſo is your ſecond *Product* 205.

III. For the third Figure of the *Multiplier* 3, ſay, 3 times 4 is 12, for which bear 1, and ſay, 3 times 3 is 9, and 1 in mind is 10, ſet down 10, for your third Product.

IV. Say

IV. Say, 2 times 3 is 6, which (being above 5) set down 1 for your fourth and last *Product*.

V. Draw a Line, and add all the *Products* together, their Sum will be 559. *Integers*, free from all *Decimal Parts*.

We may have occasion hereafter to make use of this *Compendious* way of *Multiplication* of *Mixt Numbers*; And that the Learner may be perfect therein, take these following *Examples* ready wrought.

Example III. Let it be required to Multiply 72.2543, by 5.1642, so that there may be but two Places of Parts in the Product.

72.2543	Multiplicand
2461.5	Multiplier
36127	Here 8 Places should have been cut off but two was sufficient.
723	
433	
29	
1	
373.13	Product

Example IV. Let it be required to Multiply 259879.890625, by 1.1173698.

Multiplicand	259879.890625
Multiplier	8963711.1
	259879.891
	25987.989
	2598.799
	1819.159
	77.964
	15.592
	2.338
	.207
Product	290381.939

V. Of

V. *Of Division of* Decimals.

AS *Division* of whole Numbers is the hardest of the four *Species* of *Vulgar Arithmetick*, so the *Division* of *Decimals* is the most difficult of the four kinds of *Decimal Arithmetick*, but I hope to make it plain to the understanding of the meanest capacity.

The several varieties that may happen in Division, are principally (if not only these) four. Namely, First, *To divide whole Numbers and Fractions.* Secondly, *To divide whole Numbers by Mixt, or Mixt Numbers by whole.* Thirdly, *To divide a greater Fraction by a less*; and Lastly, *To divide a lesser Fraction by a greater.*

In Division of Decimals this Rule is general, *if the* Dividend *be greater than the* Divisor, *the* Quotient *will be either a* whole Number *or a* mixt, *but if the* Dividend *be less than the* Divisor, *the* Quotient *will be a* Decimal. And (for convenience in working, if there be need) any Number of Cyphers may be annexed to the Dividend, that thereby the Quotient may extend to as many places as the tenour of the Question shall require.

The manner of the working of *Division* in Decimals, is the same with that before delivered in whole Numbers in the first part of Vulgar Arithmetick, as will appear by the Examples following, in every of the four premised Varieties.

The RULE for the first variety.

The Dividend *and the* Divisor, *being both* mixt Numbers, *or one of them being a* whole Number *and the other a* mixt; *or the* Dividend *being a* Decimal, *and the* Divisor *a* whole Number *or a* mixt, *the first Figure in the Quotient will be of the same* Place *or* Degree, *with that Figure or Cypher of the* Dividend, *which at the first demand standeth, or (at least) is supposed to stand directly over the place of* Unite *in the* Divisor.

Example I. *Where the terms given are both mixt Numbers.*

Let it be required to divide 559.3354125 by 16.325. Here the Terms given are both of *mixt Numbers*, which being placed according to the Rules delivered before for the Division of whole Numbers, the Figure in the Dividend, which at the first demand, standeth over 6, the place of Unites in the Divisor is 5, and because this standeth in the place of Tenths, therefore the first Figure in the Quotient is in the place of Tenths also, and the whole Number consisteth of two of the foremost Places, and the rest is a Decimal, thus the

the Quotient sought in our present Example is 34.2625, of which 34 the two first Figures is the Integer or whole Number, 2625 the Decimal Fraction.

```
 Divisor        Dividend          Quotient
 16.325)   559.3354125       (34.2625
                 • •  • • •
           ------------------------
           48975
            69585
           ---------
            65300
             42854
           -----------
             32650
             102041
           ------------
               97950
                40912
           ------------
                 32650
                  82625
                 ---------
                  81625
                   1000
```

Example II. *One of the Terms given, being a whole Number, the other mixt.*

The mixt Number 1355421.26 being divided by the whole Number 235, the Quotient will be 5767.75 and the first Figure in the place of Thousands, as by the Operation doth appear.

Divisor

Divisor	*Dividend*	*Quotient*
235)	1355421.26	(5767.75

```
        1175
         1804
        1645
         1592
         1410
          1821
          1645
           1762
           1645
            1176
            1175
               1
```

The RULE for the second variety.

When the Dividend *is a* whole *or* mixt Number, *and the* Divisor *a* Decimal, *add as many Cyphers to the* Dividend *as there are places in the* Divisor; *for the integral Part of the Quotient will consist of as many places as the* Divisor, *and the Places arising from the integral Parts of the* Dividend *added together.*

Example I. Let 348.75 be the mixt Number given, to divided by the Decimal .25, to the Number given, I add two Cyphers, the Number of Places in the Divisor, and then it will be 348.7500, which being divided by .25, the integral Part of the Quotient will be 1395, because the whole part of the Dividend 348, being divided by .25 giveth two places, and the Number of Places in the Divisor being two, giveth two more; and so the integral part consisteth of four Figures, as by the Operation.

Divisor

Divisor *Dividend* *Quotient*

```
.25) 348.7500 (1.395.00
       ....
     --------
     25
      98
     ---
      75
     237
     ---
     225
      125
     ---
      125
```

The Rule for the third Variety.

When the Terms given are both Decimals, *the* Dividend *being the greater, the integral part of the Quotient will consist of as many Places as the* Divisor *doth.*

Example. Let the Decimal .73958 be divided by the Decimal .32 the integral part of the Quotient will be 23, because the Divisor doth consist of two Places, as by the operation doth appear.

```
.32) .73958 (2311
       ....
     --------
      64
       99
      ---
       96
        35
       ---
        32
         38
        ---
         32
          6
```

The Rule for the fourth Variety.

When the Terms given are both Decimals, *consisting of equal Places, the* Dividend *being the* lesser Term, *place the* Dividend *as a* Numerator, *and the* Divisor *as* Denominator; *so is such vulgar Fraction the Quotient sought: But if the Terms consist not of equal places supply the place or places wanting in either of the Terms, by annexing a Cypher or Cyphers on the right Hand, and then proceed as before.* Thus if .27 be given to be divided by .93, the Quotient will be $\frac{27}{93}$. Also if .35 be given to be divided by .78563, the Quotient by annexing 3 Cyphers to .35, the lesser decimal given, will be $\frac{35000}{78563}$, which vulgar Fractions may be reduced into Decimals if need be, by the first Proposition in this Second part of decimal Arithmetick.

Examples for Practice.

```
44) .35673 (0081, &c.        .25) 2481.00 (9924
     352                          225
      47                          0231
      44                           225
       3                            60
                                    50
                                   100
                                   100
                                   000
```

Having given you Examples of the four foregoing Rules in the several cases of Division in Decimals. I will now bring all the forementioned *Rules* into One *general Rule*, and give you *Examples* of all the Varieties, that can possibly arise in Decimal Division.

A Supplement to Decimal Division

Containing One General Rule, *for finding the true* Value *of any* Decimal Quotient. *And* Examples *in all Cases.*

Decimals to be Divided may be either,

1			*Whole Number*
2	A *Whole Number*	by a	*Mixt Number*
3			*Decimal Fraction.*
4			*Whole Number*
5	A *Mixt Number*	by a	*Mixt Number*
6			*Decimal Fraction.*
7			*Whole Number*
8	A *Decimal Fraction*	by a	*Mixt Number*
9			*Decimal Fraction.*

Whereas in *Multiplication*, you always cut off so many *Figures* from the *Product* as there were *Decimal Parts* both in the *Multiplicand* and *Multiplier:* It thence necessarily follows; That the Number of *Decimal Parts* in the *Divisor* and *Quotient* in any *Division*, must be equal to the *Decimal Parts* in the *Dividend*. For which observe this

GENERAL RULE.

When your Work of Division is ended; *Consider how many* Decimal Parts *are in the* Dividend, *more than in the* Divisor*: For that* Excess *is the Number of* Decimal Parts *which must be separated in the* Quotient, *for* Decimal Parts. *But if there be not so many* Figures *in the* Quotient, *as the said* Excess *is, they must be supplied by prefixing of so many* Cyphers *in the* Quotient, *before the significant* Figures *thereof, towards the* left Hand, *with a* Point *or* Comma *before them.* So will the *Quotient* discover its true *Value.*

Examples in all Cases ready wrought.

I.) Divide 8976, by 23456

Divisor	Dividend	Quotient
23456)	8976.0000	(3826, &c.

II.) Divide

II.) Divide 586, by 36.4865.

Divisor	Dividend	Quotient
36.4865)	586.000000	(16.06, *&c.*

III.) Divide 72, by .0432

Divisor	Dividend	Quotient
.0432)	72.000000	(1666.66, *&c.*

IV. Divide 58.271875, by 725.

Divisor	Dividend	Quotient
725)	58.271875	(.080375, *&c.*

V.) Divide 885.69864, by 12.24.

Divisor	Dividend	Quotient
72.361)	885.69864	(12.24, *&c.*

VI.) Divide 5328.625, by .425.

Divisor	Dividend	Quotient
.425)	5328.625000	(12537.941

VII.) Divide .0683, by 23.

Divisor	Dividend	Quotient
23)	.06283	(.00273, *&c.*

VIII.) Divide .846, by 2.43.

Divisor	Dividend	Quotient
2.43)	.846000	(.3481, *&c.*

IX.) Divide .166592, by .7358,

Divisor	Dividend	Quotient
.7358)	.166592	(.24

Of

VI. *Of the* Rule of Three *in* Fractions Vulgar *and* Decimal.

WHat the Rule of Three is, and the manner of working, is already ſhewed in the firſt part, that which we here intended is only to add ſome Examples in Fractions Vulgar, as well as Decimal; that by comparing the work in both, the Excellent uſe of Decimal Arithmetick might the better appear.

And how to convert the known parts of *Money*, *Weight*, or *Meaſures Engliſh*, into *Decimals* hath been already ſhewed, both *Arithmetically* and by *Tables*; yet to prevent the ſeveral *Aditions* and *Subſtractions* in thoſe *Tables*, I have here annexed another *Decimal Table*, for the more ſpeedy Reduction of *Engliſh Money* under two Shillings, all Sums of Money above, not having Pence or Farthings annexed, being as eaſily reduced by *Memory* as by *Tables*, and this I have the rather done, becauſe the ſame *Table* will alſo reduce the *Coins* of *France*, and the parts of *Troy Weight*, if an Ounce be made the Integer which in point of Practice is much more uſeful than the Pound.

The

The Table of REDUCTION.

Peny					
	001042			026014	
	002083	Gr. 1		027083	13
	003125			028125	
1	004166	2	7	029166	14
	005208			030208	
	006250	3		031250	15
	007291			032291	
2	008333	4	8	033333	16
	009375			034375	
	010410	5		035406	17
	011458			036458	
3	012500	6	9	037500	18
	013541			038541	
	014583	7		039583	19
	015625			040625	
4	016666	8	10	041666	20
	017708			042708	
	018750	9		043750	21
	019791			044791	
5	020833	10	11	045833	22
	021874			046875	
	022956	11		047916	23
	023958			048958	
6	025000	12	12	050000	24

The Table of REDUCTION.

		051082				076014	
		052083	Gr. 1			077083	13
s.	d.	053125		s.	d.	078125	
1.	1.	054160	2	1	7	079066	14
		055208				080208	
		056250	3			081250	15
		057292				082921	
1	2	058333	4	1	8	083333	16
		059375				084375	
		060416	5			085406	17
		061458				086458	
1	3	062500	6	1	9	097500	18
		063542				088541	
		064583	7			089583	19
		065625				090625	
1	4	066666	8	1	10	091666	20
		067708				092708	
		068750	9			093750	21
		069792				094791	
1	5	070833	10	1	11	095833	22
		071874				098675	
		072916	11			099716	23
		073958		2	0	099858	
1	6	075000	12			000000	24

These things premised, we will now shew the use of the Table in some practical Questions belonging to the Rule of Three Direct.

Question I. If $\frac{7}{8}$ of a Yard of Cloth, cost $\frac{9}{12}$ of a pound: What shall 17 Yards cost at the same rate?

If $\frac{7}{8}$ cost $\frac{9}{12}$, what shall 17 cost? Ans. 14 *l.* $\frac{4}{7}$.

First, multiply $\frac{9}{12}$ by $\frac{17}{1}$ the product is $\frac{153}{12}$, then divide $\frac{153}{12}$ by $\frac{7}{8}$, the quotient is $\frac{1224}{84}$: again, if you divide 1224 by 84, the quotient is 14 $\frac{8}{84}$, or in the least terms 14 pound $\frac{4}{7}$ of a Pound.

And the value of this Fraction $\frac{4}{7}$ of a pound, will be found by the third Rule of Reduction of Fractions to be 11 Shillings 5 Pence and $\frac{4}{7}$ of a Peny, which is somewhat above two Farthings, and $\frac{2}{7}$ of a Farthing.

The same Question in Decimals.

If $\frac{7}{8}$ of a Yard of Cloth cost $\frac{9}{12}$ of a pound, what shall 17 Yards cost at the same rate?

To answer this Question $\frac{7}{8}$ of a yard, and $\frac{9}{12}$ of a pound must first be reduced into Decimals, either by Division, or by the Tables of Reduction: By both which ways of Reduction the Decimal of $\frac{7}{8}$ will be .875, and the decimals of $\frac{9}{12}$ will be .75, and then the Terms of the Question will Stand thus;

If .875 parts of a Yard cost .75 parts of a pound, what shall 17 yards cost at the same rate?

If 0875———0.75———17. Here if you multiply the second term 0.75 by 17 the third term given, the Product will be 12.75, and this Product divided by 875, gives in the quotient 14.57142, that is, 14 pound 57142 parts of a pound, or 145 Decades, that is 14 pounds 10 Shillings, and .7142 parts of a Decade (or two Shillings) which by the preceding Tables is 1 *s.* 5 *d.* 2 Farthings, and .0059 parts of a Farthing.

Question II. If a piece of Gold Plate weighing 19 Ounces 3 penny weight and 5 Grains, be worth 62 Pound 10 Shillings 6 Pence; what is one Ounce of the same Gold worth?

This Question in vulgar Fractions must be expressed thus.

If 1 *l.* $\frac{3437}{5760}$ Troy-weight, cost 62 *l.* $\frac{126}{140}$, what shall $\frac{1}{12}$ of a pound Troy cost at the same rate?

To answer this question, the Fractions 1 *l.* $\frac{3437}{5760}$ and 62 *l.* $\frac{126}{140}$, must be first reduced into improper Fractions and then the Fraction $\frac{1}{12}$ into the least known Parts of a pound Troy, and then the Question will stand thus.

If $\frac{9197}{5760}$ give $\frac{15006}{240}$, what shall $\frac{480}{5760}$ give?

Now

Now because it is necessary the terms given be reduced into their least Denominations, before the question be resolved, therefore the answer may be found, by using the terms given thus reduced as whole numbers, not having any regard to the Denominators of these Fractions; Saying thus,

If 9197 grains, cost 15006 pence, what shall 480 grains cost?

And here if you multiply 15006 by 480, the Product will be 7202880, which being divided by 9197, the Quotient will be 783 pence $\frac{1629}{9197}$ parts of a peny, and dividing 783 by 12, it will be 65 shillings 3 pence $\frac{1629}{9197}$, or 3 *l.* 5 *s.* 3 *d.* $\frac{1629}{9197}$. And although this question is thus more easily answered than it would have been, if the terms had been wrought as vulgar Fractions, yet the same terms being reduced to Decimals, the answer of the question will yet be found with more ease, as shall appear by the operation following.

The same Question in Decimals.

If a piece of Gold plate weighing 19 ounces 3 peny weight and 5 grains, be worth 62 *l.* 10 *s.* 6 *d*, what is an ounce of the same Gold worth?

The Decimal of 19 ounces 3 peny weight and 5 grains, making an ounce the Integer, is by this Table 19.16041, for that 19 ounces are 19 Integers, 2 peny weight is one tenth of an ounce, and the Decimal of 1 *d. w.* 5 grains is by this Table .06041; and the Decimal of 62 *l.* 10 *s.* 6 *d.* by the same Table is 62.525, and because an Unite or Integer is the third term given, there needs no multiplication, if therefore you divide 62.525 the second term, by 16.16041 the first term propounded, the Quotient will be 3.2632, that is 3 pounds 5 shillings 3 pence, and somewhat more, as by the operation following it doth appear.

Divisor	*Dividend*	*Quotient.*
19.16041)	62.52500000	(3.2632 *li.*

```
62.52500000
    ....
-----------
5748123
 5043770
-----------
3832082
1211688o
-----------
11496246
  6206340
-----------
  5748123
   4582170
-----------
  3832082
   750088
```

Or,

li.	*s.*	*d.*	*q.*
3	5	3	1 *fere*

III. *Queſtion.* If 5 Ells and a quarter of linnen Cloth coſt 2 *l.* 16 *s.* 8 *d.* 3 *q.* what ſhall 278 Ells and a half coſt at the ſame rate?

If you would work this Queſtion by whole Numbers, your eaſieſt way is firſt to reduce all the terms into their leaſt Denominations, that is to ſay, the Ells into quarters, and the pounds, ſhillings, pence and farthings, all into farthings, ſo ſhall your 5 Ells and a quarter be 21 quarters, and your 278 Ells and a half will be 1114 quarters, and your 2 *l.* 16 *s.* 8 *d.* 3 *q.* will be 2723 farthings, and then will your queſtion ſtand thus in whole Numbers.

quarters		*farthings*		*quarters.*	
If 21 —	coſt —	2723 —	what will —	1114 —	coſt?

Then multiplying the ſecond Number by the third, that is, 2723 by 1114, the Product will be 3033422, which divided by 12, the Quotient will be 144448 farthings, which being again reduced into pounds, ſhillings and pence, giveth 150 *l.* 9 *s.* and 4 pence, as by the operation following doth appear.

The OPERATION.

```
     2723        21) 3033422 (144448 14/21 q.
     1114            ·······
    -------          -------
    10892            21             Or,
    2723              93          l.    s.   d.
   2723              ----         150—09—4 14/21
  2723                84
  -------              93
  3033422             ----
                       84
                        94
                       ----
                        84
                        102
                       ----
                         84
                        182
                       ----
                        168
                         14
```

But

But if you would work the ſame Queſtion by Decimal Numbers, you may ſave the labour of reducing the terms to their leaſt Denominations, for 5 Ells and a quarter is in decimal Numbers 5.25, and 278 Ells and an half is 278.5, and 2 *l.* 16 *s.* 8 *d.* 3 *q.* is in Decimals 2.8364, and then your Queſtion in Decimals will ſtand thus:

Ells	*l.*	*Ells.*	*l.*
As 5.25 :	to 2.8364 ::	So 278.5 :	to 150.4642.

If you multiply (according to the Rule) the ſecond term by the third, that is 2.8364 by 278.5, the Product of that multiplication will be 789.93400, which divided by the firſt term 5.25, the Quotient will be 150.4642, which Decimal repreſenteth 150 *l.* 9 *s.* 4 *d.* and ſo much in money will 278 Ells and a half coſt.

The OPERATION..

```
Ells       l.        Ells.
5.25 - 2.8364 - 278.5
           278.5
         ---------
         141820
        226912
       198548
       56728
      ------------
5.25) 789.93740 (150.4642
        ......
      ----------
      525              Or,
      2649           l.   s.  d.
      ------        150 — 9 — 4
      2625
        2437
      ----------
        2100
          3374
        ---------
          3150
           2240
        ---------
           2100
             140
```

I have been the larger in this Rule, and eſpecially in this Example, which is incumbred with Fractions ſufficient, becauſe I would have the Reader the better diſcern the difference between the *Vulgar* and the *Decimal* way, and alſo to ſee how expeditious the one is over the other. Now this example being thus largely explained, I ſhall with the more brevity paſs over the Rules following, giving one Example or two at the moſt in each Rule. And thus much ſhall ſuffice for the *Golden Rule*, or *Rule of Three Direct* in Fractions.

VII. *Of the Rule of Three Reverſe.*

Queſtion. I.

A Lends *B.* 233 *l.* 16 *s.* 8 *d.* for a year without Intereſt, upon condition that *B.* ſhould do the like courteſie for *A.* when required. *A.* hath occaſion for money 7 months; howmuch money ought *B.* to lend *A.* to requite his courteſie, and ſave himſelf harmleſs?

I will not in this place tell you what the *Rule of Three Reverſe* is, nor the manner of working thereof, that being already ſufficiently declared in the firſt part, but give you the Example, and the working thereof which take as followeth: So will the Queſtion be thus, ſtated;

months

months	*l.*	*s.*	*d.*	*months.*
12 :	233	6	8 ::	7

Which in Decimals stands thus,

months	*l.*	*months.*
12 :	233.33 ::	7

```
          12
  -------------
       46666
      23333
  7)  2799.96  (399.99 3/7
      . . . . .
  ----------
     21          Or,
      69       l.  s.  d.
  -----        400  0  0
      63
       69
     -----
       63
        69
      -----
        63
         66
       -----
         63
          3
```

Here you see that 12 months and 7 months are whole Numbers, and so we let them alone without any Reduction, but the Decimal of 233 *l.* 6 *s.* 8 *d.* will be found by the fore-mention'd Tables and Rules to be 233.33, which is the middle term in the Question and of the same quality with that, must the fourth term sought be, therefore if (according to the Rule delivered in the first part) you multiply 233.33 by 12, the Product will 2799.96, which divided by 7, giveth in the Quotient 399.99, which is the Decimal of 400 *l.* and so much money ought *B.* to lend *A.* for 7 months.

Question II. If when the price of a Quarter of Wheat is 1 *l.* 5 *s.* 6 *d.* the penny white Loaf shall weigh 12 *Ou.* 16 *P. w.* I demand what the penny white Loaf shall Weigh, when the price of the Quarter of Wheat is 7 *s.* 6 *d.* the Quarter?

If

If you place the Numbers according to the tenor of the Question, they will ſtand as followeth.

l. s. d.	Ou. P. w.	s. d.
1—5—6	12 — 16	7—6

In Decimals, Thus,

```
      l.       Ou.
    1.275    12.8   .375
     1.28            .375)  16.3200  (43.5 75/375
   --------                  ...
    10200                ----------
    2550                  1500
   1275                    1320
   --------               --------
   16.3200                 1125
                            1950
      Or,                 --------
 Ou.  P.w.  Gr.             1875
 43    10    3                75
```

Here if you multiply 1.275, which is the Decimal of 1 *l.* 5 *s.* 6 *d.* by 12.8. which is the Decimal of 12 ounces 16 penny weight, you ſhall find the Product of that multiplication to be 16.3200, which being divided by .375, which is the Decimal of 7 *s.* 6 *d.* the Quotient will be 43.5, which is the Decimal of 43 ounces, 10 penny weight 3 grains; and ſo much ought the penny white loaf to weigh, when the quarter of wheat is ſold for 7 *s.* 6 *d.*

VIII. *Of the Rule of Proportion, conſiſting of five Numbers.*

Queſtion I.

IF 100 *l.* in 12 months yields 6 *l.* intereſt, what intereſt ſhall 264 *l.* 16 *s.* 5 *d.* yield in 15 months at the ſame rate?

Set down your numbers in Decimals, as in the Example following appeareth, ſo ſhall you find the Decimal of 264 *l.* 16 *s.* 5 *d.* to be 264.8208, all the reſt being whole numbers, having no Fractions joyned with them we neglect, and work with them as they are, ſo will the ſeveral numbers of your queſtion (if rightly diſpoſed) ſtand as followeth:

If

l. *mo.* *l.* *l.* *mo.*
If 100 in 12 gain 6, what 264.8208 gain in 15?

```
 100                 264.8208
  12                        6
----              -----------
1200                1588.9248
                           15
                  -----------
                     79446240
                    15889248
                 ------------
      1200)  23833.8720 (19.8615 9/16
              . . . . . . .
             ----------------
             1200              Or,
             11833          l.  s.  d.  q.
             -----          19  17  5   3
             10800
              10338
             -----
               9600
                7387
               -----
                7200
                 1872
                -----
                 1200
                  6720
                 -----
                  5000
                   720
```

Your numbers being thus orderly diſpoſed, you muſt according to the Rule before delivered in the firſt Part, multiply the firſt and ſecond terms together, which in this Example are 100 and 12, whoſe Product is 1200, which is your *Diviſor*; Then multiply the three laſt terms one into another, as 264.8201 (which is the Decimal of 264 *l.* 16 *s.* 5 *d.*) by 6, and the Product thereof will be 1588.9248, which Number again multiplied by 15, (which is the laſt term) the Product, will be 23833.8720 which is your ***Dividend***, and this number being divided by your former Product, giveth in the Quotient 19.8615, which is the Decimal of 19 *l.* 17 *s.* 2 *d.* 3 *q.* *ferè*, and ſo much doth the ſimple Intereſt of 264 *l.* 16 *s.* and 5 *d.* amount unto in 15 months, after the rate of ſix *per* ***Centum*** for a Year.

Queſtion II.

Question II. If the carriage of 23 hundred and 3 quarters of any thing 127 miles, cost 4 *l.* 13 *s.* 6 *d.* what shall the carriage of 47 hundred and an half of such like commodity cost, being carried 381 miles.

Place your numbers in order as in the following Example doth appear, then multiply the first and second terms together for your Divisor, and the three last one into another for your Dividend, and so will the Quotient of this Division answer the question demanded, and the work will stand as followeth.

The OPERATION.

	C.	*mil.*	*l.*	*C.*	*mil.*
If	23.75	127	4.675	37.5	381

```
  23.75                 37.5
    127                  381
  -------              -------
  16625                  475
  4750                 3800
 2375                 1425
 -------              --------
 3016.25              18097.5
                         4.675
                      ---------
                        904875
                      1266825
                     1085850
                     723900
                     ---------
3016.25)  84605.8125  (28.05
                ....
          ------------
           603250
          2428081            Or,
          ---------          l.  s.  d.
          2413000            28  1   0
            1508125
          ---------
            1508125
```

Here you see that the first and second Terms multiplied together produced 3016.25, which must be your Divisor, and the three last Terms being multiplied one into another, produce 84605.81250, which number, divided by 3016.25, giveth in the Quotient 28.050, which Decimal representeth 28 *l.* one shilling, and so much will the

the carriage of 47 hundred and a half cost being carried 381 miles.

Question III. If 24 yards of stuff of three quarters broad, cost 4 *l.* 14 *s.* what shall 328 yards of the same stuff cost being 5 quarters broad.

If you place your numbers according to the directions of this Rule, they will stand thus,

	yards		*quarters*		*l.*	*s.*		*yards*		*q.*
If	24	of	3	cost	4	14,	what shall	328	cost of	5

In Decimals thus,

yards	*broad*	*l.*	*yards*	*broad*
24	3	4.7	328	5

24
3
——
72

328
4.7
——
2296
1312
1541.6
5
——
72) 7708.0 (107.05

Or,

l.	*s.*	*d.*
107	1	1

72
508
——
504
400
——
360
40

First, multiply the two first numbers, as 24 and 3 together, they make 72 for Divisor, then multiply 4.7, which is the Decimal of 4 *l.* 14 *s.* by 328, and the Product is 15416, which again multiplied by 5, the last number giveth 77080; unto this Product, (that there may be a competent number of figures in the Quotient,) I add two Cyphers, making it 7708000, which I divide by 72, and the Quotient is 107.055, which is 107 *l.* 1 *s.* 6 *d.* and so much is 328 yards of stuff worth, being 5 quarters broad.

IX. *Of Barter.*

TWo Merchants having two ſeveral Commodities, are willing to Barter, or Exchange the one with the other. The one hath Indigo, which he will ſell at 4 *s.* the pound for ready money, but in Barter he will have 4 *s.* 9 *d.* the pound, and the other Merchant hath Kerſies, which for ready money he will ſell for 3 *s.* 6 *d.* the yard. Now the queſtion is, at what price he muſt rate his Kerſies in Barter, to equalize the 9 *d.* advance upon the pound of Indigo?

The Tenor of the Queſtion is this.

If 4 *s* in Barter, require 9 *d.* what ſhall 3 *s.* 6 *d.* require?

Your Numbers placed will ſtand thus,

s.	*d.*	*s.*	*d.*
4	9	3	6

In Decimals thus,

l.	*l.*	*l.*
.2	.375	1.75

```
       .375
       1.75
      -----
       1875
      2625
      375
     ------
.2) 656.25 (328
     ...
     ------
       4          Or,
      16      l.  s.  d.  q.
     ----     0   0   7   3
      16
```

Say then by the Rule of Three Direct, if 2 Decades or 4 *s.* in Barter require .375, which is the Decimal of 9 *d.* what ſhall 1.75 require? Which is the Decimal of 3 *s.* 6 *d.*

First,

First, multiply .375 by 1.75, the product is .65625, but being it is a Fraction, I cut off the two last Figures, because we require only three Figures in the Quotient, which divided by 2, giveth in the Quotient .328, which is the Decimal of 7 *d.* 3 *q.* this 7 *d.* 3 *q.* added to this 3 *s.* 6 *d.* maketh 4 shillings 1 penny 3 farthings, and so much ought he to rate his Kersies at by the yard in Barter, to save himself harmless.

X. *Of Fellowship.*

THree Persons *A*, *B* and *C.* bought 4000 Sheep, which cost 483 *l.* 6 *s.* 8 *d.* of which money *A* paid 203 *l.* *B* paid 165 *l.* 15 *s.* 8 *d.* and *C* paid 114 *l.* 11 *s.*

First, say by the Rule of Three Direct.

1. If 483 *l.* 6 *s.* 8 *d.* buy 4000 Sheep, how many Sheep shall 203 *l.* (which is *A*'s share) buy? *Answer*, 1680.

2, Say, If 483 *l.* 6 *s.* 8 *d.* buy 4000 Sheep, how many Sheep shall 165 *l.* 15 *s.* 8 *d.* (which is *B*'s share) buy? *Answer*, 1372.

3. Say again, if 483 *l.* 6 *s.* 8 *d.* buy 4000 Sheep, how many Sheep shall 114 *l.* 11 *s.* (which is *C*'s share) buy? *Answer*, 948.

Your numbers reduced to Decimals and orderly placed will stand as in the following Operation.

First for A.

```
     l.       sheep           l.
If 483.3333 buy 4000 how many 203?
                              4000
                            ------
                            812000

483.3333)  8120000000  (1680 Sheep
                 ....
           ----------
           4833333
           32866670
           --------
           28999998
            38666720
           --------
            38666664
             00000560
```

Secondly for B.

```
    l.            sheep          l.
483.3333 ------- 4000 -------- 165.7833
                                   4000
                             ----------
                             6631333000

483.3333)  6631332000  (1372 Sheep
                 ...
           ----------
           4833333
           17979990
           --------
           14499999
            34800010
           --------
            33833331
             9666790
           --------
             9666666
                 124
```

Thirdly

Thirdly for C.

```
li.         sheep       li.
483.3333 ---4000--------114.55
                           4000
                      -----------
                       458200.00

483.3333 ) 4582000000 948 sheep.
              ...
          -----------
          43499997
           23200030
          -----------
          19333332
          38666980
          -----------
          38666664
               316
```

The manner of Working.

For *A.* multiply 203 *l.* (which is *A*'s share) by 4000 (which is the number of sheep bought) and the Product is 812000 which number should be divided by 483.3333, but being it is greater than 81200, I therefore add four Cyphers thereto, that I may have four figures in the Quotient, and it makes 8120000000, which divided by 483. 3333, giveth in the Quotient 1680, and so many sheep belong to A.

2. For *B*, multiply 165.7833 (which is the Decimal of *B*'s share) by 4000, (the number of sheep bought) and it produceth 6631332000, which divided by 483.3333, giveth in the Quotient 1372, and so many sheep belong to *B*.

3. For *C*. multiply 114. 55, (which is the Decimal of *C*'s share) by 4000, (the number of sheep bought) it produceth 45820000, which number should be divided by 483.3333, but being it is not large enough to give figures enough in the Quotient, I therefore add two Cyphers, making it 4582000000, which divided by 483.3333, giveth in the Quotient 948, and so many sheep ought *C*. to have.

Now for proof, if you add the number of sheep that *A*, *B* and *C*. should severally have, you shall find them in all to make 4000, which demonstrates the Work to be true.

A.	1680
B.	1370
C.	948
	4000

XI. *Of Loss and Gain.*

I*F one Yard of Stuff cost* 6 s. 8 d. *and I sell the same again for* 8 s. 6 d. *what shall I gain in laying out* 100 li. *upon such a Commodity?*

Take the difference between the price that your Commodity cost, and the price for which you sell it, that is, in this Example, the difference between 6 *s.* 8 *d.* and 8 *s.* 6 *d.* which is 1 *s.* 10 *d.* then say by the Rule of Three Direct,

If 6 *s.* 8 *d.* gain 1 *s.* 10 *d.* what will 100 *li.* gain?

If you place your numbers according to the Rule of Three Direct, as they are here given, they will stand as followeth,

s. d. *s. d.* *li.*
If 6 8 gain 1 10 what will 100 gain?

In Decimals, Thus

```
 li.             li.           li.
.3333 ———————— .425 ———————— 100
                100
        ————————————————————
.3333)  42500.00  (127.5
          . . . .
        ————————————————
        3333
         9170      Or
        ————————   li.   s.
        6666       127   10
        25040
        ————————
        23331
         17090
        ————————
        16765
          325
```

Your numbers being placed, multiply .425, which is the Decimal of 1 *s*, 10 *d.* by 100 *li.* and the Product is 42500, to which I add two Cyphers (that I may have a competent number of figures in the Quotient) and it makes 42500.00, which divided by .3333, the Decimal

Decimal of 6 *s.* 8 *d.* giveth in the Quotient 127.5, which is 127 *li.* 5 Livers or 10 *s* so there is 27 *l.* 10 *s.* gained in laying out of 100 *li.*

I will here prove this queſtion by the converſe.

If by one yard of Stuff which is ſold for 8 *s.* 6 *d.* there was gained 27 *li.* 10 *s.* in laying out of 100 *li.* I demand what the ſaid ſtuff coſt a yard at the firſt Hand?

Add 100 *l.* and 27 *l.* 10 *s.* together, and they make 127 *l.* 10 *s.* Then ſay by the Rule of Three Direct,

If 127 *l.* 10 *s.* give 100 *l.* what ſhall 8 *s.* 6 *d.* give?

In Decimals, Thus

127.5——100——425
425

127.5) 42.500 (.3333.

3825
4250 Or,
s. *d.*
3825 6—8
4250

Here if you multiply .425, which is the Decimal of 8 *s.* 6 *d.* by 100, you ſhall have 42.500, to which if you add a Cypher, you make it 42500.0, this number being divided by 127,5, which is the Decimal of 127 *l.* 10 *s.* giveth in the Quotient .3333, and if you had added more Cyphers to the Dividend, you ſhould have had more Threes in the Quotient, and no other Figures, but theſe four are enough, and are a Decimal Fraction repreſenting 6 *s.* 8 *d.* and ſo much did the yard of Stuff coſt at the firſt Hand.

XII. *Of Loſs and Gain upon Time, wrought by the Double Rule of Three.*

IF one Ell of Lockeram coſt me 2 *s.* 8 *d.* ready mony, and I ſell the ſame again for 2 *s.* 10 *d.* the Ell, to be paid at the expiration of three Months; I demand what I ſhall gain in 12 Months, laying out 100 *l.* upon that Commodity?

This

This and such like Questions, although they may be wrought by the Rule of Three Direct, at two Operations, yet they are best performed by the Double Rule of three compounded of five Numbers, wherefore the Question may be thus stated.

If 2 *s.* 8 *d.* in three Months, gain 2 *d.* what shall 100 *l.* gain in 12 Months?

If you take your Numbers out of your Scale, and place them according as was directed in the first part of this Book, you shall find them to stand thus,

These Numbers reduced to Decimals, and placed orderly according to the Tenor of the Rule, will stand as in the following Operation.

	s.	*mo.*	*d.*		*l.*	*mo.*
If	1.333	in 3 gain	83,	what shall	100 gain in	12
	3		100			
	3.999		8300			
			12			
			16600			
			8300			
	3.999)		99600 (25 *fere*			
			7998			
			19620			
			19995			
			375			

Your Numbers being placed according to the Tenor of the Question, if you multiply 1.333, which is the Decimal of 2 *s.* 8 *d.* by 3 months, the Product will be 3.999, which must be your Divisor, then multiply 83, which is the Decimal of 2 *d.* by 100 *l.* and it makes .8300, that again multiplied by 12 months, giveth for the Product 99600 for your Dividend, wherefore if you Divide 99600 by 3999, it will give you in the Quotient 25 almost, which is 25 *l.* for the Decimal Fraction remaining is so small, that it wanteth not near a farthing of 25 *l.* and therefore we call it 25 *l.* and so it is exactly, as you may try, if you reduce all the Numbers to their least Denominations, and work as is before taught in *Vulgar Arithmetick.*

I will

I will prove this Queſtion by the converſe.

If one Ell of Lockerham coſt me 2 *s.* 8 *d.* ready money, for what price ſhall I ſell the ſame again to be paid at the end of three months So that I may gain 25 *l.* in 100 *l.* for 12 months?

Say by the Rule of Three Direct.

If 100 *l.* in 12 Months gain 25 *l.* what ſhall 2 *s.* 8 *d.* gain in 3 Months?

If you Reduce your Numbers to Decimals, and place them according to the Double Rule of Three, they will ſtand as followeth,

```
 l.     m.   l.    s.       m.
 100——12——25——1.333——3
  12                25
 ————              ————
  200              6665
 100              2666
 ————              ————
 1200             33325
                      3
                 ——————
         1200)  99975 (.83
                ——————
                  9600
                   3975
                 ——————
                  3600
                   375.
```

Your Numbers being thus placed, if you multiply 100 *l.* by 12 months, you ſhall find the Product to be 1200, which is your Diviſor. Then multiply 25 *l.* by 1.333, which is the Decimal of 2 *s.* 8 *d.* and the Product thereof will be 33325, which multiply again by 3, and the Product will be 99975 for your Dividend, this 99975 divided by 1200, giveth in the Quotient .83, which is the Decimal of 2 *d.* which 2 *d.* added to 2 *s* 8 *d.* the price which the Ell of Lockeram coſt, giveth 2 *s.* 10 *d.* and at that price muſt you ſell the ſame at 3 months time, ſo that you may gain 25 *l.* in the 100 *l.* in 12 Months.

APPENDIX.

XIII. *Of Exchanges.*

TO give, or *Exchange* one *Commodity* for another, or *Commodity* for *Money*; or *Money* for *Commodity*; or part *Money* and part *Commodity*, is called *Barter*: But *Exchange* (according to the ordinary Notion of *Merchandizing*) is to give *Coyn* for *Coyn*; that is, to give a *Sum* of *Money* in one *Place*, for a *Bill*, ordering the payment of the like *Sum* (according to the *Value* agreed upon) in another *Place*, either at home, which is called *Inland*, or in another *Country*, which is called *Foreign Exchange*.

And as there is a *Par* of *Exchange* of *Money*, ſo there is a *Par* or *Equation* of *Weights* and *Meaſures*, whereby to value *Foreign Goods* bought or ſold in any two different *Countries*, &c.

Now to perform this Work, there is nothing required more than the *Golden Rule*, (or *Rule of Three*) if firſt the *Rate*, *Ratio* or *Proportion* between the *Coyns*, *Weights* and *Meaſures* of any *Two Countries* be firſt known, which is beſt obtain'd by Experience, rather then taken upon Truſt: All that I ſhall do in this Caſe, is to inſtruct the ingenious in the manner of Work; and make uſe of ſuch *Rates* and *Proportions*, as I find ſet down by Mr. *Lewis Roberts* in his *Map of Commerce*.

I ſhall illuſtrate this *Rule* of *Exchange*, by the Working of ſeveral Queſtions thereunto relating.

Queſtion I. How many Riders, (each Rider containing 1 *l.* 1 *s* 2 *d.* 2 *q.*) muſt I receive for 251 *l.* 6 *s.* 4 *d.* 2 *q.* Sterling?

	l.	*s.*	*d.*	*q.*		*Rider*		*l.*	*s.*	*d.*	*q.*		*Rid.*
As	1	1	2	2	:	1	::	251	6	4	2	:	237
	1	5	60	4	:	1	::	251	3	6	87	:	

Here if you reduce your Numbers to their leaſt Denominations, or ſet them down in Decimals, and multiply and divide according to the Golden Rule, you ſhall find in your Quotient 237, and ſo many *Riders* ought to be received for 251 *l.* 6 *s.* 4 *d.* 2 *q.* Sterling.

Queſtion II. How many *French Crowns* (each *French Crown* being valued at 6 *s.* Sterling) ſhall I receive for 492 *l.* 18 *s.* Sterling?

	s.		F.C.		l. s.		F.C.
As	6	:	1	::	492 18	:	1643
	3	:	1	::	492 9	:	

Multiply and divide according to the Golden Rule, and you ſhall have in your Quotient 1643, and ſo many *French Crowns* are to be received for 492 *l.* 18 *s.* Sterling.

Queſtion III. A Merchant delivered at *Paris* 1643 *Crowns* of 6 *s.* Sterling the Piece, How many *Pounds* Sterling ought to be received at *London* ?

	F.C.		s.		F.C.		l. s.
As	1	:	6	::	1643	:	492 18
	1	:	3	::	1643	:	492 9

Multiply and Divide, and you ſhall have in your Quotient 492 *l.* 18 *s.* and ſo much *Sterling Money* ought to be delivered at *London*, for 1643 *French Crowns*, of 6 *s.* the *Crown Sterling*.

Queſtion IV. If 3 yards at *London*, be 4 Ells at *Antwerp*, how many yards at *London* make 84 Ells at *Antwerp* ?

	Ell An.		*Y.* Lon.		*Ell* An.		*Y.* Lon.
As	4	:	3	::	84	:	63

And ſo many yards at *London*, are equal to 84 Ells at *Antwerp*.

Queſtion V. How many yards of *London* make 27 Ells of *Antwerp* when 100 Ells of *Antwerp* make 60 Ells of *Lions*, and 20 Ells of *Lions* make 25 yards of *London* ?

The firſt Work.

Ells Lions *Yards* London *Ells* Lions.

20 ——— 25 ——— 60

60

150|0

75 *facit* 75

That is 75 Yards of *London* is equal to 100 Ells of *Antwerp*.

The Second Work.

Ells Antwerp	*Yards* London	*Ells* Antwerp.
100————	————75————	————27

75
27
———
525
150
——— *Yards of* London
20|25 facit 20.25

Question VI. If 100 *l. Sterling* be 134 *l.* 6 *s.* 4 *d. Flemish*, what is one pound *Sterling* worth?

	l. st.		*l. s. d.*		*Fl. st.*		*l. s. d. q.*
As {	100	:	134 6 4	:	1	: }	1 6 10 2
	100	:	134 3666	:	1	: }	1 34366

Multiply and Divide, and you shall have ——— in the Quotient 1.34366, or 1 *l.* 6 *s.* 10 *d.* 2 *q. fere* and so much is one pound Sterling worth.

Question VII. How many Ells of *Franckford* make 42 $\frac{1}{4}$ Ells of *Vienna* in *Austria*, when 35 Ells of *Vienna* make 25 at *Lions*, 3 Ells of *Lions*, 5 Ells of *Antwerp*, and 100 Ells of *Antwerp*, 125 Ells at *Franckford*.

	Ell. An.		*Ell.* Fran.		*Ell.* An.		*Ell.* Fran.
1)	100	:	125	::	3	:	6.25
	Ell. Lio.		*Ell.* Fran.		*Ell.* Lio.		*Ell.* Fran.
2)	3	:	6.25	::	24	:	50
	Ell. Vi.		*Ell.* Fran.		*Ell.* Vi.		*Ell.* Fran.
3)	35	:	50	::	42.25	:	60.35

Thus have I given you a few Exchanges, I will here insert some few Tables derived from Mr. *Lewis Roberts* his *Map of Commerce* afore-said, of the truth of which I am not a competent Judge, but shall leave that to the scrutiny of such as have occasion to trade into Forreign Countries.

TABLE

A TABLE shewing what one pound of *Averdupois* Weight at *London*, maketh in divers Cities, and other remarkable places.

One Pound of Aver-du-pois-*Weight at* London *maketh at*

	lb.	
Antwerp	.9615	
Amsterdam	.9	
Abevile	.91	
Ancona	1.282	
Burdeaux	1.12	
Avigon	.91	
Burgoyne	.91	
Bolonia	1.25	
Bridges	.98	
Calabria	1.3698	
Calcis	1.07	
Constantinople	.8474 Loder;	
Deep	.91	
Dantsick	1.16	
Ferrara	1.3333	
Florence	.282	
Flanders in general	1.06	
Geneva	.9345	
Genoa	1.4084	suttle
	1.4285	groſs
Hamburg	.92	
Holland	.95	
Lisborn	.881	
Lions	1.07	common weight
	.98	silk weight
	.9	customers weight
Leghorn	1.3333	
Milan	1.4285	
Mirandola	1.3333	
Norimberg	.88	
Naples	1.4085	
Paris	1.89	
Prague	.83	
Placentia	.112	
Rochel	.13888	

		lb.
One pound of Aver-du-pois *weight at* London *makes at*	Rome	1.27
	Rouan	.875 by Vicont .9017 common w.
	Sevil	.108
	Tholousa	.112
	Turin.	.12195
	Venetia	1.5625 suttle 9433 gross
	Vienna	.813

The Use of the preceding Table.

How much weight at *Bolonia*, will 655 *li.* Averdupois make ?

Look in the Table for *Bolonia*, and right against it you shall find 1.25, which sheweth that one pound Averdupois at *London* is equal to 1.25 *l.* at *Bolonia* ; Therefore say by the Rule of Three :

If 1 *l.* Averdupois give 1.25 *l.* at *Bolonia*, what shall 635 *li.* Averdupois give ? *Answer* 818.75. As by the operation following doth appear,

	Av.		*li.* Bol.		li. *Av.*		*li.* Bol.
As	1	:	1.25	: :	655	:	818.75.

A

A TABLE shewing what one pound Weight in divers Forreign Cities, and remarkable Places, maketh at *London* of *Averdupois* Weight.

One pound Weight in	*makes at* London *of* Averdupois *weight* lb.
Antwerp	1.04
Amsterdam	1.1111
Abeville	1.0989
Ancona	.78
Avignon	8928
Burdeaux	1.0989
Burgoyne	1.0980
Bolonia	.8
Bridges	1.0204
Calabria	.73
Calais	.9345
Deep	1.0989
Dantsick	.862
Ferrara	.75
Florence in general	.78
Geneva	.9433
Genoa subtle	1.07
Genoa	.71
Genoa gross	.7
Hamburg	1.0865
Holland	1.0526
Lisbon	1.135
Lions common weight	.9345
Lions silk weight	1.0204
Lions custom weight	1.1111
Legorn	.75
Milan	.7
Mirandola	.75
Norimberg.	1.136
Naples	.71
Paris	1.1235
Prague	1.2048
Placentia	.72
Rochel	.8928
Rome	.78 4

Rouan

One Pound Weight in		makes at L. of Aver. w.	lb.
Rouan	by Vicount,		1.1428
	common weight		1.1089
Sevil			.9250
Tholousa			.8928
Turin			.82
Venetia	subtle,		.64
	gross.		1.06
Vienna			1.23.

The Use of the foregoing Table.

In 7652 *li.* weight at *Mirandola*, how many pound Weight of Averdupois ?

Look in the Table for *Mirandola*, and right against it you shall find .75, which sheweth that one pound *Averdupois* is equal to .75 or $\frac{3}{4}$ of a pound at *Mirandola*, wherefore say by the Rule of Three,

If 1 *l.* at *Mirandola*, gives .75 or $\frac{3}{4}$ of a pound *Averdupois*, what shall 7652 *l.* of *Mirandola* give? *Answer* 5739, as by the operation following doth appear.

	li. *Mi.*		.Av.		li. *Mi.*		li. Av.
As	1	:	.75	: :	7652	:	5739

A

A TABLE reducing English Ells to the Measures of divers Foreign Cities, and remarkable places.

One Ell at London *makes at*

Amsterdam	1.6949	*Ells.*
Antwerp	1.6666	
Bridges	1.64	
Arras	1.65	
Norimberg	1.74	
Colen	2.08	
Lisle	1.66	
Mastricht	1.57	
Frankford	2.0866	
Dantsick	1.3833	
Vienna	1.45	
Paris	.95	Aulns
Rouan	1.03	
Lions	1.0166	
Calais	1.57	
Venice, Linnen	1.8	Braces
Venice, Silk	1.98	
Lucques	2.	
Florence	2.04	
Milan	2.3	
Leghorn	2.	
Madera-Isles	1.0328	
Sevil	1.35	
Lisbon	1.	Vares
Castilia	1.3875	
Andaluzia	1.4625	
Granada	1.3625	
Genoa	4.8083	Palmes
Saragosa	.55	Canes
Rome	.56	
Barselona	.7225	
Valentia	1.2125	

The Uſe of this Table.

In 632 Ells at London, *how many Braces at* Florence?

Look in the Table for *Florence*, and right againſt it you ſhall find 2.04, which ſheweth, that one *Ell* at *London* maketh at *Florence* 2.204 Braces; wherefore ſay by the Rule of Three.

If one Ell at London *give* 2.04 *Braces at* Florence, *how many Braces ſhall* 632 *Ells give?* Anſwer 1289.28, as by the operation following doth appear.

	Ell *Lon.*		*Bra. Fl.*		Ell *Lon.*		*Bra. Fl.*
As	1	:	2.04	: :	632	:	1289.28

A

A TABLE reducing the Measures of divers Foreign Cities, and remarkable places, to *English* Ells.

One Ell at	Amsterdam	*Makes at London*	.59	*Ells*
	Antwerp		.6	
	Bridges		.6097	
	Arras		.606	
	Norimberg.		.5474	
	Colen		.4807	
	Lisle		.6024	
	Mastricht		.6369	
	Frankford		.4792	
	Dantsick		.7228	
	Vienna		.9896	
One Auln at	Paris		1.0526	
	Rouan		.9708	
	Lions		.9836	
	Calais		.6369	
One Brace at	Venice Linnen		.5555	
	Venice Silk		.5102	
	Lucques		.5	
	Florence		.4901	
	Milan		.4347	
	Leghorn		.5	
	Madera		.9681	
One Vare at	Sevil		.7409	
	Lisbon		1.	
	Castilia		.7207	
	Andalusia		.7339	
	Granada		.7339	
One Palm at	Genoa		.2079	
One Cane at	Saragosa		1.8181	
	Rome		1.7857	
	Barselona		1.4035	
	Valentia		.8247	

The Use of this Table.

In 5727 *Braces at* Leghorn, *how many Ells* English.

Look in the Table for *Leghorn*, and right against it you shall find .5, which sheweth that one Brace at *Leghorn* maketh at *London* .5 or half an Ell, wherefore say by the Rule of Three.

If one Brace at Leghorn *give* .5 *Ells at* London, *what shall* 5727 *Braces give?* Answer 2863.5 Ells *London.*

	Br. Le.		*Ell. Lon.*		*Br. Le.*		*Ell. Lon.*
As	1	:	.5	: :	57727	:	2863.5

XIV. *Of*

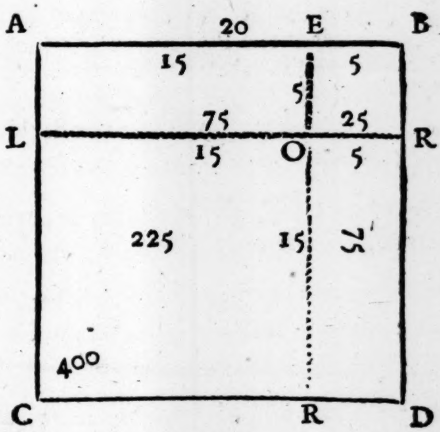
XIV. *Of Extraction of Roots.*

I. FOr the Square Root; That is, a *Square Number* being given; to find the *Root* or *Side* of it in a *Number*; which *Root* or *Side*, being *Multiplied* into it *ſelf*, muſt therefore produce that *Square Number.*

The *Reaſon* of this *Rule* is taken from the IVth *Propoſition* of the *Second Book* of *Euclid*, which ſaith;

If a Right Line *be divided by chance; the* Squares *made of the* Parts, *together with the* Rectangle *made of the* Parts *twice, is* Equal *to the* Square *of the whole.*

ILLUSTRATION.



Let the Line AB, be divided by chance in the point E, it is manifeſt that the Square of AB, that is to ſay, the Square ABDC, is equal to the Square of LO, that is of AE, and to the Square of EB, and to the two Rectangle Figures of AO and DO, that is the Rectangle AO (which is made of the Parts AE, and BE) twice; according to the *Propoſition.*

Now let AB, be ſuppoſed 20, AE 15, and BE 5.

Then the Square of AB, (which is made by multiplying the Root 20 into it ſelf) is equal to 400

And

And the Square AE, that is, 15 times 15, is equal to	225
And the Square of BE is 5 times 5	25
And the Rectangle AO, 5 times 15	75
And the Rectangle DO, 5 times 15	75
In all	400

Which is equal to the Square AB, as before.

The Genesis, *for* Extracting *of the* Square Root.

Example I. *Let it be required to find that* Square Number, *whose* Side *is* 57.

5	7	Root
25	..	
7	0	
	49	
32	49	Square

I. Write down the *Root* 57, as in the *Margine*, with the interval of one *Figure* between the 5 and the 7, and draw a *Line* under them; and also, two down-right *Lines*, the one next after the Figure 5, the other after the Figure 7; so that the *Numbers* to be found, may be orderly placed for *Addition*: Then let the *Root* given be supposed to be divided into these two *Parts*, 50 and 7; Then,

II. Multiply 5 into it self, the *Product* is 25, which set under the *Line*, and under 25, *Unite* under *Unite*.

III. Double 5, and it makes 10, which multiply by 7, makes 70; which set under 25, but one place forward towards the *Right-Hand*.

IV. Multiply 7 into it self, the *Product* is 49, which set under 7, *Unite* under *Unite*, and draw a *Line* under all.

V. Add the three Numbers between the two *Lines* together, in the same order as they stand; the *Sum* of them will be 3249, which is a *Square Number*: And the *Root* of it 57, which may be *proved* by multiplying 57 by 57, for the *Product* will be 3249.

7	.32	Root
49	...	
4	48.	
	102	4
53	582	4 Square

Example II. So if the *Number* 732 a *Root*, were broken into these two *Parts* 7 and 32; the Sum of the *Products* arising from the several Branches between the *Lines* found as the first *Example*, the *Sum* of them will be 535824, a *Square Number*, the *Root* whereof is 732.

To

To Extract the Square Root.

When a *Number* is given to have the *Square Root* thereof *Extracted*, as ſuppoſe this Number 3249: Set the *Number* down as in the *Margine*; and make a *Point* or *Prick* over 9, the place of *Unity*, and another over 2, the ſecond *Figure* from it towards the left Hand; obſerving the ſame order ſtill if there were more Secondary *Figures* in the *Number* given ——— Then, draw down-right Lines on the *Right* Side of each *Figure* that hath a *Prick* over it: And put a *Quotient Line*, (as in *Diviſion*) on the right ſide of the given *Number*, and ſo your *Number* is prepared for *Extraction*, and will ſtand as in the *Margine*: And to perform the Work of *Extraction*, follow theſe *Directions*.

	32	45	(57 Root
	24		
107)	7	49	Reſolvend
	7	49	Product
	0	00	Remain.

I. Seeing 32 (being the *Figures* of the firſt *Period*) not being a Square Number, find the neareſt *Square Number* among the nine *Digits*, which is leſs than 32, which you will find to be 25 (for 36 is greater) whoſe *Root* is 5, ſet 5 in the *Quotient*, and 25, the *Square* of it, under 32, and draw a Line under all: And Subſtracting 25 from 32 the remainder 7 ſet under the Line.

II. To this *Remainder* 7, bring down 49 (the two *Figures* of the next *Period*) ſo will the Number be 749, which you may call the *Reſolvend* ——— Then on the *Left-hand* of 749 make a *Crooked Line* for a *Quotient*, in which put the double of the Figure 5 in the *Quotient*, which is 10, and ask, how many times 10 can you have in 74, the anſwer will be 7 times; put 7 in the *Quotient*, to 5, making it 57, and alſo in the other *Quotient*, making that 107; Then multiply 107, by 7 (the *Figure* laſt put in the *Quotients*) and the *Product* will be 749, which ſet under 749, and ſubſtracting it from 749 above, the *Remainder* will be *nothing*; which ſhews that the Number 3249 is a *Square Number*, and that the *Root* thereof is 57. As in *Example* 1 of the *Geneſis*.

Example II. *Let it be required to find the* Square Root *of* 27846729.

Your *Number* being ſet down, with *Points* over the proper *Figures*; and down-right and *Quotient* lines drawn, as was before directed, and as you ſee here done in the *Margine*: You may begin your *Extraction* in this manner.

I. The

I. The Figures of the *Firſt Period* being 27, the neareſt *Square Number* thereto is 25, whoſe *Root* is 5; ſet 5 in the *Quotient*, and 25 under 27, and under it draw a Line, and ſubſtracting 25 from 27, the Remainder will be 2, which ſet under 25.

	.	.	.	.	
	27	84	67	29	(5277 Root
	25				
102)	2	84			Reſolvend
	2	04			Product
1047)		80	67		Reſolvend
		73	29		Product
10547)		7	38	29	Reſolvend
		7	38	29	Product
		0	00	00	Remainder.

II. To this *Remainder* 2, bring down the two *Figures* of the next *Period*, viz. 84, making it 284 for the firſt *Reſolvend*.

III. Double the *Figure* in the Quotient 5, it makes 10, which ſet in a Quotient, on the Left-hand of 284, and aſk, how many times 10 in 28, the Anſwer will be 2; ſet 2 in the Quotient for the *Root*, and alſo in the other Quotient by 10, making it 102.

IV. Multiply 102, by 2, the laſt *Figure* in the *Root-Quotient* and the *Product* will be 204, which ſet under 284, and Subſtracting 204 from 284, the *Remainder* will be 80, which ſet under 204.

V. To this *Remainder* 80, bring down the two *Figures* of the next *Period*, viz. 67, making the ſecond *Reſolvend* to be 8067.

VI. Double the Quotient 52, and it makes 104, which ſet in a Quotient on the Left-hand of the Reſolvend 8067, and aſk, how many times 104, may be had in 806, the anſwer will be 7 times; put 7 in the Root Quotient, and alſo in the other Quotient on the Left-hand, making it to be 1047.

VII. Multiply this Quotient 1047, by 7, the laſt Figure put into the Root Quotient and the Product will be 7329, which ſet under 8067, and Subſtracting 7329 from 8067, there will remain 738, which ſet under the Line.

VIII. To this *Remainder* 738, bring down the two *Figures* of the next *Period*, viz. 29, making the third *Reſolvend* to be 73829.

IX. Double the Root Quotient, which is now 527, and it makes 1054, which ſet in a Quotient on the Left-hand of the Reſolvend 73829, and aſk how many times 1054 may be had in 7382; the Anſwer will be 7 times, put 7 in the Root Quotient, and alſo in the other Quotient, on the Left-hand of the third Reſolvend, making that Quotient to be 10547.

X. Multiply this Quotient 10547, by 7, the laſt Figure put in the Root Quotient, and the Product will be 73829, which Subſtracted from the Third Reſovend 73829, the Remainder will be nothing; which ſhews that the given Number 27846729 is a Square Number;

ber; and the Root of it is 5277; which may be easily proved by Multiplying 5277 into it self for the Product of that Multiplication will be 27846729.

Other Examples for Practice.

(I.)	5	03	50	87	21	(22439 Root
	4					
42)	1	03				Resolvend
		84				Product
444)		19	50			Resolvend
		17	76			Product
4483)		1	74	87		Resolvend
		1	34	49		Product
44869)			40	38	21	Resolvend
			40	38	21	Product
			00	00	00	Remainder.

(II.)	9	42	49	(307 Root
	9			
60)	0	42	49	Resolvend
		09		Product
607)		42	49	Product
		00	00	Remainer

A Third Example of a Number not Square.

(III.)	64	28	03	06	(8017 Root
	64				
1601)		28	03		Resolvend
		16	01		Product
16027)		12	02	06	Resolvend
		11	21	89	Product
			80	17	Remainder.

This Given Number 64280306 is not a Square Number, for you see, that after the Extraction is ended, there remains 8017, which makes the Numerator of a Fraction; and then, double the Root and add a Unite to the double, and and it will be 16035 for the Denominator; and so the near Root of 64280306 will be $8017\frac{8017}{16035}$.

I. The Figures of the *Firſt Period* being 27, the neareſt *Square Number* thereto is 25, whoſe *Root* is 5; ſet 5 in the *Quotient*, and 25 under 27, and under it draw a Line, and ſubſtracting 25 from 27, the Remainder will be 2, which ſet under 25.

	·	·	·	·	
	27	84	67	29	(5277 Root
	25				
102)	2	84			Reſolvend
	2	04			Product
1047)		80	67		Reſolvend
		73	29		Product
10547)		7	38	29	Reſolvend
		7	38	29	Product
		0	00	00	Remainder.

II. To this *Remainder* 2, bring down the two *Figures* of the next *Period*, viz. 84, making it 284 for the firſt *Reſolvend*.

III. Double the *Figure* in the Quotient 5, it makes 10, which ſet in a Quotient, on the Left-hand of 284, and aſk, how many times 10 in 28, the Anſwer will be 2; ſet 2 in the Quotient for the *Root*, and alſo in the other Quotient by 10, making it 102.

IV. Multiply 102, by 2, the laſt *Figure* in the *Root-Quotient* and the *Product* will be 204, which ſet under 284, and Subſtracting 204 from 284, the *Remainder* will be 80, which ſet under 204.

V. To this *Remainder* 80, bring down the two *Figures* of the next *Period*, viz. 67, making the ſecond *Reſolvend* to be 8067.

VI. Double the Quotient 52, and it makes 104, which ſet in a Quotient on the Left-hand of the Reſolvend 8067, and aſk, how many times 104, may be had in 806, the anſwer will be 7 times; put 7 in the Root Quotient, and alſo in the other Quotient on the Left-hand, making it to be 1047.

VII. Multiply this Quotient 1047, by 7, the laſt Figure put into the Root Quotient and the Product will be 7329, which ſet under 8067, and Subſtracting 7329 from 8067, there will remain 738, which ſet under the Line.

VIII. To this *Remainder* 738, bring down the two *Figures* of the next *Period*, viz. 29, making the third *Reſolvend* to be 73829.

IX. Double the Root Quotient, which is now 527, and it makes 1054, which ſet in a Quotient on the Left-hand of the Reſolvend 73829, and aſk how many times 1054 may be had in 7382; the Anſwer will be 7 times, put 7 in the Root Quotient, and alſo in the other Quotient, on the Left-hand of the third Reſolvend, making that Quotient to be 10547.

X. Multiply this Quotient 10547, by 7, the laſt Figure put in the Root Quotient, and the Product will be 73829, which Subſtracted from the Third Reſovend 73829, the Remainder will be nothing; which ſhews that the given Number 27846729 is a Square Number;

ber; and the Root of it is 5277; which may be easily proved by Multiplying 5277 into it self for the Product of that Multiplication will be 27846729.

Other Examples for Practice.

(I.)	5	03	50	87	21	(22439 Root
	4					
42)	1	03				Resolvend
		84				Product
444)		19	50			Resolvend
		17	76			Product
4483)		1	74	87		Resolvend
		1	74	49		Product
44869)			40	38	21	Resolvend
			40	38	21	Product
			00	00	00	Remainder.

(II.)	9	42	49	(307 Root
	9			
60)	0	42	49	Resolvend
		09		Product
607)		42	49	Product
		00	00	Remainer

A Third Example of a Number not Square.

(III.)	64	28	03	06	(8017 Root
	64				
1601)		28	03		Resolvend
		16	01		Product
16027)		12	02	06	Resolvend
		11	21	89	Product
			80	17	Remainder.

This Given Number 64280306 is not a Square Nnmber, for you see, that after the Extraction is ended, there remains 8017, which makes the Numerator of a Fraction; and then, double the Root and add a Unite to the double, and and it will be 16035 for the Denominator; and so the near Root of 64280306 will be $8017\frac{8017}{16035}$.

This is the usual and common Way; But to find the broken parts of the Root more exactly, you must add a competent Number of Pair of Cyphers, to the given Number; as if you would have the Root to the

Tenth Hundred Thousand Ten Thousand	Part of a Unite, you must then add to the Number.	2 4 6 8	Cyphers, &c.

And in so doing, the Broken part of the Root is always a Decimal, consisting of so many Places, as there are Pair of Cyphers annexed.

So, if this Number 43623 were a Number given, to find the Square Root of it, to the Thousand part of an Unite.

First, set the whole Number down, and Point it as before; And because you would have the Root to the Thousand part of an Unite; add to the Number Three Pair of Cyphers, and point them also, so will the Work stand as in this Example.

(IV.)	4	36	23	00	00	00	(208.861 Root
	4						
408)		36	23				Resolvend
		32	64				Product
4168)		3	59	00			Resolvend
		3	33	44			Product
41766)			25	56	00		Resolvend
			25	05	96		Product
4177222)				50	04	00	Resolvend
				41	77	21	Product
				8	26	79	Remainder.

And then if you Extract according to the former directions (and as you see here done) you shall find the Root to be 208.861, and yet there is a Remainder of 82679, to which if two Cyphers were added, the next Figure in the Quotient will not amount quite to 2.

And if according to this Artifice, you would Extract the Square Root of 10, you will find it to be 3.162, &c.

To find the Square Root of a Vulgar Fraction which is Commensurable.

You must first Reduce the Fraction into its Least Terms; for it may so fall out, that the Fraction in its given Terms may be Incommensurable; but being reduced to its Least, it may be Commensurable: And then, This is the

RULE.

RULE.

Extract the Square Root of the Fraction's Numerator for Numerator of the Root : and the Square Root of the Denominator of the Fraction, for the Denominator of the Root.

Example. Let it be required to find the Square Root of this Fraction $\frac{80}{245}$.

This Fraction reduced to it's least terms is $\frac{16}{49}$, and now, the Square Root of 16 is 4 for the Denominator; and the Square Root of 49, is 7 for the Denominator; so the Square Root $\frac{80}{245}$ or $\frac{16}{49}$ is $\frac{4}{7}$.

Also, the Square Root of $\frac{25}{36}$ will be found to be $\frac{5}{6}$, and of $\frac{9}{81}$, to be $\frac{3}{9}$ or $\frac{1}{3}$, &c.

To find the Square Root of a Commensurable Mixt Number.

For the Effecting hereof, this is the

RULE.

Reduce the Mixt Number into an Improper Fraction; And then, the Square Root of the Numerator and Denominator of that Improper Fraction, shall be the Square Root of the given Mixt Number.

Example. Let it be required to find the Square Root of this Mixt Number $34\frac{33}{64}$.

This Mixt Number reduced into an Improper Fraction, will be $\frac{2209}{64}$ which being in it's Least Terms, needs no more reducing: So then, the Square Root of 2209 is 47, and the Square Root of 64 is 8, and the Square Root of the Mixt Number $34\frac{33}{64}$ (or of $\frac{2209}{64}$) is $\frac{47}{8}$ or $5\frac{7}{8}$.

Note. When a Mixt Number, or a Proper Fraction is Incommensurable to its Square Root, prefix this Character [$\surd$] before it, so the Square Root of $7\frac{2}{3}$ will be thus expressed, $\surd q\, 7\frac{2}{3}$—Also, the Square Root of $\frac{13}{16}$ must be thus expressed $\surd q \frac{13}{16}$. For these and such like cannot be expressed by any Rational Numbers whatsoever.

II. *Of the Cube Root.*

A Cube *is a Solid Figure, contained under Six equal* Squares. (Euclid, Lib. 11. Def. 25) *and may be fitly represented by a* Dye.

Of Cube Numbers.

Of *Cube Numbers* there are Three distinct kinds or Species; *viz.* *Single*, *Compound* and *Irrational*.

 1. *Single*,

1. *Single*, Such are called *Single Cube Numbers*, which are made of any one Single ſignificant Figure Multiplied Twice into it ſelf: As 1 Multiplies nothing, and ſo is both *Root* and *Cube*: But 2 times 2 is 4, and 2 times 4 is 8: ſo that 2 is the Root, and 8 the Cube: alſo, 3 times 3 is 9; and 3 times 9 is 27; and here 3 is the *Root*, and 27 the *Cube*. And ſo of all the Nine *Digit Numbers*, as in this Table.

The Root or Side.		*Multiplied into it ſelf, produceth the Square Number.*		*And that Multiplied again into the Side produceth the Cube Number.*	
	1		1		1
	2		4		8
	3		9		27
	4		16		64
	5		25		125
	6		36		226
	7		49		343
	8		64		512
	9		81		729

2. *Compound*, ſuch are called *Compound Cube Numbers*, whoſe *Roots* conſiſt of more Figures than One; So, if 12 be the *Root*, then 12 times 12, *viz.* 144 is the *Square*; and 12 times 144 is 1728 the *Cube*; Alſo, the *Cube* of 22 is 10648, *&c.*

3. *Irrational*: Thoſe are called *Irrational Cube Numbers*; whoſe exact *Cube Root* cannot be found out by any Artifice yet diſcovered, either in *Whole Numbers*, *Fractions* or *Decimals*; and ſuch are the *Cube Roots* of 2,4,7,10, and infinite others.

The Geneſis *for Extracting the* Cube Root.

Example, *Let it be required to find that* Cube Number; *whoſe* ſide, *or* Root *is* 57.

5	..7
125	...
52	5..
7	35.
	343
185	193

1. Set down the *Root* 57, with the interval of two Figures, between 5 and 7, as in the Margine; and draw a Line under them, and alſo two down-right Lines, one next after 7, the other after 5, for the more orderly placing of the Numbers to be added: Then let the *Root* given be ſuppoſed to be divided into two parts, *viz.* 500 and 7.

2. Set the *Cube* of 5, which is 125 under 5 (*Unites* under *Unites*, &c.) and the *Cube* of 7, which is 343, under 7, two Lines, or Places, below the *Cube* of 25, (*Unites* under *Unites*.)

3. Triple the *Square* of 5, which is 25, and it makes 75, which Multiply by 7, and it makes 525, which ſet under 125, and place forwarder to the right Hand.

4. Triple 5, it makes 15, which Multiply by 49, (the *Square* of 7) it

it makes 735, which set under 525, one place forwarder towards the right Hand.

5. Draw a Line, and add all the Numbers together, in the same order as they stand, and the *Sum* of them will be 185193, which is a *Cube Number*, of which 57 is the *Root*: Which may be easily proved, by Multiplying 57 into it self, and that *Product* again by 57, so the last *Product* will be 185193.

Other Examples for Practice.

4	..8		9	..9
64	...		729	...
38	4..		218	7.
7	68.		21	87.
	512			729
10	992		970	299

To Extract the Cube Root.

When a Number is given, to have the Cube Root thereof found; You must, First, write the Number down; then put a Prick over the first Figure towards the right hand, which is the Place of Unites; and so over every Third Figure from that Place of Unites, towards the left hand: Then, by every Pointed Figure, draw a down right Line, for the more orderly setting of the Figures to be added and Substracted; and also a crooked Line for a Quotient, on the right Hand, and so is your Number prepared for Extraction; For the performance whereof, observe these following Directions.

Example I. *Let it be required to find the Cube Root of this Compound Cube Number* 185193.

1. Set down the Number given 185193 and make a Prick over 3, the Place of Unites, and missing two Places, make another Prick over 5: Then make a Crooked Line for a Quotient, and by 3 and 5 draw two down right Lines: So will your Number stand thus,

$$18\dot{5} \mid 19\dot{3} \mid ($$

2. The three first Figures of the given Number towards the Left Hand, are 185, which is the First Period; seek (by the foregoing Table) the nearest Cube Number to it, being less; which you will find to be 125, whose Cube Root is 5 in the Quotient, and the Cube thereof 125, whose Cube Root is 5, place the Root 5 in the Quotient, and the Cube thereof 125 under 185, drawing a Line under 125, and Substracting it from 185, there will remain 60, which set

ſet under the Line: And ſo is the Firſt operation, for the Firſt Period, ended.

3. To the 60 which remained, bring down the three Figures of the next Period, *viz.* 193, making the 60 to be 60193, which is called the Reſolvend: under which draw a Line; and then the work will ſtand thus.

185	193	(5 Reſolvend
125		
60	193	

4. Triple the Root in the Quotient 5, it makes 15, which ſet under the Reſolvend, in ſuch order, the place of Unites in this Triple, may ſtand under the Place of Tens in the Reſolvend; ſo the Triple of the Root 5, being the 15, ſet the Unite 5 under 9, the Place of Tens in the Reſolvend and then the Work will ſtand thus.

185	193	(5
125		
60	193	Reſolvend
	15	The Triple of the Root 5

5. Triple the Square of the Root 5, and it makes 75 (for 5 times 5 is 25, and 3 times 25 is 75) which place under the Triple of the Roo t, in ſuch order that the pla ce of Unites in this, may ſtand under the place of Tens in the Triple; And then the Work will ſtand thus.

185	193	(5
125		
60	193	Reſolvend
	15	Triple of the Root 5
7	5	Triple of the Square of the Root 5

6. Draw a Line under the Triple of the Square of the Root, and add that and the Triple of the Root together, in the ſame order as they ſtand, ſo ſhall their Sum be 765, for a Diviſor; under which alſo draw a Line, and the Work will ſtand thus.

185	193	(5
125		
60	193	Reſolvend
	15	Triple of the Square of the Root 5
7	65	Diviſor

7 Draw

7. Draw a Crooked Line on the Left-hand of the Resolvend, in which place this Divisor last found, *viz.* 765. to which the whole Resolvend (except the place of Unity) must be a Dividend; that is, ask how many times 765 you can have in 6019; the answer will be 7 times, which put in the Quotient: And this 7 is the Second Figure of the Root: And then the Work will stand thus,

	185 125	193	(57
765)	60	193	Resolvend
	7	15 5	Triple of the Root 5 Triple of the Square of the Root 5
	7	65	Divisor

8. Cube 7, the last Figure in the Root Quotient, and it is 343, which place under the Resolvend, in such order, that Unites may stand under Unites; so will 343, the Cube now found stand under 193 of the Resolvend: And the Work will stand thus,

	185 125	193	(57
765)	60	193	Resolvend
	7	15	Triple of the Root 5 Triple of the Square of the Root 5
	7	5	Divisor
		343	The Cube of 7, the last Fig. in Quo.

9. Multiply the Square of 7, the last Figure of the Root, namely 49; by the Triple Root next under the Resolvend, *viz.* by 15, and the Product will be 735, which place under 343, (the Cube last set down) in such order, that the place of Tens in that: And then will the Work stand thus,

	185	193	(57
	125		
765)	60	193	Reſolvend
		15	Triple of the Root 5
	7	5	Triple of the Square of the Root 5
	7	65	Diviſor
		343	The Cube of 7, the laſt Figure of the Root.
	7	35	The Square of 7, by the Triple Square of 5

10. Multiply the Triple Square of the Root 5, *viz.* 75, by 7, the ſecond Figure of the Root, the Product will be 525, which place under 735, in ſuch order, that the place of Unites in this; may ſtand under the place of Tens in that: And then will the Work ſtand thus,

	185	193	(57
	125		
765)	60	193	Reſolvend
		15	Triple of the Root 5
	7	5	Triple of the Square of the Root 5
	7	65	Diviſor
		343	The Cube of 7, the laſt Figure of the Root
	7	35	The Square of 7, by the Triple Square of 5
	52	5	Triple Square of Root 5, in the Root 7

Laſtly, draw a Line under the three laſt Numbers, and add them together in the ſame order as they ſtand, and their Sum will be 60193, which may be called the Subtrahend, becauſe it is always to be Subſtracted from the Reſolvend; But in the Example, they are equal one to the other and the Remainder is 00000, which ſhews, that the given Number 185193 is a Compound Cube Number, and that the Cube Root of it is 57. And ſo the whole Work will ſtand as here you ſee.

	185 125	193	(57 The Root
785)	60	193	Resolvend
		15	Triple of Root 5
	7	5	Triple of the Square of Root 5
	7	65	Divisor
		343	The Cube of 7, the last Figure of the Root
	7	35	Square of 7, by the Triple Square of 5
	52	5	Triple Square of Root 5, in Root 7
	60	193	Subtrahend
	00	000	Remainder.

Now, If you compare this Example, with the former *Genesis*, you shall find that the first Number there 125, will be equal to the nearest Square less than 185: And the Second Number there, 525 is the same with the Triple Square of the Root 5, multiplied in the Root 7 — And again, the third Number there 735, is the same with the Square of the Root 7, multiplied in the Triple of Root 5: And lastly, the Number 343 there, is the same with the Cube of the Root 7.

Another Example for Practice.

	95 64	256	152	263	(4567 Cube Root.
492)	31	256			First Resolvend
		12			Triple of the Quotient 4
	4	8			Triple Square of the Quotient 4
	4	92			Divisor
		125			Cube of last Figure in Quotient 5
	3	00			Sq. of 5, in the Trip. Sq. of 4
	24	0			Triple Sq. of Root 4 in Root 5
	27	125			Subtrahend
60885)	4	131	152		Second Resolvend
		1	35		Triple of the Quotient 45
		607	5		TripleSquare of the Quotient 45
		608	85		Divisor
			216		Cube of 6, the last Fig. in Quot.
		48	60		Squ. of 6, by the Trip. Sq. of 45
	3	645	0		Trip. Sq. of 45, in the last Fig. 6
	3	693	816		Subtrahend
6239448)		437	336	263	Third Resolvend

6239448)		437	336	263	Third Resolvend (456
			13	68	Triple of the Quotient 456
		62	380	8	Triple Square of the Quotient
		62	394	48	Divisor
				343	Cube of 7, the last Fig. in Quot.
			670	32	Sq. of 7, by the Trip. Sq. of 456.
		436	665	6	Tri. Sq. of 456, in last Fig. in Quo. 7
		437	336	263	Subtrahend
		000	000	000	Remainder.

Note, when at any time in your Extraction, you find a Subtrahend to be greater than the Resolvend next before (from whence it is, always to be Substracted) your work is erroneous; and must be rectified by putting a less Figure in the Quotient.

In the two foregoing Examples, both the Numbers proposed proved to be exact Cubical Numbers: Wherefore, I will here add another Example (ready wrought) of a Number not Cubical, and shew how to find the Fraction part of the Root in Decimal Parts, to the 10, 100, 1000, &c. parts of a Unite, or farther if the niceity of the work do so require.

Let therefore the Cube Root of this Number 23456789, be required to be found.

The Number being written down, Pointed, and the Cube Root thereof Extracted, as is done in the following Operation: The Cube Root will be found to be 286, and then there will be a Remainder of 63133, now to find the Root nearer; add to the Number given, 3, 6, 9 or 12 Cyphers, whereby to attain more Decimal Parts in the Root: — So in this Example I have added twice three Cyphers; with which, I go on with the Extraction till I have two Decimal Parts in the Root: For the Root is 286 Integers, and by the adding of the six Cyphers, it hath two Decimal Parts in the Root, namely .25 which is too little: For that Work being ended, you will find a Remainder of 1732359375, to which, if you add three Cyphers more, there will be 3 Places of Decimal Parts in the Root; but this gives the Root to $\frac{1}{100}$ part of a Unite and if more Triples of Cyphers were added, it would give the Root to the $\frac{1}{1000}$, or $\frac{1}{10000}$ Part of a Unite, &c.

The

The OPERATION.

	23 8	456	789	000	000	(286.25
126)	15	456				Resolvend
		06				
	1	2				
	1	26				Divisor
		512				
	3	84				
	9	6				
	13	952				Subtrahend
23604)	1	504	789			Resolvend
			84			
		235	2			
		236	04			Divisor
			216			
		30	24			
	1	411	2			
	1	441	656			Subtrahend
2454738)		63	133	000		Resolvend
			8	58		
		24	538	8		
		24	547	38		Divisor
				8		
			34	32		
		49	077	6		
		49	111	928		Subtrahend
245739906)		14	021	072	000	Resolvend
				85	86	
		2	457	313	2	
		2	457	399	06	Divisor
					125	
			2	146	50	
		12	286	566	0	
		12	288	712	625	Subtrahend
		1	732	359	375	000, &c.

To find the Cube Root of a Vulgar Fraction, or Mixt Number.

Let the Fraction or Mixt Number be first Reduced into its Least Terms; And then, this is the

RULE.

The Cube Roots of the Numerators and Denominators of Vulgar Fractions or Mixt Numbers; shall be the Numerators and Denominators of the Cube Roots of these Fractions or Mixt Numbers.

So, this Proper Fraction $\frac{40}{135}$, neither the Numerator nor Denominator are Cube Numbers; but Reduced to its least Terms it becomes $\frac{8}{27}$, and then, the Cube Root of 8 is 2, and the Cube Root of 27 is 3, so that the Cube Root of the Proper Fraction, is $\frac{2}{3}$.

Allso, the Cube Root of this Mixt Number $20\frac{51}{64}$, will be found to be $\frac{11}{4}$ or $2\frac{3}{4}$. For the Mixt Number $20\frac{51}{64}$ or Improper Fraction $\frac{1331}{64}$ being in its Least Terms; the Cube Root of 1331, is 11 for the Numerator of the Root, and the Cube Root of 64 is 4, for the Denominator of the Cube Root.

Note, When a Vulgar Fraction or Mixt Number is Incommensurable to its Root; such are usually thus expressed, *viz.* The Cube Root of $\frac{2}{3}$ thus $\sqrt{c}\,\frac{2}{3}$; and the Cube Root of $2\frac{3}{8}$ thus $\sqrt{c}\,2\frac{3}{8}$, &c.

SECT. II.

Of Interest, Simple and Compound; Discount or Rebate of Money, and of Æquation of Payments, &c. *With Tables of all of them.*

MOney put out to Use, is divided into Three Parts, *viz.* Principal, Time and Interest: The First signifies the Sum, or Value, of the Money, or Goods so Lent — Time is the Forbearance of it; as Years, Months, Weeks or Days — Interest, is the Profit that ariseth from the other Two.

Use or Interest, is either Simple or Compound — Simple Interest is Computed from the Principal and Time only, upon a Certain Rate agreed upon, But — Compound Interest (after the First Year, or other Time Limited for the first Payment) attracts a Proporti-

onal

onal Use imposed upon the 6 *l.* or other Rate due at the first Years End, (if continued longer) And therefore is called Compound, or Interest upon Interest.

I. *Of Simple Interest.*

FOr the Arithmetical Working of Questions whereof this is the

PROPORTION.

As the Principal and Time, for which a Loan is allow'd,
Is to the Interest thereof;
So will any other Sum of Money to be Borrowed,
Be to the Interest for the same Time.

Example I. What is the Interest of 145 *l.* for a Year, at 6 *l. per Cent.* for a Year.

Proport.] As 100 *l.* : is to 6 *l.* :: So is 145 *l.* : to 8.70 *l.*

RULE.

Multiply 145 *l. by* 6 *l. the Product will be* 870, *which divided by* 100 *l.* (*by cutting off two Figures*) *the Quotient will be* 8.70; *which Reduced is* 8 l. 14 s. *for the Interest of* 145 l. *for a Year.*

Example II. What is the Interest of 250 *l.* for 5 Months, at 8 *l. per Cent. per An.*

Proport. { As 100 *l.* in 12 Months : Is to 8 *l.*
So is 250 *l.* in 5 Months; To 8 *l.* 6 *s.* 8 *d.*

RULE.

Multiply 100 l. *by* 12 *The Product will be* 1200 *for a Divisor: Then Multiply* 250 l. *by* 8 l. *the Product will be* 2000, *and that Multiplied by* 5 *Months; produceth* 10000, *for a Dividend: Then* 10000 *divided by* 1200 (*adding Cyphers*) *giveth in the Quotient* 8.333 l.

Which 8.333 Reduced, is 8 *l.* 6 *s.* 8 *d.* for the Interest of 250 *l.* in 5 Months, at 8 *l. per Cent.*

Example III. One lent 650 *l.* which was re-paid again at the end of 6 Months, 3 Weeks and 3 Days: What came the Interest thereof to at 6 *per Cent.*

Proport.

Proport. { As 100 *l.* in 365 Days: Is to 6 *l.*
So is 650 *l.* in 192 Days: To 20 *l.* 10 *s.* 3 *d.* 3 *q.* *fere.*

RULE.

Multiply 365 *by* 100, *the Product will be* 36500 *for a Divisor: Also, Multiply* 650 *by* 6, *and the Product of that by* 192, *the last Product will be* 748800; *For a Dividend: Which Divided by* 36500, *will give in the Quotient* 20.515. *And Answers the Question.*

For the Quotient 20.515 Reduced, is 20 *l.* 10 *s.* 3 *d.* 3 *q.* and so much doth the Interest of 650 *l.* amount unto in 6 Months 3 Weeks and 3 Days: (or 192 Days) at 6 *l. per Cent. per An.*

Example IV. What will the Simple Interest of 265 *l.* 13 *s.* 4 *d.* 1 *q.* amount unto in a Year at 6 *l. per Cent. per An.*

Proport. { As 100 *l.* Is to 6 *l.* in a Year,
So is 265 *l.* 13 *s.* 4 *d.* 1 *q.* To 15 *l.* 18 *s.* 9 *d.* 2 *q.* in a Year.

RULE.

Reduce the 265 *l.* 13 *s.* 4 *d.* 1 *q. into a Decimal, and it will be* 265.667, *This multiplied by* 6 *l.* (*the Rate of Interest*) *produceth* 1594, *which divided by* 100 (*by cutting off two Figures*) *the Quotient will be* 15.94 *l.*

Which Reduced is 15 *l.* 18 *s.* 9 *d.* 2 *q.* for the Interest of 265 *l.* 13 *s.* 4 *d.* 1 *q.* for a Year.

PROBLEM.

How to find the Interest due upon any Sum of Money, for any Number of Days, and at any Rate of Interest:

For the Working of Questions of this Nature, This is a

GENERAL RULE.

Multiply the Principal Sum by the Rate of Interest; and that Product by the Number of Days proposed: This last Product Divide (*always*) *by* 36500, (*the Number of Days in one Year, with two Cyphers added*) *the Quotient will answer the Question demanded.*

Example I. What will the Interest of 750 *l.* amount unto, in 232 Days, at 6 *l. per Cent. per An.?*

Multiply 752 (the Principal Sum) by 6 *l.* the Rate of Interest; the Product will be 4512; which Multiplied by 232 (the Number of Days) produceth 1046784: Which Divided by 36500 (adding Cyphers

Cyphers if need be) the Quotient will be 28.677, which Decimal Fraction Reduced, is 28 *l.* 13 *s.* 7 *d.* And so much will the Interest of 752 *l.* amount unto in 232 Days.

Example II. What will the Interest of 8 *l.* 10 *s.* amount unto, in 120 Days, at 9 *l.* *per Cent. per An.*

	l.
The Principal Sum ———	80.5
The Rate of Interest ———	9
The Product is ———	724.5
Which Multiplied by the Number of Days, ———	120
The Product thereof is ———	86940.0

Divisor	*Dividend*	*Quotient*
36500)	86940.000	(2.382

Which Quotient Reduced, is 2 *l.* 7 *s.* 8 *d.* for the Interest of 80 *l.* 10 *s.* in 120 Days.

Pounds

	1 Month.				2 Months.				3 Months.			
Pou	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*
1	0	0	1	1	0	0	2	2	0	0	3	2
2	0	0	2	2	0	0	4	3	0	0	7	1
3	0	0	3	2	0	0	7	1	0	0	10	3
4	0	0	4	3	0	0	9	2	0	1	2	1
5	0	0	6	0	0	1	0	0	0	1	6	0
6	0	0	7	1	0	1	2	2	0	1	9	2
7	0	0	8	2	0	1	4	3	0	2	1	1
8	0	0	9	2	0	1	7	1	0	2	4	3
9	0	0	10	3	0	1	9	2	0	2	8	2
10	0	1	0	0	0	2	0	0	0	3	0	0
11	0	1	1	1	0	2	2	2	0	3	3	2
12	0	1	2	2	0	2	4	3	0	3	7	1
13	0	1	3	2	0	2	7	1	0	3	10	3
14	0	1	4	3	0	2	9	2	0	4	2	2
15	0	1	6	0	0	3	0	0	0	4	6	0
16	0	1	7	1	0	3	2	2	0	4	9	2
17	0	1	8	2	0	3	4	3	0	5	1	1
18	0	1	9	2	0	3	7	1	0	5	4	3
19	0	1	10	3	0	3	9	2	0	5	8	2
20	0	2	0	0	0	4	0	0	0	6	0	0
30	0	3	0	0	0	6	0	0	0	9	0	0
40	0	4	0	0	0	8	0	0	0	12	0	0
50	0	5	0	0	0	10	0	0	0	15	0	0
60	0	6	0	0	0	12	0	0	0	18	0	0
70	0	7	0	0	0	14	0	0	1	1	0	0
80	0	8	0	0	0	16	0	0	1	4	0	0
90	0	9	0	0	0	18	0	0	1	7	0	0
100	0	10	0	0	1	0	0	0	1	10	0	0
200	1	0	0	0	2	0	0	0	3	0	0	0
300	1	10	0	0	3	0	0	0	4	10	0	0
400	2	0	0	0	4	0	0	0	6	0	0	0
500	2	10	0	0	6	0	0	0	7	10	0	0
600	3	0	0	0	7	0	0	0	9	0	0	0
700	3	10	0	0	8	0	0	0	10	10	0	0
800	4	0	0	0	9	0	0	0	12	0	0	0

Pou	4 months.				5 months.				6 months.			
	l.	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*
1	0	0	4	3	0	0	6	0	0	0	7	1
2	0	0	9	2	0	1	0	0	0	1	2	2
3	0	1	2	2	0	1	6	0	0	1	9	2
4	0	1	7	1	0	2	0	0	0	2	4	3
5	0	2	0	0	0	2	6	0	0	3	0	0
6	0	2	4	3	0	3	0	0	[illegible]	3	7	1
7	0	2	9	2	0	3	6	[illegible]	[illegible]	4	2	2
8	0	3	2	2	0	4	0	0	0	4	9	2
9	0	3	7	1	0	4	6	0	0	5	4	3
10	0	4	0	0	0	5	0	0	0	6	0	0
11	0	4	4	3	0	5	6	0	0	6	7	1
12	0	4	9	2	0	6	0	0	0	7	2	2
13	0	5	2	2	0	6	6	0	0	7	9	2
14	0	5	7	1	0	7	0	0	0	8	4	3
15	0	6	0	0	0	7	6	0	0	9	0	0
16	0	6	4	3	0	8	0	0	0	9	7	1
17	0	6	9	2	0	8	6	0	0	10	2	2
18	0	7	2	2	0	9	0	0	0	10	9	2
19	0	7	7	1	0	9	6	0	0	11	4	3
20	0	8	0	0	0	10	0	0	0	12	0	0
30	0	12	0	0	0	10	6	0	0	18	0	0
40	0	16	0	0	1	0	0	0	1	4	0	0
50	1	0	0	0	1	5	0	0	1	10	0	0
60	1	4	0	0	1	10	0	0	1	16	0	0
70	1	8	0	0	1	15	0	0	2	2	0	0
80	1	12	0	0	2	0	0	0	2	8	0	0
90	1	16	0	0	2	5	0	0	2	14	0	0
100	2	0	0	0	2	10	0	0	3	0	0	0
200	4	0	0	0	5	0	0	0	6	0	0	0
300	6	0	0	0	7	10	0	0	9	0	0	0
400	8	0	0	0	10	0	0	0	12	0	0	0
500	10	0	0	0	12	10	0	0	15	0	0	0
600	12	0	0	0	15	0	0	0	18	0	0	0
700	14	0	0	0	17	10	0	0	21	0	0	0
800	16	0	0	0	20	0	0	0	24	0	0	0

	7 months.				8 months.				9 months.			
Pou	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*
1	0	0	8	2	0	0	9	2	0	0	10	3
2	0	1	4	3	0	1	7	1	0	1	9	2
3	0	2	1	1	0	2	4	3	0	2	8	2
4	0	2	9	2	0	3	2	2	0	3	7	1
5	0	3	6	0	0	4	0	0	0	4	6	0
6	0	4	[illegible]	2	0	4	9	2	0	5	4	3
7	0	4	[illegible]	3	0	5	7	1	0	6	3	2
8	0	5	7	1	0	6	4	3	0	7	2	2
9	0	6	3	2	0	7	2	2	0	8	1	1
10	0	7	0	0	0	8	0	0	0	9	0	0
11	0	7	8	2	0	8	9	2	0	9	10	3
12	0	8	4	3	0	9	7	1	0	10	9	2
13	0	9	1	1	0	10	4	3	0	11	8	2
14	0	9	9	2	0	11	2	2	0	12	7	1
15	0	10	6	0	0	12	0	0	0	13	6	0
16	0	11	2	2	0	12	9	2	0	14	4	3
17	0	11	10	3	0	13	7	1	0	15	3	2
18	0	12	7	1	0	14	4	3	0	16	2	2
19	0	13	3	2	0	15	2	2	0	17	1	1
20	0	14	0	0	0	16	0	0	0	18	0	0
30	1	1	0	0	1	4	0	0	1	7	0	0
40	1	8	0	0	1	12	0	0	1	16	0	0
50	1	15	0	0	2	0	0	0	2	5	0	0
60	2	2	0	0	2	8	0	0	2	14	0	0
70	2	9	0	0	2	16	0	0	3	3	0	0
80	2	16	0	0	3	4	0	0	3	12	0	0
90	3	3	0	0	3	12	0	0	4	1	0	0
100	3	10	0	0	4	0	0	0	4	10	0	0
200	7	0	0	0	8	0	0	0	9	0	0	0
300	10	10	0	0	12	0	0	0	13	10	0	0
400	14	0	0	0	16	0	0	0	18	0	0	0
500	17	10	0	0	20	0	0	0	22	10	0	0
600	21	0	0	0	24	0	0	0	27	0	0	0
700	24	10	0	0	28	0	0	0	31	10	0	0
800	28	0	0	0	32	0	0	0	36	0	0	0

	10 months.				11 months.				A Year.			
Pou	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*
1	0	1	0	0	0	1	1	1	0	1	2	2
2	0	2	0	0	0	2	2	2	0	2	4	3
3	0	3	0	0	0	3	3	2	0	3	7	1
4	0	4	0	0	0	4	4	3	0	4	9	2
5	0	5	0	0	0	5	6	0	0	6	0	0
6	0	6	0	0	0	6	7	1	0	7	2	2
7	0	7	0	0	0	7	8	2	0	8	4	3
8	0	8	0	0	0	8	9	2	0	9	7	1
9	0	9	0	0	0	9	10	3	0	10	9	2
10	0	10	0	0	0	11	0	0	0	12	0	0
11	0	11	0	0	0	12	1	1	0	13	2	2
12	0	12	0	0	0	13	2	2	0	14	4	3
13	0	13	0	0	0	14	3	2	0	15	7	1
14	0	14	0	0	0	15	4	3	0	16	9	2
15	0	15	0	0	0	16	6	0	0	18	0	0
16	0	16	0	0	0	17	7	1	0	19	2	2
17	0	17	0	0	0	18	8	2	1	0	4	3
18	0	18	0	0	0	19	9	2	1	1	7	1
19	0	19	0	0	1	0	10	3	1	2	9	2
20	1	0	0	0	1	2	0	0	1	4	0	0
30	1	10	0	0	1	13	0	0	1	16	0	0
40	2	0	0	0	2	4	0	0	2	8	0	0
50	2	10	0	0	2	15	0	0	3	0	0	0
60	3	0	0	0	3	6	0	0	3	12	0	0
70	3	10	0	0	3	17	0	0	4	4	0	0
80	4	0	0	0	4	8	0	0	4	16	0	0
90	4	10	0	0	4	19	0	0	5	8	0	0
100	5	0	0	0	5	10	0	0	6	0	0	0
200	10	0	0	0	11	0	0	0	12	0	0	0
300	15	0	0	0	16	10	0	0	18	0	0	0
400	20	0	0	0	22	0	0	0	24	0	0	0
500	25	0	0	0	27	10	0	0	30	0	0	0
600	30	0	0	0	33	0	0	0	36	0	0	0
700	35	0	0	0	38	10	0	0	42	0	0	0
800	40	0	0	0	44	0	0	0	48	0	0	0

II. *Of Discount, or Rebate of Money.*

FOr the Discount, or Rebate of Money; This is the

PROPORTION.

As 100 *l.* and its Interest at a Years end,
Is to 100 *l.*
So is 100 *l.*
To the Present Worth of 100 *l.*
Suppose at VI. *per Cent.*
As 106 *l.* is to 100 *l.*
So is 100 *l.* to 9434 *l.*

Divisor	*Dividend*	*Quotient*		*s.*	*d.*	*q.*
106)	100.00000	(94.3396	Or,	18	10	2

So that 100 *l.* to be paid at a Years end, is worth in present Money 94 *l.* 6 *s.* 9 *d.* 2*q.*

And this is the difference between Interest and Rebate: For you are not to Rebate 6 *l.* out of 100 *l.* to be paid at the Years end, but 5 *l.* 13 *s.* 2 *d.* 2 *q.* which is less than 6 *l.* by 6 *s.* 9 *d.* 2 *q.* Thus for a Year.

But if you would know the Rebate of any other Sum, at any other Rate, and for any other Time, More or Less than a Year; Then use this

PROPORTION.

As 100 *l.* with the Interest thereof for any Time required,
Is to 100 *l.*
So is the Debt due to be Paid at the Expiration of that Time
To the present Worth thereof.

Example. If a Legacy, or other Sum of Money, as 345 *l.* become due to be Paid at six Months end: What Sum of Money will discharge the same?

Proport.] As 103 *l.* is to 100 *l.* So is 345 *l.* to 334. 951 *l.*

Divisor	*Dividend*	*Quotient*
103)	34500.000	(334.951

Which Quotient 334.951 reduced, is 334 *l.* 19 *s.* 0 *d.* 1 *q.* and so much ought to be paid for the 345 *l.* abating for the said 6 Months 10 *l.* 0 *s.* 11 *d.* 2 *q.* whereas the Interest thereof for the same Time, is 10 *l.* 7 *s.* the Difference being 6 *s.* 2 *q.* And according to this Proportion, is the following Table of Discount or Rebate, Calculated.

Pou	1 month.				2 months.				3 months.			
	l.	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*
1	0	19	10	3	0	19	9	2	1	19	8	2
2	1	19	9	2	1	19	7	1	1	19	5	0
3	2	19	8	2	2	19	4	3	2	19	1	1
4	3	19	7	1	3	19	2	2	3	18	9	3
5	4	19	6	0	4	19	0	0	4	18	6	1
6	5	19	4	3	5	18	9	3	5	18	2	3
7	6	19	3	3	6	18	7	1	6	17	11	1
8	7	19	2	2	7	18	5	0	7	17	7	2
9	8	19	1	1	8	18	2	2	8	17	4	0
10	9	19	0	0	9	18	0	1	9	17	0	2
11	10	18	11	0	10	17	9	3	10	16	9	0
12	11	19	9	3	11	17	7	2	11	16	5	2
13	12	18	8	2	12	17	5	0	12	16	1	3
14	13	18	7	1	13	17	2	3	13	15	10	2
15	14	18	6	0	14	17	0	2	14	15	6	3
16	15	18	5	0	15	16	10	0	15	15	3	1
17	16	18	3	3	16	16	7	3	16	14	11	3
18	17	18	2	2	17	16	5	2	17	14	8	1
19	18	18	1	1	18	16	3	0	18	14	4	2
20	19	18	0	0	19	16	0	2	19	14	1	0
30	29	17	0	1	29	14	0	3	29	11	1	2
40	39	16	0	1	39	12	1	0	38	8	2	1
50	49	15	0	1	49	10	1	1	45	5	2	2
60	59	14	0	1	59	8	1	2	59	2	2	1
70	69	13	0	2	69	6	1	3	68	19	3	3
80	79	12	0	2	79	4	2	0	78	16	4	1
90	89	11	0	2	89	2	2	1	84	13	4	3
100	99	10	0	2	99	0	2	2	98	10	5	1
200	199	0	1	1	198	0	4	3	197	0	10	2
300	298	10	1	3	297	0	7	1	295	11	4	0
400	398	0	2	2	396	0	9	2	394	1	9	1
500	497	19	3	0	495	0	11	3	492	12	2	2
600	597	0	3	2	594	1	2	1	591	2	8	0
700	696	10	4	1	693	1	4	3	689	13	1	1
800	796	0	4	3	792	1	7	0	788	3	6	2

	4 months.				5 months.				6 months.			
Pou.	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*
1	0	19	7	1	0	19	6	1	0	19	5	0
2	1	19	2	2	1	19	0	1	1	18	11	0
3	2	18	9	3	2	18	6	2	2	18	3	0
4	3	18	5	1	3	15	0	2	3	17	8	0
5	4	18	0	2	4	15	6	1	4	17	1	0
6	5	17	7	3	5	15	0	3	5	16	6	0
7	6	17	3	0	6	16	7	0	6	15	11	0
8	7	16	10	1	7	16	1	1	7	15	4	0
9	8	16	5	3	8	15	7	1	8	14	9	0
10	9	16	1	0	9	15	1	2	9	14	2	0
11	10	15	6	1	10	14	7	2	10	13	7	0
12	11	15	3	2	11	14	1	3	11	13	0	0
13	12	14	10	3	12	13	8	0	12	12	5	1
14	13	14	6	0	13	13	2	0	13	11	10	1
15	14	14	1	2	14	12	8	1	14	11	3	1
16	15	13	8	3	15	12	2	1	15	10	8	1
17	16	13	4	0	16	11	8	2	16	10	1	1
18	17	12	11	1	17	8	2	2	17	9	6	1
19	18	12	6	2	18	10	8	3	18	8	11	1
20	19	12	2	0	10	10	3	0	19	8	4	1
30	29	8	2	3	29	5	4	1	29	2	6	1
40	39	4	3	3	39	0	5	3	38	18	8	2
50	49	0	4	3	48	15	7	1	48	10	10	2
60	58	16	5	3	58	10	8	3	58	5	0	2
70	68	12	6	3	68	5	10	1	67	19	2	3
80	78	8	7	2	78	0	11	3	77	13	4	3
90	84	4	8	2	87	16	1	1	87	7	6	3
100	98	0	9	2	97	11	2	3	97	1	9	0
200	196	1	6	3	195	2	5	1	194	3	6	0
300	294	2	4	1	292	13	8	0	291	5	3	0
400	392	3	1	3	390	4	10	2	388	7	9	0
500	490	3	11	0	487	16	1	1	485	8	8	3
600	588	4	8	2	585	7	3	3	582	10	5	3
700	686	5	5	3	682	18	6	2	679	12	2	3
800	784	6	3	1	780	9	9	0	776	13	11	2

	7 months.				8 months.				9 months.			
Pou	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*
1	0	19	4	0	0	19	2	3	0	19	1	3
2	1	18	7	3	1	18	5	2	1	18	3	1
3	2	17	11	3	2	17	8	1	2	17	5	0
4	3	17	3	2	3	16	11	0	3	16	6	3
5	4	16	5	2	4	16	1	3	4	15	8	1
6	5	15	11	1	5	15	4	2	5	14	10	0
7	6	15	3	1	6	14	7	2	6	13	11	3
8	7	14	7	0	7	13	10	1	7	13	1	1
9	8	13	11	0	8	13	1	0	8	12	3	0
10	9	13	2	3	9	12	3	3	9	11	4	3
11	10	12	6	3	10	11	6	2	10	10	6	1
12	11	11	10	2	11	10	9	1	11	9	8	0
13	12	11	2	2	12	10	0	0	12	8	9	3
14	13	10	6	1	13	9	2	3	13	7	11	1
15	14	9	10	1	14	8	5	0	14	7	1	0
16	15	9	2	1	15	7	8	1	15	6	2	3
17	16	8	6	0	16	6	11	0	16	5	4	1
18	17	7	10	0	17	6	1	3	17	4	6	0
19	18	7	1	3	18	5	4	2	18	3	7	3
20	19	6	5	3	19	4	7	2	19	2	9	1
30	28	19	8	2	28	16	11	0	28	14	2	0
40	38	12	11	1	38	9	2	3	38	5	6	2
50	48	6	2	1	48	1	6	2	47	6	11	1
60	57	19	7	0	57	13	10	1	57	8	4	0
70	67	12	8	0	67	6	1	3	66	19	8	2
80	77	5	10	3	76	18	5	2	76	11	1	1
90	86	19	1	2	86	10	9	1	86	2	5	3
100	96	12	4	2	96	3	1	0	95	13	10	2
200	193	4	8	3	192	6	1	3	191	7	9	0
300	289	17	1	1	288	9	2	3	287	1	7	2
400	386	9	5	2	384	12	3	3	382	15	6	0
500	483	1	10	0	480	15	4	2	478	9	4	2
600	579	14	2	2	576	10	5	2	574	3	3	0
700	676	6	6	3	673	1	6	2	669	17	1	2
800	772	18	11	1	769	4	7	2	765	11	0	0

Pou	10 months.				11 months.				A year.			
	l.	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*
1	0	19	0	2	0	18	11	2	0	18	10	2
2	1	18	1	1	1	17	11	0	1	17	8	3
3	2	17	1	3	2	16	10	2	2	16	7	1
4	3	16	2	1	3	15	10	0	3	15	5	3
5	4	15	2	3	4	14	9	2	4	14	4	0
6	5	14	3	2	5	13	9	0	5	13	2	2
7	6	13	4	0	6	12	8	2	6	12	1	0
8	7	12	4	2	7	11	8	0	7	10	11	1
9	8	11	5	1	8	10	7	2	8	9	9	3
10	9	10	5	3	9	9	7	0	9	8	8	1
11	10	9	6	1	10	8	6	1	10	7	6	2
12	11	8	6	3	11	7	5	3	11	6	5	0
13	12	7	7	2	12	6	5	1	12	5	3	2
14	13	6	8	0	13	5	4	3	13	4	1	3
15	14	5	8	2	14	4	4	1	14	3	0	1
16	15	4	9	1	15	3	3	3	15	1	10	3
17	16	3	9	3	16	2	3	1	16	0	9	0
18	17	2	10	1	17	1	2	3	16	19	7	2
19	18	1	10	3	18	0	2	1	17	18	6	0
20	19	0	11	3	18	19	1	3	18	17	4	1
30	28	11	5	1	28	8	8	3	28	6	0	2
40	38	1	10	3	37	18	3	2	37	14	8	2
50	47	12	4	2	47	7	10	2	47	3	4	3
60	57	2	10	1	56	17	5	1	56	12	1	0
70	66	13	4	0	66	7	0	1	66	0	9	0
80	76	3	9	3	76	16	7	0	75	9	5	1
90	85	14	3	2	85	6	2	0	84	18	1	1
100	95	4	9	1	94	15	8	3	94	6	9	2
200	190	9	6	1	189	11	5	3	188	13	7	0
300	285	14	3	2	284	4	2	2	283	0	4	2
400	380	19	0	2	379	2	11	1	377	7	2	0
500	476	3	9	3	473	18	8	0	471	13	11	2
600	571	8	6	3	568	14	5	0	566	0	9	0
700	666	13	4	0	663	10	1	3	660	7	6	2
800	761	18	1	1	758	5	10	2	754	14	4	0

III. Of Æquation of Payments.

ÆQuations of Payments, are of Two kinds; *viz.*

1. Several Equal Payments, due at Equidistant Times.

2. Of several Unequal Payments, at several times not Equidistant.

For the First of these: [Æquation of Equal Payments at Equidistant times; This is the General

RULE.

From the whole Account of the Annuity, or, Monthly Payment, Substract the Aggregate of the several Payments; and the Remainder (if yearly) multiply by 365. Or (if Monthly) by 30.416 Days. Then, Divide that Product by the Annual or Monthly Interest of the said Aggregate, and the Quotient will give the Number of Days before the End (or Term) of the Annual or Monthly Payment.

Example I. If A, be to Pay unto B 100 *l. per An.* for Five Years; and they agree, that A shall Pay to B, the whole 500 *l* at one intire Payment, at 6 *per Cent*: The Question is, What time before the Expiration of the whole Five Years, A must Pay B the 500 *l.*

According to the RULE,

	l.
From the whole Account of the Annuity in the Five Years.	560
Substract the Aggregate	500
The Remainder is	60
Which multiplied by	365
The Product will be	21900
That Divided by 30 (the Five Years Interest) the Quot. is	730

So that the 500 *l.* must be Paid 730 days before the expiration of the Five Years: Divide then 730 by 365 days, the Quotient will be 2 Years: So that A must Pay unto B, the 500 *l.* at the end of Three Years.

Example II. A is to Pay unto B, 62 *l. per Annum* for Four Years, but they mutually agree, that A shall Pay the Aggregate of the whole Sum, *viz.* 248 at one intire Payment, at 6 *l. per Cent.*

How many days before the expiration of the whole Four Years, must this Payment be made?

Look in the following Table of 6 *per Cent.* for the Account of 100 *l.* in 4 Years, which you will find to be 436: This multiplied by 62 (the Annual Payment) the Product will be 27032: And that Divided by 100, the Quotient will be 270.32 *l.* for the Account of 62 *l.* in 4 Years: —Being thus prepared; Proceed according to the RULE; as followeth

1. The Annual Payment for 4 Years, is	62.00
2. The Account of that in 4 Years, is	270.32
3. The Aggregate of 4 Years Payment, is	248.00
4. The Annual Interest of the Aggregate, is	14.88
Out of the whole Account	270.32
Substract the Aggregate,	248.00
The Remainder is	22.32
Which multiplied by 365 days	365
The Product is	8146.80
Which Divided by the Annual Interest	14.88
The Quotient is	547.50

And so many Dayes before the Expiration of the Four Years, must the Sum of 248 *l.* be Paid, at One intire Payment.

For Proof hereof I say,

If 248 *l.* be put out at Interest at 6 *l. per Cent.* for One Year, and 172.5 Days: I say, If the Interest thereof in that time do amount unto 270.32 *l.* the Work is Proved Thus;

The Interest of 1 *l.* for 1 day, is	.000164383
Which multipled by	547.5
The Product is	.0899996925
Which multiplied by	248
The Product is	22.3199997680
To which add	248.
The Sum is	270.3199997680

Proving

Proving the Work to be Exact.

Hitherto of Yearly Payments: Now for Monthly Payments.

Example III. *A* is to Pay unto *B* 100 *l.* a Month, for Five Months: At what time must he pay it at one intire Payment, at *6 l. per Cent?*

1. The Interest of 1 *l.* for one Month, is	.005
Which multiplied by 500, is the Monthly Interest of 500 *l. Viz.*	2.500
2. The full Amount of 5 Monthly Payments of 100 *l.* is	500
The Double of the Monthly Interest. is	5.000
The Whole Amount	505
3. From the Whole Amount 505, Deduct the Agreg.	500
The Remainder is	.005

4. Multiply 30.416 (the Days in a compleat Month) by 5, the Product will be 152.080; which Divided by 2.500 (the Monthly Interest of 500 *l.*) the Quotient will be 60.832 days; that is 60 days, and about 20 hours, which is two compleat Months before the 5 Months: Or, 3 Months after the day of Agreement.

Of the Æquation of Unequal Payments, at Times not Equi-distant.

This manner of Work will best be understood by Example, Therefore,

Example I. One Owes 500 *l.* which he is to Pay, at three several Unequal Payments; *Viz.* At the end of Four Months 300 *l.* at the end of Six Months 100 *l.* And at the end of 12 Months 100 *l.* But the Debter agrees with the Creditor to Discharge the Debt (*viz*, 500 *l.*) at one intire Payment. The Question is; At what time 500 *l.* may be Paid, without Dammage or Prejudice to either of the two Parties?

To Resolve Questions of this Nature ; This is the General

RULE.

First, find the true amount of each of the Sums, from the First day of the Agreement, to the Last day of Payment; as supposing them to be forborn to the Last. Then out of that, Deduct the Aggregate of the Respective Payments, and multiply the Remainder (if Annual) by 365 Days, (if Monthly) by 30.416 ; and the Product, Divide by the Annual or Monthly Interest of the said Aggregate ; And the Quotient gives the Number of Days, from the Last day of Payment ; Accounting backwards.

The manner of the Work is as followeth.

First, The Length of Time from the Day of Agreement, to the Last Day of the Payment, is just 12 Months.

So Then,

	l.
I. 300 *l.* Payable after 4 Months, and being forborn to the end of 12 Months, hath 8 Months Interest to Account for, *viz.*	12.000
II. 100 *l.* Payable after 6 Months, and being forborn to the end of 12 Months, hath 6 Months Interest to Account for, *viz.*	3.000
III. To these Sums add the	500.000
The Whole amount is	515.000
IV. From the Whole amount 515 *l.* Substract the Aggregate of the Sums	500.000
The Remainder is	15.000

Being thus far prepared : The Proportion is,
As 2.5 *l.* (the Interest of 500 *l.* for 1 Month)
Is to 1 Month ;
So is 15. (the Remainder)
To 6 Months.

As 2.5 *l*: is to 1 Mon : : So is 15 *l*: to 6 Months: And that Resolves the Question ; So that if the whole 500 *l.* be Paid at 6 Months end, there will be no Loss or Dammage to either Debtor or Creditor.

For

For the Proof of this,

	l.
I. 300 *l.* was Due at 4 Months end, and being continued 2 Months longer, the Interest for 2 Months is 3 *l.* and the Whole amount is——— ——— ———	303.00
II. 100 *l.* Paid at 6 Months end, which is the time it was Due, is therefore just——— ——— ———	100.00
III. The other 100 *l.* Paid at 6 Months before the time there must be an Abatement made of 3 *l.* and is	97.00
The Total	500.0.0

So that as there is in the First Sum an Increase of 3 *l.* and in the Last a Decrease of 3 *l.* which are to be set one against the other, and the Whole amount is the Aggregate of the Respective Sums; and being Paid at the end of 6 Months, makes the Æquation just 500 *l.*

Example II. *A* Owes to *B* 100 *l. per Annum* for Five Years, and they agree, that *A* shall Pay it off at the end of any of the Four Years; for at the end of Five Years nothing Less than the whole 560 *l.* will Pay the Debt.

			l.
Thus the Present Worth of the whole Amount 560 *l.* at the	First	Years end is	451.6129
	Second		474.5762
	Third		500.0000
	Fourth		528.3018
	Fifth		560.0000

These Numbers are found, by help of the Numbers in the Third and Fourth Columns of a Table in *Cursus Mathematicus*, Pag. 116.

And the Proportion for finding them is

As 100 *l.* (the Amount of the Annual Payment)
Is to 94.33.962 (the present Worth of the First year)
So is 560 *l.* (the Whole Amount in Five Years)
To 528.3018, (the present worth at the Fourth Year's end.

And so of the rest.

	l.				l.
For, As	100	Is to	94.33962	So is 560 *l.* to	528.3018
	206		183.92856		500.0000
	318		269.49152		474.5762
	436		351.61290		451.6129
	560		430.76923		430.7690

IV. Other

IV. *Other Rules for Æquations of Payments.*

I Shall here insert some other Rules for the Æquations of Payments; which (although not so exact,) are performed with more ease: And although in great Sums and long times of Payment, there may be some small Difference: Yet small Sums and short times being of more frequent Use, and the Difference but small; I shall here shew the manner of Working them this other way.

I. *When the Terms of Payment are equal.*

Example I. *A* Owes to *B* 400 *l.* to be Paid at Four Six Months, that is, 100 *l.* at 6 Months, 100 *l.* at 12 Months; 100 *l.* at 18 Months; and 100 *l.* at 24 Months: But it is agreed to Pay the whole Mony at One intire Payment. What is the true time of Payment?

In this Question, both the Sums to be Paid, and the Times of Payment are Equal; and to Resolve the Question, This is the

RULE.

Add One Term of Time of Payment to the Terms of Time given; And half that Number of Terms shall be the Time of Payment.

In this Question, the Terms of Payment are Four; and the distance of each of them Six Months; in all 24 Months; to which Add One Term more, and it makes 30 Months; the half whereof is 15 *Months*: At the Expiration of which Time, the Payment is to be made.

Example II. If *A* be to Pay unto *B*, 100 *l.* a year for Five years; At what Term of Time must *A* Pay *B* the Whole 500 *l.* at One intire Payment?

Here the Terms of Time are Five, to which add One, they make 6; the half whereof is Three years; at the end whereof, *A* ought to Pay unto *B* the whole 500 *l.* at One intire Payment. And this agrees exactly with the First Example of the former way of Working.

Example III. *A* is to Pay unto *B* 248 *l.* at Four Equal Payments; *viz.* 62 *l.* at 12 Months, 62 *l.* at 24 *Months*; 62 *l.* at 36 Months: and 62 *l.* at 48 *Months*: At what Time ought it to be paid at One intire payment?

Add

Add 12 *Months* to the 48 *Months*, and they make 60 *Months*; the half whereof is 30 *Months*; at the end of which Time, *A* is to pay unto *B*, the 248 *l.* at One entire payment: And 30 *Months* contains 547.5 Days; exactly agreeing with the foregoing Second *Example*.

II. *When the Sums of Mony, and Times of Payment are Different.*

Example I. *A* is to pay unto *B*, 400 *l.* in this manner: *viz.* 100 *l.* at 3 *Months*; 100 *l.* at 6 *Months*; 100 *l.* at 12 *Months*; and 100 *l.* at 24 *Months*: At what time will it become payable at One intire payment?

To Resolve this, or the like, Question; This is the

RULE.

Multiply the several Sums, by the Times of their Respective Payment; and add them altogether: Then Divide that Sum by the Whole Debt; and the Quotient shall give you the Term of Time, for the Entire Payment at Once.

$$\text{Thus, } 100\ l. \text{ Multiplied by} \left\{\begin{matrix} 3 \\ 6 \\ 12 \\ 24 \end{matrix}\right\} \text{Months, is} \left\{\begin{matrix} 300 \\ 600 \\ 1200 \\ 2400 \end{matrix}\right\} \textit{Pound.}$$

The Sum is —4500

And this Sum Divided by 400 *l.* the Whole Debt, giveth in the Quotient 11.25, or 11 *Months* and One Quarter of a *Month*; At the Expiration of which Time (after the Time of Contract) must the Whole 400 *l.* be paid at Once?

Example II. *A* is to pay unto *B*, a Legacy of 500 *l.* in this manner; 300 *l.* at 4 *Months*; 100 *l.* at 6 *Months*; and 100 *l.* at 12 *Months*: At what Time must it be paid at Once?

$$\left.\begin{matrix} 300 \\ 100 \\ 100 \end{matrix}\right\} \text{Multiplied by} \left\{\begin{matrix} 4 \\ 6 \\ 12 \end{matrix}\right\} \text{makes} \left\{\begin{matrix} 1200 \\ 600 \\ 1200 \end{matrix}\right\} \textit{Pounds.}$$

500 The Sum 3000

Which Sum Divided by 500, the Quotient is 6 *Months*, and at that

that Time ought the 500 *l.* to be paid at Once: And this agrees with the same Question Wrought by the Preceeding *Example* I.]

And this Resolves the Question according to the Interest Table; For,

$$\text{The Interest of}\left\{\begin{array}{c}300\\100\\100\end{array}\right\}\text{For}\left\{\begin{array}{c}4\\6\\12\end{array}\right\}\text{Months, is}\left\{\begin{array}{c}6\\3\\6\end{array}\right\}\textit{Pounds.}$$

$$\overline{\quad 15 \quad}$$

And the Interest of 500 *l.* in 6 Months, is 15 *l.*

But to Resolve this or the like Question more exactly, is by the Rebate, and not by the Interest of the *Mony*; And then,

$$\text{The Rebate of}\left\{\begin{array}{c}l.\\300\\100\\100\end{array}\right\}\text{For}\left\{\begin{array}{c}4\\6\\12\end{array}\right\}\text{Months, is}\left\{\begin{array}{rrrr}l. & s. & d. & q.\\294. & 2. & 4. & 1\\97. & 1. & 9. & 0\\94. & 6. & 9. & 2\end{array}\right.$$

	l.	s.	d.	q.
Total ——	485.	10.	10.	3
But the Rebate of 500 *l.* for 6 Mon. is but ——	485.	8.	8.	3
The Difference ——	000.	2.	2.	0

And thus much concerning Simple Interest and Rebate.

IV. *Of Compound Interest: With the Constructi-on and Use of the Tables thereof.*

THe Tables, which I shall here shew the Construction and Use of are in Number Five; And all of them Calculated according to the Present-Rate of Interest by Law now established in *England*; that is to say, at Six Pound *per Cent. per Annum* for a year, Compound Interest: And from 1 year to 31 years: The two First of the Tables (as being of more frequent Use than the other three, which are deduced from them) are Caculated for Years, Months, Weeks and Days: And the other three but for Years only; And by one or other of these Five Tables, all Questions of Compound Interest Mony may be comprehended and Solved; The Tables here follow; And after them their Construction and Uses.

TABLE

TABLE I.

Shewing what One Pound being forborn any number of years, months, weeks and days, (under 31 years) will amount unto: Accounting Intereſt upon Intereſt at VI. *per Cent.*

Y.	Dec. P.
1	1.06000
2	1.12360
3	1.19102
4	1.26248
5	1.33823
6	1.41851
7	1.50363
8	1.59384
9	1.68947
10	1.79084
11	1.89829
12	2.01210
13	2.13292
14	2.26090
15	2.39655
16	2.54035
17	2.69277
18	2.85433
19	3.02559
20	3.20713
21	3.39956
22	3.60353
23	3.81975
24	4.04893
25	4.29187
26	4.54938
27	4.82234
28	5.11168
29	5.41838
30	5.74349

	Months.
1	1.004867
2	1.009760
3	1.014674
4	1.019613
5	1.024576
6	1.029563
7	1.034574
8	1.039610
9	1.044671
10	1.049756
11	1.054865

	Weeks.
1	1.001118
2	1.002237
3	1.003358

	Days.
1	1.000160
2	1.000319
3	1.000479
4	1.000369
5	1.000798
6	1.000958

TABLE II.

Shewing the Decreaſe of One Pound, Or, what One Pound at the End of any Number of Years, Months, Weeks and Days (under 31 years) is Worth in ready Mony, Rebating Intereſt upon Intereſt, at VI. *per Cent.*

Y.	Dec. P.
1	.943396
2	.889996
3	.839919
4	.792093
5	.747958
6	.704960
7	.665057
8	.627412
9	.591898
10	.558394
11	.526787
12	.496989
13	.468839
14	.442300
15	.417265
16	.393646
17	.371364
18	.350343
19	.330512
20	.311804
21	.294155
22	.277505
23	.261797
24	.246978
25	.232998
26	.219810
27	.207367
28	.195630
29	.184556
30	.174110

	Months.
1	.995156
2	.990335
3	.985538
4	.980764
5	.976013
6	.971286
7	.966581
8	.961859
9	.957239
10	.952643
11	.947988

	Weeks.
1	.998883
2	.997767
3	.996653

	Days.
1	.999840
2	.999681
3	.999521
4	.999361
5	.999202
6	.999042

TABLE III.

Shewing what 1 *l.* will amount unto, it being Forborn any number of years under 31 at VI. *per Cent.* Compound Interest.

Y.	Dec.Par.
1	1.00000
2	2.06000
3	3.18360
4	4.37461
5	5.63709
6	6.97531
7	8.39383
8	9.89746
9	11.49131
10	13.18079
11	14.97164
12	16.86994
13	18.88213
14	21.01506
15	23.27596
16	25.67252
17	28.21287
18	30.90565
19	33.75999
20	36.78559
21	39.99272
22	43.39228
23	46.99582
24	50.81557
25	54.86451
26	59.15638
27	63.70576
28	68.52810
29	73.63979
30	79.05818

TABLE IV.

Shewing the present worth of 1 *l.* annuity, to be paid yearly to continue any Number of years under 31, at VI *per Cent.* Compound Interest.

Y.	Dec.Par.
1	0.94340
2	1.83339
3	2.67301
4	3.46510
5	4.21236
6	4.91732
7	5.58238
8	6.20979
9	6.80169
10	7.36008
11	7.88687
12	8.38384
13	8.85268
14	9.29498
15	9.71224
16	10.10589
17	10.47725
18	10.82760
19	11.15811
20	11.46992
21	11.76407
22	12.04158
23	12.30337
24	12.55035
25	12.78335
26	13.00316
27	13.21053
28	13.40616
29	13.59071
30	13.76482

TABLE V.

Shewing what Annuity (payable yearly) 1 pound will Purchase, for any Number of years under 31. at VI *per Cent.* Compound Interest.

Y.	Dec.Par.
1	1.06000
2	.54544
3	.37411
4	.28859
5	.23740
6	.20336
7	.17913
8	.16103
9	.14702
10	.13586
11	.12679
12	.11927
13	.11296
14	.10758
15	.10296
16	.09895
17	.09544
18	.09235
19	.08962
20	.08718
21	.08 00
22	.08304
23	.08127
24	.07969
25	.07822
26	.07690
27	.07569
28	.07459
29	.07375
30	.07264

The

The Construction of TABLE I.

THe first Column of this Table, having [Years] at the head thereof, begins at 1, and so proceeds to 30; and the Numbers in the next Column standing against any number of Years, are Decimal Numbers, which shew what one Pound (or 20 *s.*) is worth (or will amount unto) being forborn any number of Years under 31: And this Table is made according to this Proportion.

As 100 *l.*
Is to 100 *l.* and the increase thereof in one Year, *viz.* 106 *l.*
So is 1 *l.* (or 20 *s.*)
To 1 *l.* and the increase of it in a Year, *viz.* 1.06000.

Which is the Decimal of 1 *l.* and the Increase of it in a Year, and is the first Number in the second Column of the Table against 1 Year.

Then for the Second Number,

As 100 *l.* Is to 106*l.*
So is 1.06 *l.* To 1.12360 for the Second Year.

Then for the Third Year.

As 100 *l.* Is to 106 *l.*
So is 1.1236 to 1.19102

For the Third Year, *Et sic ad Infinitum:* Thus for the whole Years. But,

To find Decimal Numbers, for any part of a Year; as Months, Weeks, Days; or for Half-years or Quarterly-Payments.

Take the Decimal for one Years Increase, *viz.* 1.06000 the Square Root whereof is 1.02956, and is the Decimal of the Increase of 1 *l.* in 6 Months: And the Mean proportional between 1.02956 and 1.06000 will be 1.04467, and is the Decimal for the Increase of 1 *l.* in 9 Months: And the mean Proportional between an Unite (or 1) and 1.02956, will be 1.01467 and is the Decimal of the Increase of 1 *l.* in 3 Months. And thus you may with facility discover all the o-Numbers in this first Table.

The Use of TABLE I.

Example I. What will 136 *l.* 15 *s.* 6 *s.* amount unto, it being forborn 20 years; at 6 *l. per Cent. per Annum* Compound Interest?

Look in the Table for 20 years, and right against it you shall find 3.207136, which is the Increase of 1 *l.* in 20 years; Multiply this

Number by 136.775 the Decimal of 136 *l.* 15 *s.* 6 *d.* the Product will be 438.65520 which Reduced is 438 *l.* 13 *s.* 1 *d.* 1 *q.* and unto so much will 136 *l.* 15 *s.* 6 *d.* be augmented unto in 20 years.

Example II. What will 200 *l.* increase unto, if forborn 6 Months at 6 *l. per Cent. per Annum* Interest upon Interest?

The Increase of 1 *l.* in 6 months is 1.029563, which multiplyed by 200 *l.* the Product will 205.912600 *l.* which Reduced is 205 *l.* 18 *s.* 3 *d.* and to so much will 200 *l.* be Increased in 6 months.

Example III. What will the Increase of 300 *l.* amount unto, if forborn 3 Weeks, at 6 *l. per Cent.* Compound Interest.

The Increase of 1 *l.* in 3 Weeks is 1.003358, which multiplyed by 300 *l.* the Product is 301.007400, and that Reduced is 301 *l.* 0 *s.* 1 *d.* 3 *q.* from which Substract 300 *l.* and the remainder 1 *l.* 0 *s.* 1 *d.* 3 *q.* is the Increase of the 300 *l.* in 3 Weeks.

Example IV. If 3600 *l.* be forborn 5 days, what shall be the Increase of it, at 6 *l. per Cent. per Annum* Compound Interest?

The Increase of 1 *l.* in 5 days is 1.000798 which multiplyed by 3600 *l.* the Principal, the Product will be 3602.872800, which reduced is 3602 *l.* 17 *s.* 5 *d.* 2 *q.* from which Substract 3600, the Remainder 2 *l.* 17 *s.* 5 *d.* 2 *q.* is the Increase in five Days.

The Construction of TABLE II.

The PROPORTIONS.

AS 100 *l.* with the Increase of it due at a years End, *viz.* 106 *l.*
Is to the Principal, 100 *l.*
So is 1 *l.* due at the same time:
To .943396, the discount of 1 *l.* for a year, *viz.* 18 *s* 10 *d.* 1 *q.*

Then,

As 106 *l.*
Is to the Decimal last found .943396
So is 100 *l.* present pay
To .889996 the Decimal for the second year, *viz.* 17 *s.* 8 *d.* 2 *q.*
Et sic ad infinitum, for whole years; But,

To find the Decimal Numbers for parts of a year upon Discount; or for half years or quarterly-payments.

These are composed after the same manner as the Table of money forborn, excepting only in the Pointing of the Numbers for Extracting the Roots; the Decimals in the first Table being all Mixt Numbers

numbers; and these for Discount are every one proper Fractions; having a Point prefixed: Wherefore, in these proper Fractions, make the first Point under the second Figure on the Left-hand: As for Example .943396 is the Decimal for the years Rebate of 1 *l.* put the first point over the Figure 4, and so in order to the Right hand; the Root thus Extracted will be .971286, for the discount of 1 *l.* in 6 months. The Square Root of that again will be .985538, for 3 months; and thus proceed with mean Proportionals until the Places are all compleat between the Radius and the Decimal last found: As for half yearly and quarterly Payments, they are discovered as were those before in the Forbearance of Money; of which you may see variety of ways in my *Cursus Mathematicus.*

The Use of TABLE II.

Example I. If 356 *l.* be payable at the end of 7 Years, what is it worth in present Money, Discount or Rebating, after the Rate of 6 *l. per Cent. per Annum* Compound Interest?

Look in the Table II. for 7 years, against which is .665057 and so much ready Money is 1 *l.* or 20 *s.* due at 7 years End worth presently; multiply .665057 by 356, the whole Sum, the Product will be 236.761292 which Reduced is 236 *l.* 15 *s.* 2 *d.* 1 *q.* and so much is the 356 *l.* worth in present Money.

Example II. At the end of 6 months, *A* is to pay unto *B* 500 *l.* but they agree that it shall be paid presently upon Discount after the rate of 6 *l. per Cent.* Interest upon Interest.

Look into Table II, for 6 months, against which stands .971286, the Decimal of the worth of 1 *l.* 6 months hence: This Multiplied by the Sum to be paid 500 *l.* the Product is 485,643000, which Reduced is 485 *l.* 12 *s.* 10 *d.* 3 *q.* And so much present Money will discharge the Debt of 500 *l.* due at 6 months end.

Example III. *A* hath a Lease in Reversion, which at the Expiration of 7 years is valued to be worth 1200 *l.* which Lease *B* would Purchase with present ready Money: Rebating after the Rate of 6 *per Cent. per Annum* Compound Interest, what ready Money must *B* give *A* for this Lease?

Look in Table II. for 7 years, and against it is .665057 the Decimal for the worth of 1 *l.* or 20 *s.* due at 7 years end: This Decimal multiply by the worth of the Lease after 7 years 1200 *l.* the Product will be 798.068400: which Reduced is 798 *l.* 1 *s.* 4 *d.* 2 *q.* and so much present Money must *B* disburse, to purchase the Lease.

Example

Example IV. *A* is to pay unto *B*, a Legacie of 1800 *l.* at three ſeveral Payments, *viz.* 600 *l.* at the end of ſix months, 600 *l.* more at 12 months, and 600 *l.* more at 18 months: *B* deſires the money preſently; and *A* is willing upon Diſcount of 6 *per Cent. per Annum* Compound Intereſt: What preſent money will ſatisfie the whole Legacie?

Look in Table II. for the Decimal of the Diſcount for 6 months, where you ſhall find it to be .971286, which multiplyed by 600 *l.* (the firſt Payment to be due at the half years end) the Product will

Decimals	*l.* — *s.* — *d.*	
582.771600	582—15—5	1
566.637600	566—00—9	2
549.784200	549—15—8	3
1698.593400	1698—11—10	

be 582.771600, which Reduced is 582 *l.* 15 *s.* 5 *d.* Then is there 600 *l.* upon a years Rebate, the Decimal for 1 year is .943396, which multiplied by 600 *l.* the Product is 566.037600; and that Reduced is 566 *l.* 0 *s.* 9 *d.* due upon the years Rebate, as appears in the Table above. Now the laſt payment is 600 *l.* upon 18 months Rebate; now to find a Decimal Number for this; do thus: This Decimal for one years Diſcount is .943396, and for 2 years .889996, theſe two multiplied, the product will be 839618666416 and the Square Root thereof .916307, and this multiplied by 600 *l.* the laſt payment, the product will be 549.784200, which Reduced is 549 *l.* 15 *s.* 8 *d*, and the Total 1698 *l.* 11 *s.* 10 *d.* which Sum will diſcharge all the Three payments at one Time.

The Conſtruction of TABLE III.

THis Table is deduced from Table I. For if you add the principal 1.000000 *l.* to the firſt Number in the firſt Table 1.060000 (which is the principal and Intereſt of 1 *l.* for a year,) the Sum of them will be 2.060000 *l.* and that is the Number ſtanding againſt the ſecond year in this third Table — Again, to this ſecond Number in Table III, add the ſecond Number in Table I, *viz.* 1.123600; the Sum will be 3.183600, for the third Number in Table III.— And to this third Number in Table III, add the third Number in Table I, the Sum will be 4.374616, for the fourth Number in Table III, and ſo of all the reſt.

The

The Use of TABLE III.

Example I. What will an *Annuity* of 20 *l.* payable yearly, be augmented unto in 12 years; being all that time forborn? Accounting Interest upon Interest at 6 *l. per Cent per Annum.*

Look for 12 years in the Table III. against which stands 16.86994, which shews that an *Annuity* of 1 *l.* a year, in 12 years will amount unto 16.86994 *l.* wherefore multiply 16.86994 by 20, the product will be—337.9980 *l.* which Reduced is 337 *l.* 7 *s.* 11 *d.* 3 *q.* And so much will the *Annuity* of 20 *l. per An.* be augmented unto, if forborn 12 years.

Example II. If an *Annuity* of 60 *l.* a year, be forborn 7 years; How much will it amount unto when that term of years is expired?

The Number standing against 7 years in this Table III. is 8.39383, which multiplied by 60, the product will 503.62980 *l.* which Decimal Reduced is 503 *l.* 12 *s.* 7 *d.* And unto so much will the *Annuity* be augmented unto in 7 years.

The Construction of TABLE IV.

As the Third *Table* was Deduced from *Table* I. So this Fourth *Table* is Deduced from *Table* II. Now the Number standing against 1 year in *Table* II. is .943396 and this must also be the first Number in *Table* IV. to that Add .889996 (the second Number in *Table* II.) the Sum 1.833392 is the second Number in *Table* IV.— Again, to this second Number in *Table* IV. add the third Number in *Table* II. *Viz.* .839619, the Sum will be 2.673011, which must be the third Number in *Table* IV. *& sic*, &c.

The Use of TABLE IV.

Example I. What is the present Worth of an *Annuity*, or Rent of 50 l. *per Annum*, payable yearly, for 21 years, accounting Compound Interest, after the Rate of 6 l. *per Cent.*

The Number standing against 21 years, in *Table* IV. is 11.76407, the present Worth of 1 l. *Annuity* for 21 years: Wherefore multiply 11.76407, by 50, the product will be 588.20350 l. which Decimal Reduced is 588 l. 4 s. 0 d. 3 q. And so much is the present Worth of the 50 l. a year Worth for 21 years in present *Mony*?

The

The Construction of TABLE V.

For the Construction of this Table, this is the

PROPORTION.

AS 1.83339, the Decimal for 2 years Rent Rebated (as in *Table* IV.)
Is equal in Value to 1 l. *Annuity* for 2 years,
So is 1 l. of *Annual Annuity* for the same term of time in proportion to the Decimal Purchased by 1 l.

Or,

As 1.83339 : is to 1 l. : : So is 1.00000 : to .54544.
And this is the second Number in *Table* V.
Now for the third years Decimal.
As 2.67301, the Decimal for 3 years Rent Rebate
Is to an Unite with Cyphers 1.0000000000 :
So will 1 l. for a Purchase, be in proportion
To .37411. The Number standing against 3 years in *Table* V.
Et sic &c.

The Use of TABLE V.

Example I. What *Annuity* to begin presently and to continue 28 years, will 640 l. Purchase : Accounting Compound Interest after the Rate 6 l. *per Cent. per Annum.*

The Number standing against 28 years in *Table* V. is .07459 ; which shews, that 1 l. (or 20 s.) will Purchase an *Annuity* Worth .07459 (or 1 s. 5 d. 3 q. to continue 28 years ;) Wherefore, multiply .07459, by 640 l. the product will be 47.73760 l. Which Reduced is 47 l. 14 s. 9 d. And so much *per Annum* will 640 l. Purchase for 28 years to Commence presently.

Example II. What *Annuity* Rent or Pension, will 250 l. in ready *Mony*, Purchase, for 7 years : Compound Interest allowed at 6 l. *per Cent. per Annum.*

The Decimal standing against 7 years, is .17913, which multiplied by 250 l. the product will be 44.78250 l. which Reduced is 44 l. 15 s. 8 d. 1 q. And such *Annuity* will 250 l. Purchase for 7 years.

Ex-

Example III. A Citizen giving over Trade Reſignes over to a Servant of his,his Shop ready Furniſhed, the Wares Prized, and the Leaſe of his Houſe valued,all together being Worth 1658 l. which the Maſter was willing to Receive in equal and Annual payments in 7 years; the Compound Intereſt agreed upon, at 6 l. *per Cent. per An.* VVhat muſt the Annual payment be?

The whole Stock being Valued at 1658l. and the term of time for payment 7 years, multiply the Decimal ſtanding againſt 7 years, which is .19613, by 1658l. the product will be 296.99754, which Reduced is 296l. 19s. 11q. (which you may call 279l.) and ſo much paid yearly for 7 years, will Diſcharge the Debt.

SECT. III.

Of the Menſuration of Superficies and Solids; And of the Works of the ſeveral Artificers relating to Building.

I. *Of the Menſuration of Plain, or Superficial Figures.*

PLain, or Superficial Figures are ſuch as conſiſt of Length and Breadth only; not having any Commenſurable Depth or Thickneſs: As Board, Glaſs, Pavements, Land, &c.

1. *How to Meaſure a Square.* Figure I.

The RULE.

Multiply the Length of any of the Sides of the Square (the Dimenſion thereof being taken in any kind of Meaſure) as Feet, Inches, Yards, Poles, or Perches. &c. *into it ſelf: The Product of that Multiplication ſhall give the Superficial Content, (or Area of that Square,) in ſuch Meaſure as the Side of the Square was Meaſured by: Whether, Inches, Feet, Perches, or any other Meaſure.*

Let *A B C D*, be a Square,whoſe Side is 24 foot and a half,(or 24.5 foot:) multiply 24.5, by 24.5 the product will be 600.25 foot,which is 600 foot and a quarter, and ſo many Square feet are contained in that Square whoſe Side is 24.5 foot.

2. *How to Measure a Parallelogram, or Long Square.* Fig. II.

The RULE.

Multiply the Length by the Breadth, the Product shall give the Area, or Superficial Content of the Figure.

Let *E F G H*, be a Long Square or Parallelogram ; whose Length *E F*, is 36.25 Pole or Perches ; and the Breadth *E F* 12.5 Perches.

Multiply 36.25 by 12.5, the product will be 453.125 Perches, for the Area, or Superficial Content of the Parallelogram.

3. *How to Measure a Triangle.* Fig. III.

The RULE,

For the Measuring of any Triangle, there are Three ways ; all which are comprised in this One

GENERAL RULE.

From the Angle which is opposite to the Longest or shortest Side of any Triangle, let fall a Perpendicular ; Then, (1.) Half the Length of the Longest or Shortest Side, Multiplied into the Length of the Perpendicular, the Product of that Multiplication shall give the Area of the Triangle. Or, (2.) The Length of the Longest or Shortest Side of the Triangle, being Multiplied by half the Length of the Perpendicular ; the Product shall give the Area or Superficial Content of the Triangle.

Let *K L M* be a Triangle, whose $\left\{\begin{array}{l}\text{Longest}\\ \text{Shortest}\end{array}\right\}$ Side is *L M*, from the Angle *K*, opposite to the $\left\{\begin{array}{l}\text{Longest}\\ \text{Shortest}\end{array}\right\}$ Side *L M*, let fall the Perpendicular *K N*. Then,

In the First Triangle, the Longest Side *L M*, is 22 Inches, and the Perpendicular *K N* 9 Inches, half the Longest side 11, multiplyed by 9, the Perpendicular, produceth 99 Inches for the Area of the First Triangle.

Or, 22, the Length of the Longest Side multiplied by 4.5 Inches, half the Length of the Perpendicular, the product will also be 99.00 for the Superficial Content of the Triangle in Inches. Again,

In the Second Triangle, Let the Shortest Side *L M* be 7 Inches and a Quarter (or 7.25 Inches) and the Perpendicular *K N* 24 Inches ; I say, that 3.625 (Half the Length of the Shortest Side,) multiplied by 24, (the Length of the Perpendicular) the product will be 87 Inches for the Content of the Triangle. Or, 7.25, (the Length of the Shortest Side,)

Side,) multiplied by 12, (half the *Perpendicular*) the product will be 87 inches also, for the Content of the Triangle.

4. *How to Measure a Figure of Four un-equal Sides, commonly called a Trapezia.* Fig. IV.

Let *D E F G* be a Trapezia, from *D* to *F*, the Longest Diagonal of the Figure, draw the Right Line *D F*; which shall Reduce the Trapezia *G D E F* into two Triangles, *E D F*, and *G D F*; this Line or Diagonal being common to both Triangles: Then from the Angles *E* and *G* Let fall two *Perpendiculars E H*, and *G K* upon the Diagonal *D F*. Now, to find the Area of this Trapezia, this is the

RULE.

Multiply the whole Length of the Diagonal by the half Sum of the two Perpendiculars; that Product shall be the Area of the Trapezia: —Or, Multiply half the Length of the Diagonal, by the Sum of the two Perpendiculars, and that Product shall give you the Area, or Content of the Trapezia.

In the Trapezia *D E F G*, the Diagonal *D F*, is Common to both the Triangles, and is in length 73 *Perches*: the *Perpendicular G K* is 28 *Perches*; and the *Perpendicular E H* 18 *Perches*: their Sum is 46, and their half Sum 23 *Perches*. Now,

If you multiply 73 (the whole Diagonal) by 23, (half the Sum of the *Perpendiculars*) the product will be 1679 for the Area, or Superficial Content of the Trapezia, in *Perches*.—Or, 36.5 (half the Length of the Diagonal) multiplied by 46 (the Sum of the *Perpendiculars*) the product will be 1679 as before, for the Area, of the Trapezia.

5. *How to Measure an Irregular Superficies having many unequal Sides.* Fig. V.

Let *A B C D E F G* be an Irregular *Piece* of Land (as a Wood, or the like) to be Measured. Before this *Piece* can be Measured it must be Reduced into some of the foregoing Regular Figures, as into Trapezia's or Triangles, by drawing of Lines from Angle to Angle within the *Plot*; And by such means, this Irregular *Plot* is Divided into two Trapezia's, and One Triangle; For, the Line drawn from *F* to *C*, cuts off the Trapezia *A B C F*; Also another Trapezia *F C D E* is cut off by drawing the Line *F E*, and then there is left the Triangle *E F G*.

The Figure being thus Reduced, let fall the several *Perpendiculars D K, F L, B H, F I*, and *F O*, which Measured, let each Diagonal and *Perpendicular*, be such Number of *Perches* as are set to them in the Figure.

And now are you ready for Casting up of the Content or Area in *Perches.*

First, For the *Triangle F G E,* whose Base *E G* is 34, and Perpendicular *F O,* 10.

Now 34 multiplied by 5 (or 17 by 10) the product will be 170 *Perches.*

Secondly, For the *Tapezia A B C F,* where the Diagonal or Common Base to the two *Triangles,* is *A C,* and whose Length is 35 *Perches;* and the two Perpendiculars and *F I* 24 *Perches,* and *B H* 8 *Perches,* their Sum is 32 *Perches,* half whereof 16 multiplied by the Common Base 35, the product will be 560 *Perches* for the *Trapezia A B C F.*

Thirdly, For the *Trapezia C D E F,* the Diagonal or Common Base, whereof is *C E,* and contains 32 *Perches:* And the two Perpendiculars are *F L* 17, and *D K* 13 *Perches,* the Sum of both being 30 *Perches,* half whereof 15, multiplied by 32, the product will be 460 *Perches;* for the *Trapezia C D E F.*

			Perches
The *Triangle*	*F G E*	Contains	170
The *Trapezia*	*A B C F*	Contains	560
The *Trapezia*	*C D E F*	Contains	460
		In all	1190

6. *How to Measure any Regular Polygon, or Figure of any Number of Equal Sides.* Fig. VI.

The RULE.

Multiply the Length of one of the Sides of the Polygon, into half the Number of the Sides; and that Product Multiply by half the Length of a Perpendicular let faln from the Centre of the Polygon upon any of the Sides; this Second Product shall be the Area or Centre of the Polygon.

Let *A B C D E* be a Regular Polygon of Five Sides, each Side containing 25 Inches, and let the Length of the Perpendicular *F G* be 17.2 Inches: ——(1.) Multiply the Length of the Side *C D* 25, by 5 the product is 105, the half whereof is 62.5 and that product multiplied by 17.2, the last product will be 1075 Inches for the Area or Content of the Polygon.

7. *How to Measure Circles, and Parts of Circles.* Fig. VII.

The Proportion that the Diameter of any Circle bears to the Circumference of that Circle, is, As 7 to 22 (in the least terms) Or, As 113 to 355 (in more exact terms:) And by either of these Proportions, may the following Problems be Resolved.

8. *The*

8. *The Diameter of a Circle being given to find the Circumference.*

Let the Diameter of a Circle *A*, be 27.6 Inches;

Multiply 27.6 by 22, the product will be 607.2; which Divided by 7, the Quotient will be 86.7 Inches for the Circumference. Or more exact,

Multiply 27.6, by 355, the product will be 9798.0, which Divided by 113, the Quotient will be 86.707 Inches for the Circumference.

9. *The Circumference of a Circle being given, to find the Diameter.*

Multiply the Circumference by (7 or 113,) and Divide the product by 22, (or 355) the Quotient will give the Diameter.

So the Circumference being 86.7 (or 86.707) the Diameter will be found to be 27.6 Inches.

Multiply 86.7 by 7 the product will be 606.9, which Divide by 22, the Quotient will be 27.6 for the Diameter. Or,

Multiply 86.7 by 113, the product will be 9797.1, which divided by 355, the Quotient will be 27.62 Inches for the Diameter, as before, but nearer.

10. *The Diameter of a Circle being given to find the Area, or Superficial Content thereof.*

The RULE.

Multiply the Diameter into it self, and that Product again, by 22 (or 355) Divide this second Product by 28 (or 452) the Quotient will be the Area, or Superficial Content of the Circle.

So, the Diameter of a Circle being 27.6 Inches; That multiplied into it self produceth 761.76, and that multiplied by 22 produceth 16758.72; Which divided by 28, gives in the Quotient 598.52 Inches for the Area of the Circle.——Or 761.76 multiplied by 355 produceth 270424.80, and that divided by 452, giveth in the Quotient 598.284 the Area more exactly.

11. *The Circumference of a Circle given to find the Area.*

The RULE.

Multiply the Circumference into it self, and that Product Multiply by 7 (or 113:) Divide this second Product by 88 (or 1420) The Quotient will give the Area; or Content of the Circle.

So the Circumference of a Circle being 86.7 Inches. That multiplied into it self produceth 7516.89, which multiplied by 7 the product

duct is 52618.23, and that divided by 88, giveth in the Quotient 597.82 Inches for the Area of the Circle.

12. *The Diameter of a Circle being given, to find the Side of a Square which shall be equal in Area to that Circle.*

The RULE.

Multiply the Diameter of the given Circle (always) by this Decimal .886227, the Product shall be the side of a Square Equal in Area to that Circle.

So, the Diameter of a Circle being 27.6, that multiplied into .886227, the product will be 24.4598652; That is 24.46 Inches *ferè*, And that is the Side of a Square, Equal in Area to the Circle whose Diameter is 27.6 Inches; For 24.46 multiplied into it self produceth 598.2916 Inches.

Note. These small differences do arise partly, for want of more places in the Decimal Parts; and partly into the Difference of the Proportions between 7 to 22 and 113 and 355.

13. *To Measure a Semicirle.*

The RULE.

Multiply half the Diameter of the Circle into it self and that Product (always) by 22, and that product divide (always) by 14. The Quotient will be the Area of the Semicircle.

So that the Diameter being 13.8, that multiplied into it self produceth 190.44, which multiplied by 22, produceth 4189.68, which divided by 14, giveth in the Quotient 299.26 Inches, for the Area of the Semicircle.

14. *To Measure a Quadrant,*

The RULE.

Multiply half the Diamteer into it self and that Product (always) by 11 and that Product Divide (always) by 14, the Quotient gives the Quadrants Area.

So 13.8, half the Diameter, multiplied into it self produceth 190.44, which multiplied by 11, produceth 2094.84, and that divided by 14 giveth in the Quotient 149.63 Inches, for the Area of the Quadrant.

15. How

15. *How to measure any Part, Portion or Segment of a Circle.*

The RULE.

Multiply half the Diameter, by half the Arch-line; and that Product will be the Area of the Sector, or Portion.

So, 13. 8, half the Diameter, being multiplied by 5.32 the Length of half the Arch-line: the product will be 73.41, for the Area of the Sector.

16. *How to find a Line equal to half the Arch-line*, F G C.

Draw the right Line *FC*, and divide it into Four equal parts in *H*, *K*, *L*, take one of those parts, as *LC*, and set it from *C* to *G*; then a right Line drawn from *G* to *H*, shall be equal to half the Arch-line *FGC*.

II. *Of the Mensuration of Solids.*

SOlid Figures or Bodies, are such as do consist of Three Dimensions as Length, Breadth and Depth, or Thickness: As Stone, Timber, Columns, Globes, Bullets, *&c.*

17. *How to Measure a Cube.* Fig. VIII.

RULE.

Multiply the Side of the Cube into it self; and that Product again by the Side: The last Product will be the Solidity or Solid content of the Cube.

So, let *ABCDEFG* be a Cube, the Side whereof *AB*, &c. let be 5.2 Inches, (or any other Measure) 5.2 multiplied into it self, produceth 27.04 and that multiplied again by 5.2, the last product will be 140.608 Inches; which is the Solid Content of that Cube, whose Side is 5.2 Inches.

18. *How to Measure a Long Cube or Parallelipipedon.* Fig. IX.

RULE.

Multiply the Side of the Square at the End, into it self, and that Product by the Length, the last Product shall give the Solid Content.

So, let *HKLMNOP*, be a Long Cube, the Side of whose Square at the End let be 6.7 Inches, and the Length thereof 3.5 Foot, (or 42 Inches) 6.7 multiplied into it self, produceth 44.89, and that mult-

multiplied by 42, the Length, makes the Second product to be 1885.38 Inches : Which is the Solid Content of the Long Cube.

19. *How to Measure an Oblong Parallelipidedon.* Fig. X.

RULE.

Multiply the two un-equal Sides at the end of the Piece, one by the other, and the Product of them Multiply by the Length of the Peice; the last Product shall be the Solid Content of the Oblong Parallelipipedon.

So let *QRSTUZ* be an Oblong Parallelipipedon, whose Breadth at the End *QZ* Let be 6. 5, and Depth at the End *UZ*, Let be 3.4 Inches, and let the Length thereof be 16 Foot 3 Inches (or 192 Inches. (Multiply 6.5 the Breadth, by 5.4 Inches the Depth, the Product thereof is 35.1, which multiplied by 192 Inches the Length; the product will be 6739.2 Inches the Solid Content of the Oblong Parallelipipedon in Inches: Which if you Divide by 1728 (the Solid Inches contained in One Solid Foot of Timber or Stone) the Quotient will be $3.\frac{1545}{1728}$ Foot. And so much Solid Stone or Timber is in that Piece.

Note, Forasmuch as in the following Mensurations of Solids, and in Measuring of the Works of Artificers : Dimensions are generally taken in Feet and Inches, but the Content is, for the more part, required to be given in Feet and Parts of a Foot, and therefore, it will be necessary here to shew,

20. *How to Reduce Inches, and Parts of Inches into Decimal Parts of a Foot.*

Note that $\left\{\begin{array}{l}\text{1 quarter}\\ \text{2 quarters}\\ \text{3 quarters}\\ \text{1 Inch}\end{array}\right\}$ of an Inch is $\left\{\begin{array}{c}\frac{1}{48}\\ \frac{2}{48}\\ \frac{3}{48}\\ \frac{4}{48}\end{array}\right\}$ of a Foot

And these Fractions Reduced into Decimal Parts, as hath been Taught in the beginning of the Second Part of the Decimal Part of Arithmetick, *Problem* I. And according to the *Rule* there given, the Decimal Parts Answering to,

$\left.\begin{array}{l}\text{1 quarter}\\ \text{2 quarters}\\ \text{3 quarters}\\ \text{1 Inch}\end{array}\right\}$ Of an Inch, will be $\left\{\begin{array}{l}.021\\ .042\\ .063\\ .083\end{array}\right\}$ Of a Foot

And so of all the rest as in the Table following.

A Table ſhewing the Decimal Parts Anſwering to every Inch and Quarter of an Inch in One Foot, the Foot being Divided into 1000 Parts

Inch.	*Quart. Inch.*	*Dec. Parts.*	*Inch.*	*Quart. Inch.*	*Dec. Parts.*
0	0		6	0	.5
	1	.021		1	.521
	2	.042		2	.542
	3	.063		3	.563
1	0	.080	7	0	.58
	1	.101		1	.601
	2	.122		2	.622
	3	.143		3	.643
2	0	.16	8	0	.66
	1	.181		1	.681
	2	.202		2	.702
	3	.223		3	.723
3	0	.25	9	0	.75
	1	.271		1	.771
	2	.292		2	.792
	3	.313		3	.813
4	0	.34	10	0	.84
	1	.361		1	.861
	2	.382		2	.882
	3	.403		3	.903
5	0	.42	11	0	.92
	1	.441		1	.941
	2	.462		2	.962
	3	.483		3	.983

One Foot 1.000

21. *To Measure the Solid content of a Squared Stone or Piece of Timber, whose End is a Regular Polygon, of 5, 6, 8, or 9 equal sides.* Fig. XI.

RULE.

Find the Area, *or* Superficial Content, *of the* Polygon *at the End (by the* 6th *before-going) which had, multiply that* Area, *by the Length of the* Piece, *that shall give you the* Solidity.

So let *ABCDEFGHK*, be a Square Piece of Stone or Timber of 7 Sides, each Side being 4 Inches and a Quarter (or .361 Parts) and the Perpendicular from the Centre ⊙, 3 Inches and a half, (or .292 Parts) and let the Length thereof be 8 Foot 3 Inches, (or 8.25 Dec. Parts.)

(1.) 4 and a Quarter Inches (or .361 Parts) multiplied by 3 Inches and a half (or 3.5) (half the Number of the Sides,) the product will be 1.2635 which multiplied by 8 Foot 3 Inches, (or 8.25) the last product will be 104.23875 (or 104 Foot 2 Inches and 3 Quarters) for the Solidity.

22. *How to Measure a* Prisme. Fig. XII.

RULE.

22. *Multiply the Area of the Base (or End) by the Length: the Product is the Solid Content.*

Let *ABC EHO* be a *Prisme*, the Side of the Triangle at the End *BE*, is 18 Inches (or 1.5 Parts) and the Perpendicular *CD* 13 Inches (or 1.021 Parts) and the Length *GC* 7 Foot 9 Inches or (7.75 Parts.)

(1.) Multiply 1.021 the Perpendicular by half the Side of the Triangle *BE, viz.* .75, the product will be .765752. (2.) Multiply .76575 by 7.75 the Length the Product will be 5.9345625 (but by contracted multiplication 5.935) for the Solid Content: Which is 5 Foot 11 Inches, and almost a Quarter of an Inch.

22. *To Measure a* Pyramid. Fig. XIII.

RULE.

Multiply the Area of the Base, by One third Part of the Altitude; the product shall be the Solid Content.

In the Pyramid *ABCDE*, the Base at the Bottom being a Square, and each side thereof is 18, and the height thereof 540.

(1.) Multiply 18 by 18, the product is 324, which multiplied by 180 (one third part of the height) the product is 58320, for the Solid content of the Pyramid.

24. *To*

24. *To Measure a Cone.* Fig. XIV.

RULE.

Multiply the Area, or Content of the Circular Base; by One third part of the height of the Cone, the product shall be the Solid Content of the Cone.

Let *O X Y Z* be a Cone, whose Base *X Y Z* is found (by the 10*th*. or 11*th* hereof) to be 346.5; and the Height thereof *O S* 540: Multiply 346. (the Base) by 180 (One third Part of the height) the product will be 62370 for the Solid Content of the Cone.

25. *To Measure a Segment or Frustum of a Pyramid or Cone.* Fig. XIII.

The Frustum of a Pyramid, or Cone, is when the Smaller End thereof is cut off; and in this form are most Timber Trees either Round or Squared; and such is the Frustum of the Pyramid *R S T V, B C D E*: whose Solidity let be required.

RULE.

Multiply the Areas of the two Ends one into the other; find the Square Root of that product; Then add the two Areas, and this Root together; and Multiply their Sum by One third part of the Length of the Frustum, and the product shall be the Solid Content of the Frustum.

So, the Square of the greater End *BCDE*, (the side being 18) is 324, and the Square of the Lesser End *R S T V* (the side being 12) is, 144; these two Squares multiplied together, the product is 46656; the Sq. Root of which is 216: Add these three together, *viz*. 324, 144 and 216; their Sum will be 684; which multiplied by 60 (One third Part of 180, the Altitude of the Frustum) the product will be 41040 which is the Solid Content of the Frustum: And these (if they were Inches would make $23\frac{1136}{1728}$ Foot.

26. *How to Measure a Cylinder or Column.* Fig. XV.

RULE.

Multiply the Area or Content of the Circle at the Base (or End) of the Cylinder; by the Height (or Length) thereof; the product will be the Solid Content of the Cylinder or Column.

So in the Cylinder *A B C D*, Let the Diameter of the Circle at the End *A B*, be 7.00, then will the Area of the Circle be found to be 38.5: This multiplied by the Length thereof *A C* or *B D*, 12.00 produceth 462, for the Solid Content of the Cylinder.

27. *How to Measure Spheres, Globes or Bullets.* Fig. XVI.

The Axis or Diameter of a Globe (or Bullet) given; to find the Superficial Content thereof.

RULE.

Multiply the Square of the Diameter (always) by 355; *and divide that product (always) by* 113, *the Quotient shall shew the Superficial Content of that Globe.*

Let *ABCD*, be a Sphere or Bullet, whose Diameter *AB* is 21: this multiplied into it self produceth 441 for the Square of the Diameter; This multiplied by 355 the product will be 156555: And this product divided by 113, giveth in the Quotient 1385.44 for the Superficial Content of the Globe or Bullet.

28. *The* Axis, *or* Diameter *of a* Globe (or Bullet) *given to find the* Solid Content *thereof.*

RULE.

Multiply the Cube of the Diameter (always) by 355 *and the product thereof (always) divide by* 678, *and the Quotient shall be the Solidity of the Globe or Bullet;*

So the Axis *AB*, being 21, the Cube thereof is 9261, which multiplied by 355, the Product will be 3287655, which divided by 678; giveth in the Quotient 4849.048; which is the Solid Content of that Globe or Bullet.

According to these Rules may any Regular Piece of Stone or Timber be Measured; I shall conclude this of Measuring of Solids with shewing you how to Measure any Tapering Timber Tree, which is not exactly Square at both Ends in Foot Measure.

29. Let there be a Timber Tree, whose Breadth at one End is 2.3 Foot and the Depth 1.8 Foot: And the Breadth at the other End 3.5 Foot and the Depth 2.1 Foot, and let the Length of the Tree be 42.6 Foot.

First, 2.3 multiplied by 1.8, gives 4.14 for the Area of the Lesser End: And 3.5 by 2.1 gives 7.25, for the Area of the Greater End.

Secondly, multiply 7.25 by 4.14, the product will be 30.0150: the Square Root whereof is 5.48: Add those three Numbers together, *viz.* 4.14—5.48 and 7.25 and their Sum will be 16.87: This multiplied by 14.2 (One third Part of the 42.6, the Length) the product will be 239,55 Foot, and so many Solid Foot of Timber is there in the Tree.

III. *Of*

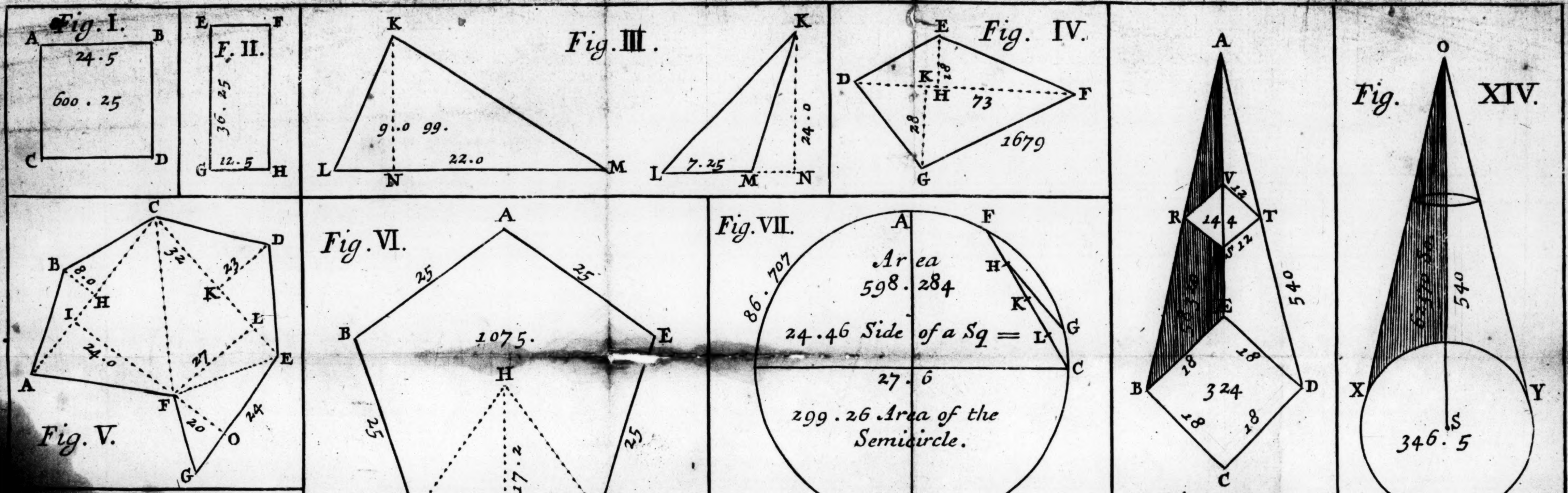
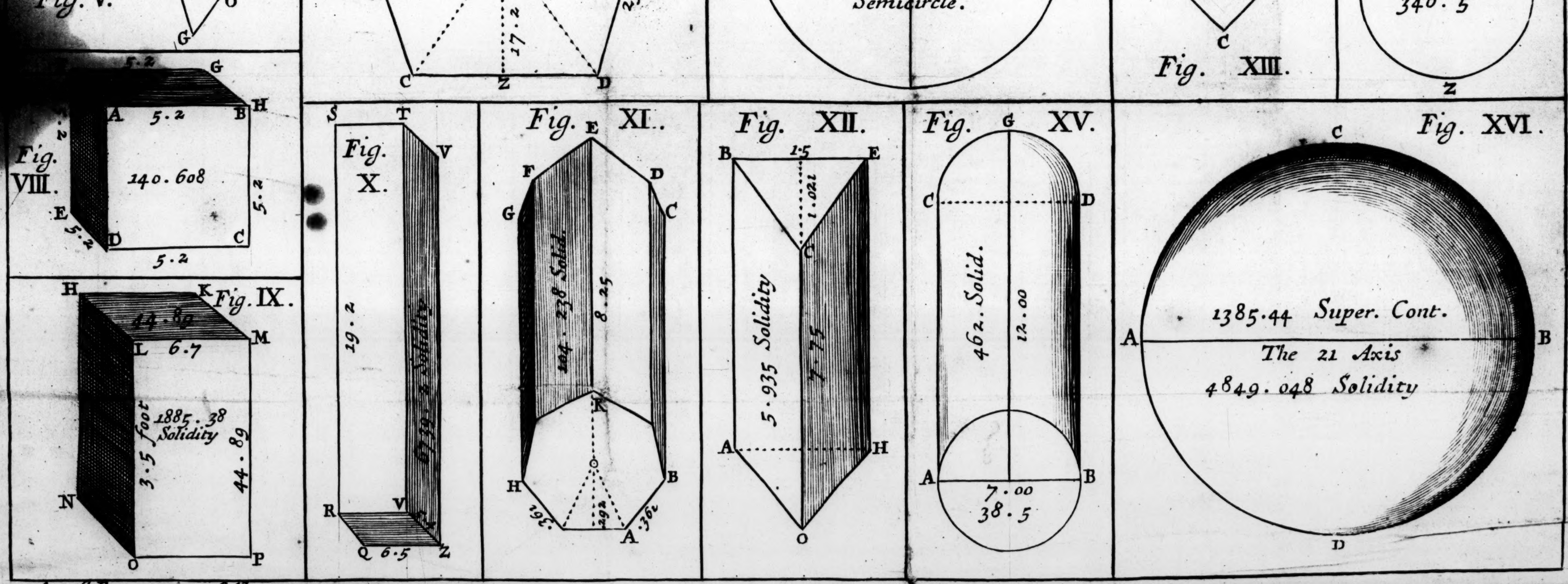


Fig. I.
24.5
600 . 25
F. II.
36 . 25
12 . 5
Fig. III.
9 . 0 99.
22.0
7 . 25
24 . 0
Fig. IV.
73
1679
Fig. V.
Fig. VI.
1075.
Fig. VII.
Area
598 . 284
24 . 46 Side of a Sq =
27 . 6
299 . 26 Area of the Semicircle.
86 . 707
324
540
Fig. XIV.
346 . 5



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III. *Of the Measuring of the Works of the several Artificers relating to Building: and what Method and Customs are Observed therein.*

THE Principal Artificers relating to Building (or repairing) of Houses and other Edifices, are the Carpenter, Bricklayer, Plaisterer, Mason, Joyner, Painter and Glasier, &c. Of which, some of their Works are Measured by the Square of Ten Foot (or 100 Square Foot.)— Some by the Rod, Pole or Perch of 16 Foot and a Half ($272\frac{1}{4}$, Square Foot making a Rod:)—Some by the Yard Square of 3 Foot, 9 Square Feet making a Yard Square:—Some by the Foot Superficial containing 144 Square Inches: Or by the Foot Solid containing 1728 Solid Inches.

Of the Carpenters Work.

The Carpenters Works which are Measureable are Roofing, Flooring, Partitioning, &c. All which are Measured by the Square of 100 Superficial Feet.

I. Of *Roofing.*] It is a Rule received among Work-men; that the Flat of the Ground upon which any House is Erected, and Half the same Flat (the Dimensions taken within the Walls) is equal to the Measure of the Roof that will Cover that House, as to the Carpenters Work.

So that, If a House within the Walls be 46 Foot Long, and 18 Foot Broad, it will contain 1242 Square Feet; For 46 multiplied by 18 is 828 and Half that is 414 their Sum 1242 Superficial Feet: that is, 12 Square 42 Foot, or 12 Square 1 Quarter and 17 Foot; and so many Square feet will that Roof Contain.

$$\left.\begin{array}{l}\textit{Note}, \text{ 25 foot is one Quarter} \\ \text{50 foot is Half} \\ \text{75 foot is 3 Quarters}\end{array}\right\} \text{of a Square}$$

II. Of *Flooring.* The Length multiplied by the Breadth gives the Content. So, If a Floor be 57 foot 3 Inches (or 57.25) Long, and 28 foot 6 Inches (or 28.5) Broad multiply 57.25 by 285, the product will be 1531.625 Feet, that is 16 Square 1 Quarter, 6 foot $7\frac{1}{2}$ Inches; and so much doth that Floor Contain.

III. of

III. Of *Partitioning*. Partitioning is Measured by multiplying the Length thereof into the Height; so that if a Partition be 82 foot 6 Inches (or 82.5) Long, and 12 foot 3 Inches, (or 12.25) High: 82.5, multiplied by 12.25, the Product will be 1010.625 feet, that is 10 Square foot and $7\frac{1}{2}$ Inches, for the Content of that Partitioning.

There are other Works about a Building performed by the Carpenter, which are Measured by the Foot, Running Measure, that is, by the Number of Feet in Length only,

As	Cornices	Guttering	Skirt-Board
	Moldings	Lintale	Rails and Balasters
		Brest-somers	Timber-Fronts
		Shelfing	Dressing, &c.

There are other things also, Valued *per* Peice, &c.

Doors and Cases	Balcony Doors	Stairs and Cases
Window Lights	Cellar Doors	Mantle-trees
Lantern Lights	Columns, Pilasters	Pediments

And divers other things not here mentioned.

Note, 1. In Measuring of Flooring, you must deduct out thereof for the Well-holes, for the Stairs, and Chimney-ways: And in Partitioning for Doors, &c. except (by Contract) they are to be included. But in Roofing, seldom any Deductions are made for Holes for the Chimney Shafts, the Vacancies for Lutheren Lights or Skie Lights, there being more trouble in the ordering of them then the Stuff is Worth to Cover them.

II. *Of the Brick-layers Work.*

The Principal are *Tiling*, *Walling* and *Chimnies*.

I. Of *Tiling*. Tiling is Measured by the Square of 10 Foot, as Flooring and Roofing were, so that between the Roofing and the Tiling the Difference will not be much, yet the Tiling will always be the greater. And besides, sometimes the Bricklayers will require Running Measure for Hips and Valleys, which in some Cases may be allowed, but in Plain Roofs not.

An Example of this kind is needless.

II. Of *Walling*: All Walls in Brickwork whether for Inclosing of Gardens or for Houses, are Measured by the Rod of $16\frac{1}{2}$ foot, Measured upon the Superficies of the Wall, each Rod containing $272\frac{1}{4}$ Superficial feet: but this is not all, For all Brick-walls of what

Thick-

Thickneſs ſoever, they muſt be Reduced (all the parts them) to a Standard Thickneſs, that is, to One Brick and a half Thick.

Example I. *If a Brick-wall be* 192 *Foot Long, and* 12 *Foot High, how many Rod is contained therein ?*

Multiply 192 by 12, the product will be 2304, which divided by 272 (for the $\frac{1}{4}$ of a foot is always rejected,)] the Quotient will give 8 Rod, and 128 Remaining, which divide by 68 (the feet in a quarter of a Rod,) and the Quotient will be 1 quarter, and 60 foot over; So that the whole Wall contains 8 Rod, 1 quarter, and 60 foot; which wants but 8 foot of 8 Rod and a half.

But there is yet ſomething elſe to be conſidered in the Meaſuring of Brick-work; Namely, the Thickneſs of the Walls; for the Thicker the Wall is, the more Rods will be Contained in any Number of feet Meaſured upon the Superficies: For, If the Wall be one Brick and a half Thick, then 272$\frac{1}{4}$ foot Meaſured upon the Superficies of the Wall, will be a Rod Standard: But if the Wall be Thicker than One Brick and a half, the 272$\frac{1}{4}$ foot Meaſured upon the Superficies of the Wall, will make more than a Standard Rod: And if Thinner than a Brick and half 272$\frac{1}{2}$ feet will be Leſs than a Standard Rod: And therefore, For the ready Reducing of Brick-work of any Thickneſs, to that of a Standard Thickneſs, *viz.* One Brick and a half; Obſerve this

GENERAL RULE.

Multiply the Number of Superficial Feet, that are found to be contained upon the Superficies of any Wall, by the Number of half Bricks which that Wall is in Thickneſs; One Third Part of that Product, ſhall be the Number of Rods and Parts of a Rod contained in that Wall Reduced to Standard Thickneſs.

Example II. If a Wall be 72 foot Long 19 foot High, and be Five Bricks and a half Thick; How many Rod of Brick-work (Reduced to Standard) is there Contained in that Wall?

Multiply 72 by 19, the Product will be 1368, and ſo many Superficial feet there are upon the Superficies of the Walls——Multiply 1368 by 11 (the Number of half Bricks that the Wall is in thickneſs) the product will be 15048, One third Part whereof is 5016, and ſo many feet doth that Wall contain, it being Reduced to Standard-Thickneſs—Now if you divide theſe 5016 feet by 272 the Quotient will be 18 Rod, and 120 Remaining; which divided by 68, the Quotient is 1 quarter, and 52 foot Remaining: So that the whole Wall Reduced to Standard-Thickneſs, Contains, 18 Rod, 1 quarter and 52 feet.

Other

Other Brick-layers Work, *viz.*

Chimnies, The Chimnies in most Buildings are agreed for by the Hearth in each Room, and sometimes they are included in the Building, and Paid by the Rod; and Measured in this manner — If the Chimnies stand single, the Usual way is to Girt it about; and if the Jaums are but One Brick thick, and wrought upright over the Mantle-tree to the next Floor; then Girt it about for a Length, and the Height of the Story for a Breadth, at One Brick thick. ——— But if the Chimny stand against a Wall that is before Measured, then the Breadth of the Brest and the Breadth of the Two Jaums is the Length, and the Height of the Story the Breadth at One Brick and half, if the Jaums be so Thick, and nothing to be Deducted for the Area between the Hearth and Mantle-tree; because of the Wyths, and Thickening of the Hearth above. For the Chimny Shafts, Girt them about in the smallest Place, for the Breadth, and the Height of the Shaft for the Length, at One Brick thick, in consideration of Wyths, Pargetting and Scaffolding.

Other Brick-layers Works which are Measured by the Foot Running Measure, are

Cornices of all sorts	Streight Arches	Hyps and Valley's
Facices	Skeen Arches	Water Courses, &c.

Other Works valued by the Piece, *viz.*

Peers, Pilasters, Plain or Rustick, Pediments, &c. Paving also is Brick-layers Work, with Brick or Tiles; and is Measured by the Yard Square, or 9 Foot, so, if a Cellar be Paved with Brick 32 foot Long, and 19 foot Broad; multiply 32 by 19, the product will be 608, which divided by 9, the Quotient is 67 Yards, and 5 foot.

Note, That in Measuring of the Brick-work in Houses, if you take the Dimensions of the sides on the Outside, you must take the Dimensions of the Ends on the Inside, And Note also that Deductions, are to be made, for all Doors, Windows, &c. according to the several thicknesses of the Walls in which such Doors or Windows are.

III. *Of the Masons Work.*

Masons do Measure all their Works by the Foot, either Superficial or Solid, and therefore I need give no Examples, for the Rules before delivered in this Section for Measuring of Superficies and Solids, are sufficient to perform any Work done by the Masons.

IV. *Of*

IV. Of the Plaisterers Work.

Plaisterers Works are principally of Two kinds, Namely, (1.) Work Lath'd and Plaistered, which they call Ceiling—(2.) Work Rendered, which is Principally of Two kinds, viz. upon Brick-walls or between Quarters, both which are Measured by the Yard Square.

1. Of Ceiling; If a Ceiling be 58 foot 9 inches, (or 58.75) Long and 23 foot 7 inches (or 23.58) Broad;— Multiply 58.75 by 23.58, the product will be 1914.07, foot, which divided by 9 gives in the Quotient 212 yards and 6 foot, for the content of that Ceiling.

2. Of Partitioning; If the Partitioning between the Rooms on one Floor be 132 foot about, and the Story 12 foot High—Multiply 132 by 12, the product will be 1584, which divided by 9 gives in the Quotient 176 yards, for the Content of that Partition.

Note 1. If there be any Doors, Windows, or the like, in your Partitioning. you must make a Deduction, for them.

Note 2. When you Measure Rendring upon Brick-work, you are to make no deductions, but to Measure all where your Trowel goes.— But when you Measure Rendring between Quarters, you may very well deduct, one 5*th*. Part for the Quarters, Braces and Interstices, &c.

Note 3. Whiting and Colouring are both Measured by the Yard; and in Measuring the Whiting of Cieling Partitioning and Rendring upon Walls will be the same with the Plaistering, yet in Whiting and Colouring between Quarters, you may very well add one Fourth or Fifth part more.

V. Of the Joyners Work.

Joyners do Measure their Work by the yard Square of 9 Foot, but they have a Custom, and say, we ought to Measure where our Plain touches: And therefore; In taking the Dimensions of a Room where there is a Cornice about, and swelling Pannels with Moldings; they with a Line, do Girt over every Member of the Cornice and swelling Pannels and Moldings Downwards, and take that Length for the Height: But for the Girt of the Room about, they take that only as Flat.

Example, If a Room Wantscoted were thus Girt downwards it should be found by the string to be 15 foot 7 inches, (or 15.58) and in Compass about 286 foot, how many yards doth that Room contain of Girt Measure?

Multiply 286 foot, by 15.58, the product will be 4455.88, or 4456 foot, which divided by 9, the Quotient will be 495 yards and One foot, for the content.

In Measuring of Joyners Work, there is another thing Observable; that is, in the Measuring of Doors, Window-shutters, Cup-

boards, Doors and Drawers, and ſuch Works as are Wrought on both ſides; for theſe they do Account to be paid for Work and half-work, for indeed the Work is more, though the Stuff be the ſame.

Example, Let the Window-ſhutters about a Room be 78 foot 4 inches (or 78.34.) and the Height of them 6 feet 6 inches, (or 6.5) how many yards are contained in theſe ſhutters, at Work and half?

Multiply 78.34, by 6.5, the product will be 509.21, the half whereof is 254.60 and theſe two added make 763.81 foot, (or 764 foot,) which divided by 9, the Quotient is 84 yards 8 foot, (or 85 yards.)

Note, that Deductions are to be made for all Window-Lights; But you muſt Meaſure the Window Boards, Safeta and Jaum Boards by themſelves, as VVork only ſingle.

VI. *Of the Painters Work.*

The taking of the Dimenſions for Painters-work is the ſame as for the Joyners, by Girting of the Moldings, &c. The Dimenſions ſo taken; the Caſting up, and Reducing of the Feet into yards, is altogether the ſame as the Plaiſterers and Joyners were: And they never ſay VVork and half; but, Once, Twice or Thrice Primed or Coloured.

Examples herein are unneceſſary, Only take Notice, that VVindow-Lights, VVindow Bars, Caſements, and ſuch like things, they reckon at ſo much *per* Piece.

VII. *Of the Glaſiers Work.*

Glaſiers do Meaſure their Work by the Foot Superficial, ſo that the Length and Breadth multiplied into each other produceth the quantity of feet in that Pain of Glaſs.

So if a Pain of Glaſs be 4 foot 9 inches (or 4.75) Long, and 3 foot 3 inch. (or 3.25) Broad: If you multiply 4.75 by 3.25, the product will be 15.4375, that is 15 foot, 5 inches and a quarter, for the Content.

Note, That when VVindows have half Rounds at the Top, they Meaſure them at the full Height and Breadth as if they were Square. Alſo Round and Oval VVindows are meaſured at the full Length and Breadth of their Diameter. Likewiſe Crocket-windows in Stone-work, are all meaſured at their full Square; And there is reaſon for ſuch allowance: For the Trouble in taking the Dimenſions to VVork by, The VVaſte of Glaſs in VVorking, and the Time ſpent extraordinary in ſetting up of ſuch VVindows, is for more than the Glaſs ſaved can be valued at. And thus much concerning meaſuring. But for farther ſatisfaction, you may conſult *Curſus Mathematicus*; and my Book for Building, &c.

The End of the Second Part.

INSTRUMENTAL ARITHMETICK.

The Third PART.

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By certain SCALES contrived, suitable to the *Coins, Weights* and *Measures* now Used in *England*. And for Extracting of the Square and Cube Roots.

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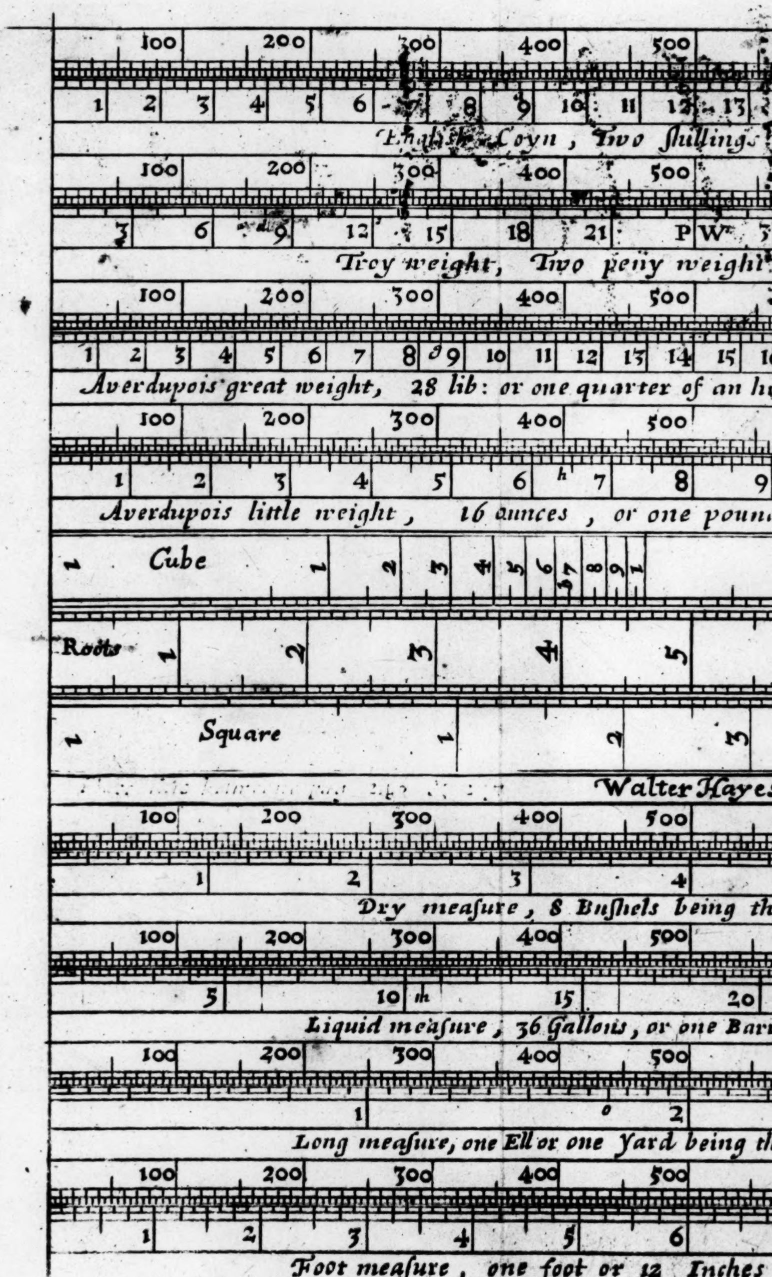
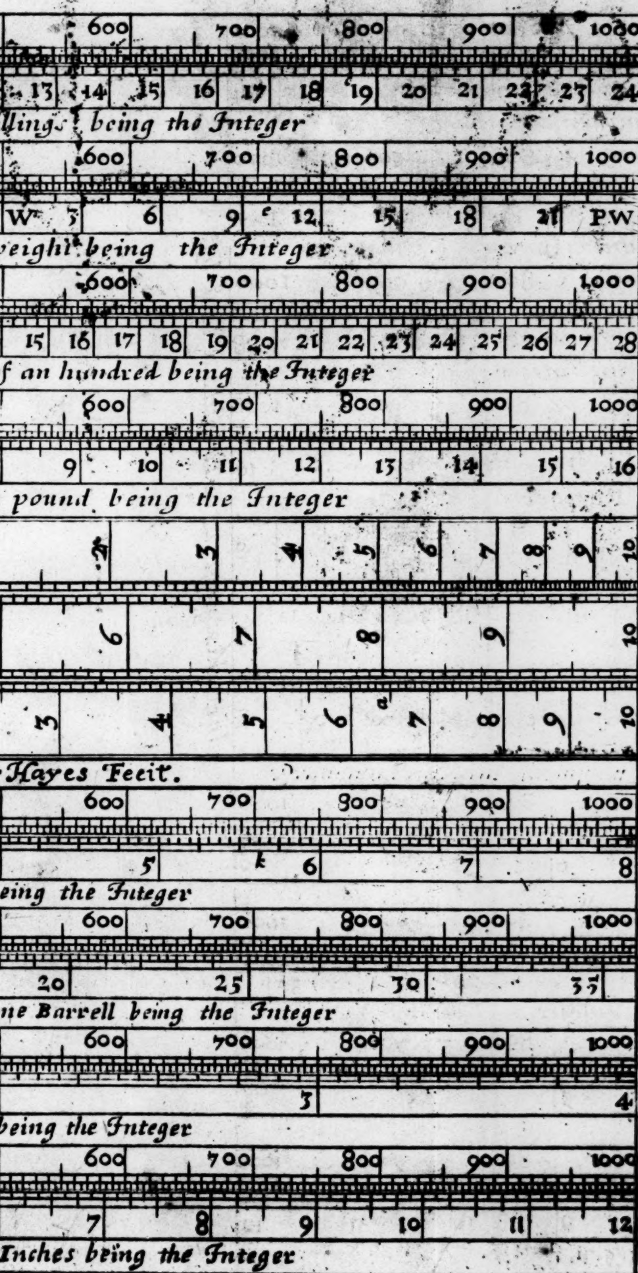


English Coyn, Two shillings
Troy weight, Two peny weight
Averdupois great weight, 28 lib: or one quarter of an
Averdupois little weight, 16 aunces, or one
Cube
Roots
Square
Walter Hayes
Dry measure, 8 Bushels being
Liquid measure, 36 Gallons, or one
Long measure, one Ell or one Yard being
Foot measure, one foot or 12 Inches



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Instrumental Arithmetick.

SECT. I.

By *Decimal Scales.*

THe *Arithemetick* of which we now come to treat, and which I call *Instrumental Arithmetick*, is not any new kind of *Arithmetick*, but is indeed the same with *Decimal Arithmetick* before taught; only, whereas in *Decimal Arithmetick* there were certain *Tables* made of *Money*, *Weight* and *Measure*, by help of which the Decimal of any Fraction of *Money*, *Weight* and *Measure*, might be set down (as it were) in whole numbers; here in this *Instrumental part*, we have contrived certain *Scales* of *Money*, *Weight* and *Measure* equally divided into the several Denominations into which the *Weights* and *Measures* for which they are contrived, may be equally divided. Unto all which Scales there is joyned a Scale of 100, 1000, or 10000, equal parts, according to the length of the Scale, so that by inspection only you may readily and exactly without Addition (as in using the forementioned *Tables* you must necessarily do) set down the *Decimal Fraction* of any part of *Money*, *Weight* or *Measure*, with great celerity and exactness, if the Scale be any thing well divided, and be but of a reasonable length.

Now the Scales which I have chiefly made choice of in this Work; as being of most Use with *English*-men; (though other Scales may be made for the *Coins*, *Weights* or *Measures* of any other Country as well, and upon the same ground) are chiefly these, *viz.*

1 Of Money.
2 Of Troy Weight.
3 Of Averdupois great Weight.
4 Of Averdupois little Weight.
5 Of Liquid Measures.
6 Of Dry Measures.
7 Of Long Measures.
8 Of Dozens.

Unto every these of Scales, is joyned another Scale of 100 or 1000 equal parts, these Scales are made to face one another, so that

that if you look upon any one Division in the one, you shall also discern plainly what Division or part of a Division answereth thereunto in the other.

These Scales being thus disposed, as they may easily be upon any *Ruler* of *Silver*, *Brass* or *Wood*; but best of all upon a *square Ruler*, made in form of a *Parallelepipedon*, will by inspection only give you any Decimal fraction required without Addition, (or on the contrary) reduce the fraction into the known parts of the Integer, by inspection also, without *Substraction*.

Let thus much suffice for a general description of what I mean by Scales, the particular description of them will more plainly appear, when we treat of Numeration upon the Scales, unto which we shall now proceed. But first take a view of the Scales as they are here disposed, and as they may be set upon such a Ruler as I have here mentioned.

Numeration upon the Scales.

THe Scale here to be described are in number Eight, as hath been already shewed, and as by the figure of them appears. *Now* Numeration *upon a* Scale, *is to find upon what part of the* Scale *any number upon the same* Scale *will fall.*

We will begin with the first, and so proceed till we have given an Example in every one,

1. The first Scale is of *English* Money, and is divided into 24 equal parts, which represent 24 pence or 2 shillings, these parts are numbred with Arithmetical figures, from the beginning thereof, by 1, 2, 3, 4, 5, &c. to 24, each division representing one peny, and the whole 24 divisions represent 24 pence, or 2 shillings; So that where the figure 1 standeth that part of the Scale representeth one peny, where the figure 2 standeth, it representeth 2 *d.* where the figure 18 standeth it representeth 18 pence, or one shilling six penceand so of any other figure of the same Scale.

Then because there are four farthings contained in a peny, each of these pence (or divisions) is sub-divided into 4 other equal parts by short Lines, every one of these representing one farthing, so is the whole Scale divided in all into 96 equal parts, which are the number of farthings contained in two shillings. Thus if you look into the Scale of Mony for 8 pence 3 farthings, you shall find it at the Letter *a*, which Letter is here put only for example sake. Also if you would find in the Scale the place of 18 *d.* half peny, you shall find it at *c*, and thus may you find the places of any number of pence and farthings under two shillings upon the Scale.

Unto

Unto this Scale of Mony (as to all the rest of the Scales) there is joyned another Scale of 1000, the use of which Scale is this. When you have found any number of pence or farthings upon the Scale of Mony, you shall find upon the Scale of 1000, what parts of a thousand is the Decimal of those pence and farthings: Thus when in the Scale of Mony, you find at the Letter *a* 8*d*. 3 *q*. if you cast your eye directly cross to the Scale of 1000, you shall find 364 to stand directly against 8*d*.3*q*. which 364 is the decimal of 8 *d*.3*q*. Also if you find upon the Scale of Mony 18 *d*. half-peny, which is at the Letter *c*, you shall find against it in the Scale of 1000, this number .771, which is the Decimal of 18 *d*. half-peny. And in this manner may the Decimal of any number of pence or farthings under two shillings be most easily and exactly obtained.

Now on the contrary, suppose a Decimal Fraction were given, representing some part of *English* Coin; if you look in the Scale of 1000 for your number given right against it in the Scale of Mony, you shall find what number of pence and farthings is represented thereby. As for Example, Suppose .364 were a Decimal given, and it were required to find what part of Coin it doth represent. Look in the Scale of 1000 for the number .364, and right against it you shall find 8 pence 3 farthings. Also if .771 were a Decimal given, if you look in the Scale of 1000 for .771, you shall find against that number 18 pence 2 farthings. And thus of any other.

By what hath been already said, it may be easily discerned of what exeeeding expedition these Scales thus disposed are of, for I dare affirm, that I will set down 2 (if not 3) numbers, by the Scale, as soon as one by the Tables, and if the Scale be but of any reasonable length, altogether with as much exactness, but if I should vary an unite in the last place, in my estimation in the Scale of 1000, it is not any thing material.

I have been very tedious in shewing the use of these Scales to find the fraction-parts of mony; but the reason is, because I intend to be the briefer in the rest, for *Weight* and *Measure*; the manner of working (when the Division of the Scale is known) being the same in all respects without the least alteration.

2. The second Scale is of *Troy-Weight*, two peny weight being the Integer, which Scale is divided into 48 equal parts or divisions, each of which divisions contains one grain, and are numbered by Arithmetical figures at every three grains by 3,6,9,12, *&c.* to 24, and at the place where 24 should stand, there standeth P. W. which signifieth one peny weight, or 24 grains, this P. W. standeth in the middle of the Line. Then is the same Scale continued farther by Arithmetical figures. 3,6,9,12, *&c.* as before to 24, and there is written P W. again, representing two peny weight, or 48 grains.

The Scale being thus divided, it is easie to find the place where any number of Grains under 48 shall be upon the Scale; As for example, if it be required to find where 8 peny weight shall fall, look

upon

upon the Scale of Troy-weight, from the beginning thereof, and count the figures 3 and 6 then count also two of the smaller Divisions, and that makes 8 grains, which you shall find to stand at the Letter *d.* which is the place of 8 grains; Also if upon the Scale you find the place of one peny weight 10 grains, you shall find it at the Letter *e*, and so of any number of grains under 48, or two peny weight.

But if you had a Decimal given, and would know what number of grains it representeth, if you seek your Decimal given in the the Scale of 1000, right against it in the Scale of Troy-weight, you shall find the number of grains represented thereby.

Example. Let .167 be a Decimal fraction given, If you look in the Scale of 1000, for 167, right against it in the Scale of Troy-weight, you shall find 8 peny weight.

Also if 708 were a Decimal fraction given, If you seek 708 in the Scale of 1000, right against it you shall find 1 peny weight 10 grains.

3 The third Scale is of *Averdupois great weight*, 28 pounds, or one quarter of an hundered, being the Integer, this Scale is numbred by 1, 2, 3, 4, &c. to 28, which 28 representeth 28 *l.* or a quarter of a hundred, and each of those is sub-divided into four small parts each representing one quarter of a pound.

Now if you would know what is the Decimal of any number of pounds or quarters under 28, if you seek the number of pounds in the Scale of *Averdupois great Weight*, right against it in the Scale of 1000, you shall find the Decimal thereof.

Thus if it were required to find the Decimal of 8 pound and an half, if you look upon the Scale for 8 pound and an half, you shall find it at Letter *g*, and right against it in the Scale of 1000 you shall find. 304, which is the Decimal of 8 pounds and an half.

4. The fourth Scale is of *Averdupois little Weight*, 16 ounces or one pound being the Integer; This Scale is first divided into 16 equal parts, and numbred by 1, 2, 3, 4, &c. to 16, each Division representing one Ounce. Then again, each of these ounces is sub-divided into 8 other smaller parts or divisions, each of which divisions representeth two Drams; but if your Scale be large enough, you may have each ounce divided into 16 equal parts or divisions, each division representing one Dram.

Now to find the Decimal belonging to any number of Ounces and Drams, repair to the Scale of *Averdupois little Weight*, and on it the quantity of ounces and drams required, and right against it in the Scale of 1000, you shall have the Decimal thereof.

Thus if it were required to find the Decimal of 6 Ounces and 6 Drams, if you look this in the Scale of *Averdupois little Weight*, you shall find it at the Letter *h*, and right against it in the Scale of 1000, you shall find 398, which is the Decimal of 6 ounces and 6 drams.

5. The

5. The fifth Scale is of *Dry Measures*, one Quarter or 8 Bushels being the Integer; this Scale is first divided into 8 equal parts, and numbered by 1, 2, 3, &c. to 8, each of which divisions representeth a Bushel, and each of those parts is again sub-divided, first into 4 equal parts or divisions, each representing one Peck, and then those again sub-divided into 4 other smaller parts, representing Quarters, Halves, and Three Quarters of a Peck.

Now if you would know the Decimal belonging to any number of Bushels (under 8 Bushels one Quarter) Pecks and parts of a Peck, if you seek the number of Bushels, Pecks, and parts of a Peck in the *Scale* of *Dry Measures*, right against it in the *Scale* of 1000, you shall have the Decimal required.

As for Example, if it were required to find the Decimal belonging to 5 Bushels 2 Pecks, and half a Peck, if you look into the Scale of *Dry Measures* you shall find 5 Bushels, 2 Pecks, and an half to stand at the Letter *k*, and right against it in the *Scale* of 1000, you shall find .702, which is the Decimal answering to 5 Bushels, 2 Pecks and a half.

6. The sixth *Scale* is of *Liquid Measures*, the Integer being 36 Gallons, or one Barrel, this *Scale* is divided first into 36 equal parts or divisions, and numbered by 5, 10, 15, &c. to 36, then every of these divisions, is again sub-divided into 4 other small divisions, each representing a quart, but (if the *Scale* be large enough) you may sub-divide each Gallon into 8 parts, so will every part represent one pint.

Now to find the Decimal belonging to any number of Gallons (under 36 Gallons or one Barrel) quarts or pints, repair to the *Scale* of *Liquid Measures*, and seek there upon the Scale, the number of gallons, quarts, pints, and against it in the *Scale* of 1000, you shall find the Decimal thereunto belonging.

So if it were required to find a Decimal representing 10 gallons and two quarts, or 4 pints, which is all one; if you seek in the Scale of *Liquid Measures* for 10 gallons, 2 quarts, you shall find it at the Letter *m*, against which in the Scale of 1000, you shall find. 292, which is the decimal of 10 gallons 4 pints.

7 The seventh *Scale* is of *Long Measures*, the Integer being *Yards* or *Ells*, this *Scale* is divided into 4 equal parts, and numbred by 1, 2, 3, 4 representing 1 quarter 2 quarters, 3 quarters or 4 quarters of a Yard, or, Ell, these are again sub-divided, first into 4 other equal parts, representing Nails, and those may be again sub-divided at pleasure if need be.

Now if you would know what decimal belongeth to any number of Quarters or Nailes of a Yard or Ell, if you seek the number of quarters and nailes in the *Scale* of *Long Measure*, the *Scale* of 1000 will give you the decimal thereof.

Thus if it be required to find the decimal belonging to 1 quarter and 3 nailes, if you ſeek this in the Scale of *Long Meaſure*, you ſhall find it ſtand at the Letter *o*, againſt which in the Scale of 1000 you ſhall find .437, which is the Decimal anſwering to 1 quarter and 3 nails of a Yard or *Ell*.

8 The Eighth and Laſt Scale is of *repreſentative Inches*, the whole Scale being divided into 12 equal parts, and numbred by 1, 2, 3, &c. to 12. and thoſe parts are again ſub-divided into halves, quarters, and half quarters, as Carpenters-Rules are uſually divided.

Unto this Scale (as unto all the other) there is joyned a Scale of 1000, this Scale will readily diſcover what is the Decimal belonging to any number of Inches, halves or quarters, and the uſe is the ſame with the Scales before mentioned.

Thus I have given you a brief Deſcription of theſe Scales, and the uſes of them, and do now ſuppoſe my Reader to be perfectly acquainted with the way of numbering or accounting upon them; wherefore I intend only to give you a Queſtion or two in the moſt uſual Rules of Arithmetick, and ſo conclude.

ADDITION.

WHat *Addition* is, and the manner of Working of it hath been already taught, both in the firſt and ſecond Parts, we will now come to an Example, which let be in *Addition of Engliſh Coin*. and let the ſums to be added be 36 *l.* 8 *s.* 8 *d.* 29 *l.* 0 *s.* 2 *d.* 31 *l.* 16 *s.* 9 *d.* and 6 *l.* 2 *s.* 5 *d.*

Firſt, ſet down 36 *l.* 29 *l.* 31 *l.* and 6 *l.* one under another, in ſuch order as you ſee herein the margine, drawing a Line by the ſide of them as you ſee done, and alſo a Line under them.

36	
29	
31	
6	

This done, ſeeing that your firſt number to be ſet down to 36 *l.* is 8 *s.* 8 *d.* you muſt for the 8 *s.* (becauſe two ſhillings, which we call a *Decade*, or the tenth part of a pound, is made the Integer, in the Scale of Mony) ſet down 4, which is done by memory, and after it make a Comma; Then your next number to be ſet by 29 *l.* being 0 *s.* 2 *d.* for the 0 *s.* ſet down a Cypher; Thirdly, for your number to be ſet by 31 *l.* being 16 *s.* 9 *d.* for the 16 *s.* ſet down 8 *Decades* with a comma after it, and Laſtly, the number to be ſet by 6 *l.* being 2 *s.* 5 *d.* for the 2 *s.* I ſet down 1 *Decade* comma after it, and then will your work ſtand, as here you ſee.

36	4,
29	0,
31	8,
6	1,

Then

Then take your Scale in hand, and seeing your first number of pence are 8 *d.* look in your Scale of money for 8 *d.* and against it in your Scale of 1000, you shall find 333, which set in the Line with 36 *l.* 8 *s.* behind the comma, then your next number of pence being 2 *d.* look in your scale for 2 *d.* and against it in the scale of 1000, you shall find 083, which set to 29 *l.* 0 *s.* behind the comma. Then your third number of Pence being 9 *d.* look in your your scale for 9 *d.* and against it the scale of 1000, you shall find 376, which set to 31 *l.* 16 *s.* and Lastly, your last number of pence being 5 *d.* look in your scale for 5 *d.* and against it you shall find 208, which set to 6 *l.* 2 *s.* and then will your whole work stand, as here you see.

36	4,333
29	0,083
31	8,376
6	1,208

Your sums being thus set down, which is done with more facility than you can imagine, till you make trial and be something perfect therein, you must then add all the numbers together, as in Addition of Decimals and you shall find the sum of them to be 103|4,000, Now to know this in mony, is as easie as it was to set several sums down, for the figures 103, which stand behind the down right line, are 103 *l.* and figure 4 which stands between the down right line and comma are, 4 *Decades* or 8 *s.* and being the rest to the right hand are all Cyphers, they signifie neither pence nor farthings, so is the total of this Addition 103 *l.* 8 *s.* 0 *d.*

36	4,333
29	0,083
31	8,376
6	1,208
103	4,000

That the manner of working may appear more plain, I will give you another short Example as difficult as I can invent, which I performed by a scale of Wood but of 8 inches long. Let the sums to be added together be these following:

li.	*s.*	*d.*	*q.*
332	17	4	1
159	6	8.	1.
217	5	3	3
709	09	4	1

First set down your several Sums of pounds one under another as before, and draw a line by the side of them, and another under them. So will they stand as here you see.

332	
159	
217	

1. Your sums of pounds being thus orderly placed and lines drawn, repair to your Scale, and seeing your first number of shillings, pence and farthings is 17 *s.* 4 *d.* 1 *q.* for your 17 *s.* set down 8 Decades which is 16 *s.* with a comma after it, then will there rest to be set down 1 *s.* 4 *d.* 1 *q.* or 16 *d.* 1 *q.* which if you seek in your

Scale of mony, you shall find to stand against it in the Scale of 1000 this number 677, which is the Decimal of 1 *s.* 4 *d.* 1 *q.*

2. You second number of shillings, pence and farthings is 6 *s.* 8 *d.* 1 *q.* for your 6 *s.* set down 3 Decades which is 6 *s.* and there will remain 8 *d.* 1 *q.* which if you seek in your Scale of mony, you shall find to stand against it in the Scale of 1000, this number .344, which is the Decimal of 8 *d.* 1 *q.*

3. Your third number of shillings, pence, and farthings, is 5 *s.* 3 *d.* 3 *q.* for your 5 *s.* set down 2 Decades, which is 4 *s.* with a comma after it, then will the rest to be set down 1 *s.* 3 *d.* 1 *q.* or 15 *d.* 3 *q.* which if you seek in your Scale of mony, you shall find to stand against it in the Scale of 1000, this number .656, which is the Decimal of 15 *d.* 3 *q.* or 1 *s.* 3 *d.* 3 *q.* and the three Sums to be added together will stand as here you see.

l.	
332	8,677
159	3,344
217	2,656
709	4,677

These Sums being added together according to the Rule for Addition of Decimals, you shall find the Sum of them to be 709|4,677 now to know what this is in mony, take notice that the 709 which stands to the left hand of the down right line are 709 pounds, and the figure 4. which stands between the down right line and the comma, are 4 decades or 8 *s.* but (because the first figure next after the comma is above (5, *viz.* 6) you must add 1 *s.* to the 4 decades, making them 9 *s*, then will there remain 177, wherefore if you look in the Scale of 1000 for 177, you shall find against it in the Scale of mony 4 *d.* 1 *q.* So is the whole Sum of this Addition 709 *l.* 9 *s.* 4 *q.* as by the preceeding work doth appear.

¶ Here note, that when you had set down your 709 *l.* 4 decades or 8 *s.* there remained beyond the comma, 677, which if had sought in your Scale of 1000 you should have found against it in the Scale of mony 15 *d.* 1 *q.* or 1 *s.* 3 *d* 1 *q.* (which is all one) as before; for it appeareth plainly by the Scale, that 500 in the line of 1000 is equal to one shilling.

SUBSTRACTION.

Substraction (as hath been before said) is the taking one or more smaller Sums out of one greater; I shall only give you an Example or two, as I have taken the numbers from a Scale.

Example. Delivered to a Gold-smith first of old Plate 297 ounces, 13 peny-weight, 19 grains.

Received of the same Gold-smith first 165 ounces, 11 peny-weight

weight and 7 grains, and after that received of the same Gold-smith 32 ounces 19 peny-weight and 32 grains, what Plate remaines in the Gold-smiths hands?

Take your Numbers out of the Scale of Troy-weight, and set them down as you here see.

	ounces
Delivered	29716,896
Received first	1655,646
Received more	329,979
Received in all	19815,625
Rests in the Gold-smiths hands	9911,271
	or

Ounces	*peny-wt.*	*gr.*
99	2	13

Then add the several weights of Plate received together, and they make 19815,625, or 198 ounces 11 pen-weight, 6 grains, which if you substract from 29716896, or 257 ounces, 13 peny-weight, 19 gr. which was the quantity of Plate delivered, there will remain 9911,271, or 99 ounces, 2 peny-weight, 13 gr. and so much Plate is still in the *Gold-smiths* hand. And let thus much suffice for *Substraction.*

Now we should proceed to *Multiplication* and *Division*, but when the numbers are taken from the Scale and set down, the manner of working doth not at all differ from *Multiplication* and *Division* of *Decimals* before taught. But before I teach you how to multiply and Divide Instrumentally, I shall shew you,

The farther use of the Decimal Scales, and how by them to find the Square or Cube Root of any Number. And also, any Root being given, to find the Square or the Cube number of that Root.

And both these by inspection only, without the help of either *Pen*, *Compasses* or any other *Motion.*

FOr the effecting hereof, there is now inserted, among the fore-mentioned Decimal Scales of Mony, Weight, Measure, &c. namely between the Scales of *Averdupois little Weight*, and that of Dry Measures, Two other Scales, one having written at the beginning there-

thereof the Word Square, and to the other there is added the Word Cube, and between them, there is a third line, which hath written upon it, the word Root. And by theſe three Scales thus united, the Square and Cube Roots of any number may be extracted by inſpection only. For,

If you find any number whoſe Square Root you require, in the Line or Scale of Squares, right againſt it, in the Line of Roots, you ſhall have the Square Root of that number. Thus,

If the number 64 were given, and it were required to find the Square Root thereof.

Find the given number 64, upon the Line or Scale of Squares (which you may do at the Letter *a*) and right againſt it, in the Scale of Roots ſtands the figure 8, which ſhews, that 8 is the Square Root of 64. And in the ſame manner you may find the Square Root of any other number.

For, againſt	81	in the Line of Squares, you ſhall find	9	in the Scale of Roots, which is the Square Root thereof,
	64		8	
	49		7	
	36		6	
	25		5	
	16		4	
	9		3	
	4		2	

In like manner,

If the number 64 were given, and it were required to find the Cube Root thereof.

Find the given number 64 in the Scale of Cubes, (which you may do, by counting the ſame number between the ſecond and third figures of 1 upon the Scale, at the letter *b*) and right againſt it, in the Line or Scale of Roots, ſtands the figure 4, which ſhews, that 4 is the Cube Root of 64. And in the ſame manner you may find the Cube Root of any other number.

For, againſt	729	in the Scale of Cubes, you ſhall find	9	in the Scale of Roots, which is the Cube Root thereof.
	512		8	
	343		7	
	216		6	
	125		5	
	64		4	
	27		3	
	8		2	

And by this Artifice, not only the Roots of direct Square and Cube numbers may be found, but in numbers that be not directly Square or Cube, the Fraction part of the Root is nearly diſcovered alſo.

I

I have hitherto given you Examples in ſuch Square and Cube numbers, as are common and familiar, and that any man may compute almoſt by memory; but by theſe the Demonſtration of the Artifice is diſcovered, the Lines of Squares, and Cubes being only Square and Cube numbers transferred to Lines. And now let us proceed to greater numbers. And

I. *For the Square Root.*

In the Extraction of a Square Root, it is uſual to ſet a Prick under the firſt figure, the third, the fifth, the ſeventh, and ſo forwards, beginning from the right hand towards the left. And as many Pricks as fall under the Square Number given, of ſo many figures will the Root of that Number conſiſt: So that if the Number given be leſs than 100, the Root ſhall be only of one figure, if leſs than 10000, it ſhall be but two figures, if leſs than 1000000, it ſhall be three figures, and ſo forward.

Hence it is, That the Line or Scale of Squares, is divided firſt into 100 parts, and if the Number given be greater than 100, the firſt diviſion (which is the place where the firſt figure of 1 ſtandeth, and which before did ſignifie One) muſt now ſignifie 100, and the whole line ſhall be 10000. If farther, the numbe be greater than 10000, you muſt count or eſteem the firſt Figure of 1 to be 10000, and then will the whole line be 1000000 parts; and if this be too little to expreſs the number given, you may proceed farther, and call the firſt 1. 1000000, and ſo increaſe the Line 100 times more; but this is ſufficient.

Thus when any Square Number is given, if you ſet it down, and prick it, (or imagine it to be ſo) if the laſt prick to the left hand ſhall fall under the laſt Figure, (which will always be when the Figures in the given Number be odd) you muſt find all ſuch Numbers upon the Line, between the two Figures of 1.———But if the laſt prick ſhall fall under the laſt Figure but one of the given Number, (which it will always do, when the Figures of the Number given are even) then you muſt find the Number given in the Line of Square, between the ſecond Figure of 1 and 10 at the end of the Line.

This being conſidered, find the Number given, whoſe Square Root is required, in its proper place upon the Line of Squares, and againſt it in the Scale of Roots you ſhall have its Square Root deſired. Thus,

The Square Root of	36	will be found to be	6
	360		19
	3600		60
	36000		189.

And

And in this manner may the nearest Root of any number not exactly Square be obtained. For

The nearest Root of	725	will be found to be about	27
	7250		85
	72500		269
	725000		851

And thus on the contrary, a Number may be Squared, as may partly appear by what hath been before delivered; for if you find the Root in the Scale of Roots, you have its Square in the Line of Squares and, so

Against	27	in the Scale of Roots, you shall find	725	the Square thereof.
	85		7250	
	269		72400	
	851		725000	

Thus much for the Square Root. Now

II. *For the Cube Root.*

In the Extraction of the Cube Root, it is usual to set Pricks under the First Figure, the Fourth, the Seventh, the Tenth, and so on, pricking always the third Figure from the right hand towards the left. And as many pricks as fall under the Cubick Number given, so many Figures shall be in the Root. So that if the Number given be less than 1000, the Root shall consist only of one Figure, if less than 1000000, it shall be only of two Figures; if it be above 1000000, and less than 1000000000, it will be only three Figures.

Hence it is, That the Line of Cubes is divided first into 1000 parts; And if the number given be greater than 1000, the first Figure of 1 (which before did signifie only One) must now signifie 1000, and the second Figures of 1, must now signifie 10000, and the third 1, must signifie 100000, and the whole Line must be esteemed to be 100000. Farther, If the number given be greater than 1000000, the first 1, must signifie 1000000, the second 1000000, the third 10000000, and the whole line 1000000000 parts. And if these be yet too little, you may proceed farther; but let these suffice.

Thus when any Cube Number is given, if you set it down, and prick it; If the last prick to the left hand shall fall under the last Figure, the Number shall be reckoned between the first and second

Fig: 1.

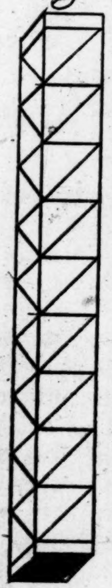

Fig: 2.

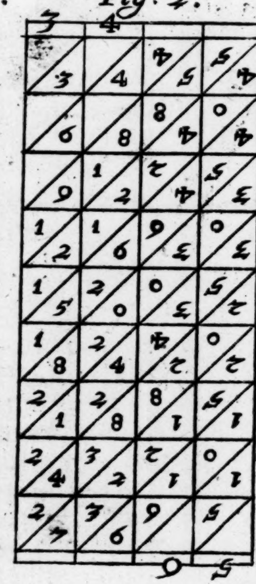

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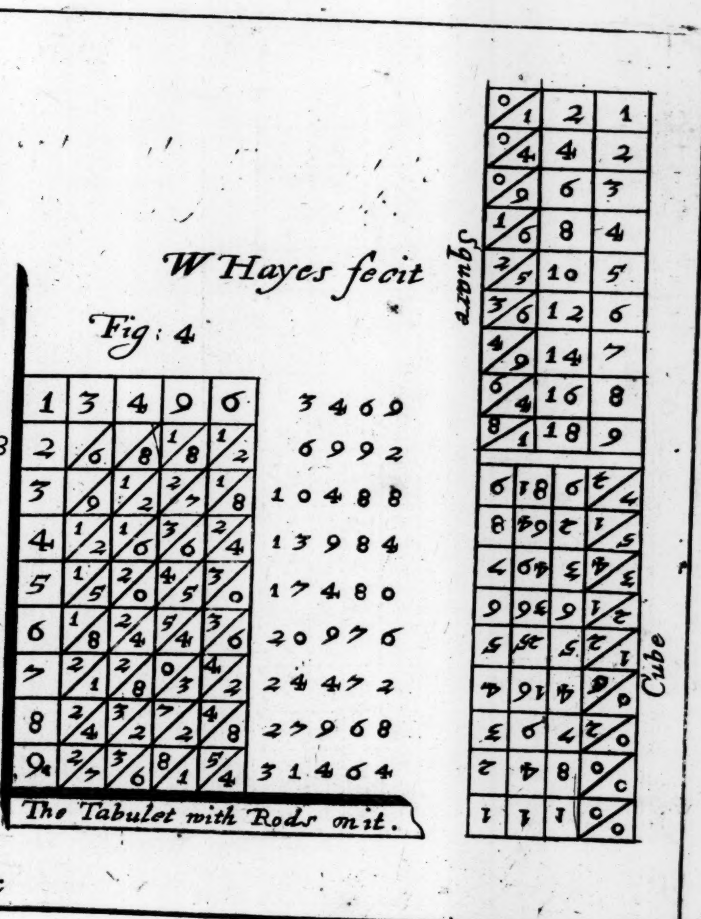
Fig: 4



cond Figures of 1, and the first Figure of the Root shall be always either 1 or 2 ——— If the last prick fall under the last Figure but one, the number given must be reckoned between the second and third Figure of 1, and the first Figures of the Root shall always be either 2, 3, or 4. ——— But if the last prick shall fall under the last Figure but two, then the Number given must be reckoned between the third Figure of 1, and so at the end of the Line.

This being considered, find the the Number given, whose Cube Root is desired, in the proper Section upon the Line or Scale of Cubes, and right against it in the Scale of Roots, you shall have its Cube Root desir'd. Thus.

849	The Cube Root of	8490000	will be found to be about	204
849		84900000		439
849		849000000		947

And the like of any other.

On the Contrary, a number may be Cubed; for if you find the number in the Line of Roots you shall have the Cube thereof right against it in the Scale of Cubes, giving the true denomination to the Cube, according as the part of the Line against which the *Root* standeth doth require.

Thus have you by this Instrumental way of working, these things, which in the ordinary course are most hard and intricate, rendred very familiar and easie: And although at all times you do not make use of them, yet they are ready helps to confirm you in your working without the tedious way of proving by Reverse working.

SECT. II.

By *NEPAIR's BONES.*

IN the foregoing Argument I told you, That the Author and Inventer of this kind of Instrument, of which I intend to shew the Use, called it *RABDOLOGIA*, and the word he thus defines:

RABDOLOGIA, *est Ars Computandi per Virgulas numeratrices.* That is, *RABDOLOGIA* is the Art of Counting by Numbering Rods.

I. *Of the Fabrick of these Rods, according to the Inventor's Description of them.*

THese Rods may be made either of Silver, Brass, Box, Ebony, or Ivory, of which last substance I suppose they were at first made, for that they are (for the most part) by all that know or use them, called *NEPAIRS BONES.*

But let the matter of which they are made be what it will, their form (according to this description) is exactly a square Parallelepipedon, the length being about three Inches, and the breadth of them about One tenth part of the length. But the length of these Rods are not confined to three Inches, but let the length be what it will, the breadth must be a tenth part thereof, but that may be accounted a competent breadth that is capable of receiving of two numerical Figures, for there is never upon one Rod required more to be set on the breadth thereof.

The breadth of these Rods being exactly one tenth part of the length thereof, when 10 of these are laid together they do exactly make a Geometrical Square, and if 20 of them be tabulated or laid together, they will make a right-angled Parallelogram, whose length is double to its breadth. If 30 be tabulated, the Figure will be still a Parallelogram, whose length will be three times the breadth, and so if 40, four times the length, & *sic*, &c.

The Rods being thus prepared of exact length and breadth, let each of them be divided into 10 equal Parts, with this *Proviso*, that Nine of the Ten parts stand in the middle of each Rod, and the other tenth part must be divided into two parts, half whereof must be set at the one end, and the other half at the other end of the same Rod. Then from side to side draw right Lines from division to division, so is your Rod divided into Squares on every side thereof. Lastly, from corner to corner of every of these Squares draw a Diagonal Line, and that will divide every Square into two Triangles. The Rods being thus prepared and lined, first into Squares, and then into Triangles, they are then fit to be numbred.

The Figure 1. at the beginning of the Book, shews the Form of one of these Rods lined as it ought to be.

II. *How*

II. *How theſe Rods are to be Number'd.*

IN the two half Squares which are at the ends of each Rod on every ſide, there are ſet one ſingle Figure, on each ſide of every Rod one, in the diviſion at the end thereof, ſo every Rod containing four ſides; Ten Rods will contain 40 ſides, and ſo conſequently will have 4 ſingle Figures at the ends of every of them; that is, there will be upon the Ten Rods among them, four Figures of each kind, that is, four Ones, 1111. four Twoes, 2222. four Threes, 3333. four Fours, 4444. four Fives, 5555. four Sixes, 6666. four Sevens, 7777. four Eights, 8888. four Nines, 9999. four Cyphers, 0000.

And here it is to be noted, That what Figure ſoever it be that ſtandeth at the top of the Rod alone, the Figure that ſtandeth alone on the other ſide of the ſame Rod, maketh that Figure up the number 9. As for Example, If 1 ſtand on one ſide, 8 will ſtand on the other ſide, ſo 2 and 7, &c. As in this Table, where,

If	1 2 3 4 5 6 7 8 9	ſtands alone at the top of any ſide of any of the Rods, then	8 7 6 5 4 3 2 1 0	ſtandeth on the other ſide of the ſame Rod.

This alſo is to be obſerved in the figuring of every Rod, that what Figure ſoever ſtandeth alone at the top or ſuperiour part of the Rod, the Figure or Figures that ſtand in the two Triangles next underneath it, is double to the Figure which ſtandeth at the top. And the Figures which ſtand in the next two Triangles below, that that is three times as much as the Figure above. And that in the fourth Place, or Triangles, is four times as much as the Figure above, &c. till you come to the loweſt Triangles in that Rod, and then the Figure or Figures that ſtand in thoſe Triangles are nine times as much as the Figure which ſtandeth at the top of the Rod.

So if a Rod have 4 at the top thereof, the two Triangles which are juſt and next under it, have 8 in them, which is double to 4: In the next two Triangles below there is 1 and 2, that is 12, which is treble to 4; In the two Triangles below them is 1 and 6,

which

which together make 16, which is four times as much as the 4 at the top; the next Triangles have in them 20, &c.

Thus have you the Fabrick, Inscription and number of these Rods, according to the Inventors contrivance of them: He makes mention of Ten of them, and hath in his Book set the Figure of the said Ten, of one of which Ten I have given you a Scheme at the beginning of the Book, which is *Figure* 2. I will now proceed to give you the description of these Rods in another more commodious form.

III. *A Description of these Rods according to their best and latest Contrivance.*

THe Description which I shall here give of these Rods, varies not at all from that before delivered in the matter of which they are made, for these may be made either of Silver, Brass, Wood, Ivory, &c. Neither do they differ in their dividing, nor yet in their numbering: Only, whereas my Lord *Nepair* maketh them Square, each Rod to contain four sides, these are made flat, consisting each Rod but of two sides, and contain in length about 2 Inches $\frac{3}{10}$
and in breadth $\frac{1}{4}$ of an Inch.
and in thickness $\frac{1}{12}$ of an Inch.

One set of these Rods consisteth of five Pieces, and therefore hath but ten Faces or Sides, whereas those of the Lord *Nepair*'s consisted of 40 Plains or Sides.

Upon one of these five Pieces (a Figure whereof is at the beginning of the Book, noted with *Figure* 3) you have a Cypher at the head of the first piece, and 9 at the bottom thereof. Upon the second of them you have 1 at the head, and 8 at the bottom: Upon the third you have 2 at the head, and 7 at the bottom: Upon the fourth 3 at top, and 6 at bottom; And upon the fifth you have 4 at the top, and 5 at the bottom. Every of the two Figures at the top and bottom together make 9; as 0 and 9 is 9, 1 and 8, 2 and 7, 3 and 6, 4 and 5. And here observe, that the Figures 9, 8, 7, 6, 5, which stand at the bottom of the Scheme, stand with their heels upwards in this manner, 6 8 ⅂ 9 ϛ, and so do all the other Figures under them, till you come to the double Line which is in the middle of the Scheme, noted with *A* and *B*, at which Line, if the Scheme were cut into two pieces, and folded or pasted on the back-side of the other half, so that the 9 at the bottom were placed upon the Cypher at the top, and so 8 upon 1, 7 upon 2, 6 upon 3, and 5 upon 4, then the Scheme cut again into five little slippets by the

the down-right Lines; these five slippets would exactly represent one set of these Rods, for upon one side of these Pieces, you should have a Cypher upon one side, and 9 on the other; Upon the next 1 and 8, upon another 2 and 7, on another 3 and 6, and on the other 5 and 4; both the Figures on either side making 9, as before was described.

These 5 slippets do now contain the whole Multiplication Table of *Pythagoras* before mentioned, but so few are not of sufficient use, neither are the Ten before mention'd of the Lord *Napiers* order; for there can be but four Figures of one kind, which in all cases is not sufficient.

Therefore, as these Rods are made now a days, they do commonly make six sets of them, that is, 30 pieces, which contain 60 Faces, and these will be of good use, and there will seldom be found a want, which in those of the Inventor's there will often be, except you have a great quantity, which will be far more cumbersome than these here described, for there is required as much Metal or Wood in one of his, as in four of these, and then for his four sides we have here Eight.

Concerning a Case for these Rods.

For the orderly keeping and ready finding of these Rods, I have often (for my self and others) had a Box made of Walnut-tree or Pear-tree, with five Partitions in it, each Partition to hold five or six Sets of these Rods, or more if more Rods were requir'd. Every of these Partitions being Figur'd on the side thereof next the Eye, with such Figures as the Rods in such a Partition had Figures at the top, so that the party that was to use them, could take them as readily out of his Partition, as a Printer can take his Letters out of his respective Boxes to make any Word.

In this Box there is also convenient Room made for one other Rod, double in breadth to these here described, but of the same length and thickness; upon the one side whereof there is a Table or Plate useful in the Extracting of the Square Root, and on the other side another for the Extracting of the Cube Root, the Figure whereof is at the beginning of the Book, noted with Square, Cube: But I shall forbear to say any thing of them, till I come to shew you how to Extract the Square and Cube Roots by the help of them and the Rods.

Of

Of a Board with a Frame, upon which to lay your Rods, when any Operation is to be wrought by them, known by the name of a TABELLET.

In the using of these Rods, care is to be had first of the orderly laying of them, and then secondly, for the keeping of them in that position till your work be ended. For the effecting whereof, both neatly and certainly; there is a little Table or Frame contriv'd, containing in breadth $\frac{1}{20}$ of an Inch more than the length of the Rods, and in length at pleasure, but it may well be about one and a half the length of the breadt h.

It ought to be made of a thin peice of Pear or Walnut-tree, or of such matter as your Box or Case is made of, and it may very commodiously be contriv'd to be put into the Box as ever I had them made to do, for that I found it convenient to carry loose.

Upon the Superficies of this board, close to one of the edges thereof, must be glewed or otherwise fastened with Pins, a small piece of the same matter, and also of the same length, breadth, and thickness of one of your Rods, which must be divided into 9 equal parts, and Lines drawn cross the piece, so will there be 9 Squares, in which you must Grave or Stamp the nine Digits, beginning with 1 at the top, and so descending by 2 3 4 to 9 at the bottom thereof: And it were necessary that these Figures (as also those which are at the head of every of your Rods) were Graven or Stamped of something a bigger Figure than the other figures of your Rods are.

Under the end of this ledge beginning at the Figures, and so continuing the whole length of the Board, must another ledge of the same matter and thickness, be glewed, or pinned, and then is your *Tabellet* finished. A Figure whereof you have at the beginning of the Book, noted with *Figure* 4, it is called a *Tabellet*, for that, when the Rods are laid thereon, for any Operation to be wrought by them, we usually say, the Rods are Tabulated.

Thus much for the Fabrick, Inscription, and Numbering of these Rods; let us now come to shew the Uses of them: Which is in Multiplication, Division and Extracting the Square and Cube Roots.

IV. *How*

IV. *How to apply to lay down any Numbers by the Rods.*

PROP. I.

Any Number being given, how to Tabulate or lay down the same by the Rods.

Let it be required to Tabulate or lay down this Number 3496. Firft, from among your Sets of Rods (or out of your Cafe) take four of them, of which let one of them have the Figure 3 at the top thereof, and lay it upon your Tabellet clofe to the edge thereof, then,

Secondly, Take another Rod from your Cafe, which hath the Figure 4 at the top of it, and lay that alfo upon your Tabellet clofe by the fide of the other.

Thirdly, Take another Rod which hath the Figure 9 at the top of it, and lay that upon your Tabellet clofe by the other two.

And laftly, take a fourth Rod, having the Figure 6 at the head thereof, and lay that alfo upon your Tabellet clofe by the reft.

Thefe four Rods thus taken, and laid upon the Tabellet, you fhall fee in the uppermoft Row (which ftandeth againft the Figure 1 on the fide of your Tabellet) thefe four Figures, 3 4 9 6, that is 3496, equal to your given Number. In the fecond Row (againft the Figure 2 of your Tabellet) you fhall find the double thereof. In the third (againft the Figure 3) you fhall find the triple thereof. In the fourth the Quadruple thereof. In the fifth the Quintuple; and fo on the ninth and laft the Noncuple of the Number given.

PROP. II.

How thefe Rods will appears when Tabulated, and being Tabulated, how to read the Multiplication of that Number fo Tabulated, by any of the Nine Digits.

The Four Rods being Tabulated according to the Precepts delivered in the preceding *Propofition*, they will appear exactly as they are reprefented in *Figure* 4, at the beginning of the Book; which Figure lively reprefents the four Rods lying upon the Tabellet, which mind well, for upon the true Tabulating, and right reading of the Rods fo Tabulated, depends the whole work.

The Rods thus Tabulated, and as you fee them in the *Figure* 4, do to the Eye, appear in the form of a Glafs-window, every Pane thereof

thereof representing a Rhomboiades or Diamond-form: In the reading of the Figures which are in these several Rhomboiades or Diamond-form, observe these few Directions following, which will fully illustrate the whole Business intended, and therefore especially to be minded.

NOTE,

I. *That the Figures upon the Rods are to be read, beginning at the right hand and reading towards the left*; which is contrary to our common course of reading and writing, which is from the left hand towards the right.

II. *That in every Rhomboiades or Diamond, there are either One Figure, or Two Figures, but never more than Two.*

III. *If there be but one Figure in a Rhombus, then that Figure is the Figure to be set down alone (be it either a Figure or a Cypher) but if there be two Figures in a Rhomboiades (as for the most part there is) then add them two Figures together, and set down their sum in one Figure.*

IV. *But if the sum of the two Figures in one Rhomboiades or Diamond do exceed Ten, then you must set down the overplus above Ten, and keep One in mind, which One you must carry to the next Rhomboiades.*

V. *Note that the first towards your right hand, and the last towards your left hand are but half Rhomboiades or Diamonds, and never have in them more than one Figure only, but all between them are whole ones, and for the most part have two Figures in them.*

VI. *If in either Rhomboiades or half Rhomboiades, you find no Figures but Cyphers, you must not neglect but set them down as if they were Figures.*

¶ These Rules being rightly understood, all that follows will be familiar and easie, and these I shall explain by Example following.

Example. For the illustration of the preceding Rules, we will make use of those Rods which were before Tabulated, therefore have recourse to *Figure* 4 at the beginning of the Book, where this Number 3496 is Tabulated.

The Figures at the top of the Four Rods are these: 3, 4, 9, 6. which signifie the former given Number 3496, and this number stands against the Figure 1 on the side of the Tabellet. Then I say, that the Figures in the next row standing against the Figure 2 of the Tabellet are double thereunto, which I thus prove.

Repair

Repair to the Rods as they lie upon the Tabellet, and in that row which lieth against the Figure 2, you shall find in the first half Rhomboiades towards your right hand (where *by Rule* 1 you must begin) the Figure 2, wherefore set down with your Pen upon Paper the Figure 2. In the next Rhomboiades in the same row you shall find 8 and 1, which added make 9, set down 9 on the left hand of 2: In the next Rhombus you shall find 8 and 1 again, which is 9 also, set down 9 on the left hand of the other, and in the last Rhomboiades you shall find only 6, wherefore set down 6 on the left hand of 9, so have you in all 6992, which is double to 3496.

Again, the Figures in the row which stands against the Figure 3 in the Tabellet, are triple to 3496; for in the first half Rhomboiades towards your right hand, you have 8, set down 8.——In the next Rhom. you have 7 and 1, which is 8, set down 8 again.——In the next you have 2 and 2, which is 4, set down 4.——In the next Rhom. you have 9 and 1, which makes 10, set down 0 and carry 1, but it is the last Rhom. and because there is never another to carry the 1 unto, you must therefore set it down, so have you this number 10488, which is triple to 3496.

Again, the Figures standing against 4 in the Tabellet, are Quadruple to 3496,—for in the half Rhom. you have 4, set it down: in the next 6 and 2, which is 8, set that down: In the next 6 and 3, which is 9, set that down: In the next 2 and 1, which is 3, set that down: and in the last half Rhom, you have 1, which also set down: so have you 13984, which is Quadruple to 3496.

Also, the Figures against 5 in the Tabellet: the first is a Cypher, therefore put down 0; the next is 5 and 3, which is 8, set down 8; the next is 0 and 4, set down 4; the next is 5 and 2, that is 7, set down 7; and the last is 1, therefore set down 1, so have you in all 17480, which is Quintuple to 3496.

Against 6 in the Tabellet, you have in the first place 6, set it down; then in the next 4 and 3, that is 7, set that down; in the next 4 and 5, that is 9, set 9 down; in the next you have 8 and 2, that is 10, set down 0 and carry 1 to the next Rhom. where you find only 1, to which add the 1, which you carried from the Rhom. before, and it makes 2, set down 2: so have you 20976, which is six times 3496.

Against 7 in the Tabellet, you have first 2, set it down; then 3 and 4, which is 7, set 7 down; in the next you have 8 and 6, which is 14, which being 4 above 10, set down 4, and carry 1 to the next Rhom. where you have 2 and 1, which is 3, and 1 which you carried makes 4, set down 4; then in the last place you have only 2, which set down, so have you in all 24472, which is Septuple to 3496, or seven times as much.

Against 8 in the Tabellet, you have first 8 which set down; then 2 and 4, which is 6, set 6 down; then 2 and 7, which is 9, set 9 down; then 4 and 3, which is 7, set 7 down; and lastly 2, set that

 down.

down; so have you 27968, which is Octuple to 3496, or eight times as much.

Lastly, against 9 in the Tabellet, you have in the first place 4, set that down; in the next you have 1 and 5, which is 6, set 6 down; in the next place you have 6 and 8, which is 14, set down 4, and carry 1 to the next-Rhom. where you find 7 and 3. that is 10, which with 1 which you carried makes 11, set down 1, and carry 1 to the next Rhom. where you find only 2, and the 1 carried makes 3, therefore set down 3, and so you have 31464 which is Noncuple to 3496, or nine times as much as the tabulated number.

Thus have I given you Examples, in shewing you how the Numbers upon the Rods are to be read and written down; and in the delivery of this *Example*, I have made the whole work which is to follow, so plain and easie, that the meanest capacity (I think) if he can but tell his Figures, and add any two Figures together, he may by this here delivered, read or write down any Number, that can be tabulated; and that you may throughly understand this Chapter before you proceed further, I will give you the Products of 7009078 multiplied by all the nine Digits, which I would have your self to tabulate, and see if you find your working by your Rods to agree with those which are here written, which Numbers if they do, you need not scruple at the most difficult that can be proposed to you, therefore study it, and try it.

			7009078
7009078 being multiplied by	2	Produceth	14018156
	3		21027234
	4		28036312
	5		35045390
	6		42054468
	7		49063546
	8		56072624
	9		63081702

Thus have I sufficiently described these Rods, *and the manner of Numbring upon them; and now I think it time to apply them to that use for which they were intended, namely, the more difficult parts of* Arithmetick, *as* Multiplication, Division, *and* Extraction *of Roots.*

V. *Multiplication by the Rods.*

IN Multiplying by the Rods, you are to consider (as in Vulgar Arithmetick) three *Terms, Things,* or *Numbers,* viz.

1. The *Multiplicand,* which is the Number to be multiplied.

2. The *Multiplier,* which is the Number by which the *Multiplicand* is multiplied.

3. The

3. The *Product*, which is the sum produced by the multiplying of the two former together.

And here Note, that the *Product* doth contain the *Multiplicand*, so many times as there be *Unites* in the *Multiplier*.

Thus much for the definition of *Multiplication*, now for the working thereof by the Rods, for which this is

THE RULE.

First, *Set down upon your Paper the Multiplicand, and orderly under it the Multiplier.* It matters not greatly of which of the two given Numbers be made *Multiplicand* or *Multiplier*, but it is usual and best to make the greatest Number *Multiplicand*, and the lesser *Multiplier*. *Then draw a Line with your Pen under them, and having Tabulated your Multiplicand (or greater-Number) look what Numbers in your Rods stand against that first Figure towards your right hand, and that Number which you shall find upon your Rods standing against that first Figure found in your Tabellet, set down under your Line which you formerly drew under your Multiplicand and Multiplier: And having so done with the first Figure of your Multiplier, do so with the rest, setting them down one under another, removing every Figure one place more towards the left hand, than that which went before it, as is done in common* Multiplication, *and as you see in the following Example.*

Example. 1. Let it required to multiply 3496, by 489. As if it were required to know how much 489 times 3496 would amount unto.

First, Set down your given Numbers 3496, and 489, one under another, and draw your Line under them, as here you see done.

Secondly, 3496 your Multiplicand being Tabulated, and 9 being the first Figure to the right hand in your Multiplier, look upon your Rods, what Figures there stand against 9 in the side of your Tabellet, and you shall find (as by the Fifth Rule) 31464, which is the Product of 3496 multiplied by 9, wherefore set down this Number 31464 under your Line, as you see in the Example.

3496	*Multiplicand,*
489	*Multiplier,*
31464	
27968	
13984	
1709544	Product

Thirdly, Look what Figures upon the Rods stand against 8, which is the second Figure of your Multiplier, and you shall find 27968; set this Number under the former, moving it one place forward towards the left hand.

Fourthly, Look what Figures upon the Rods stand against 4, which is the third Figure in your Multiplier, and you shall find 13984, which set down under the other, one place more to the left hand.

Laſtly, Under theſe three Sums draw a Line, and add the three Sums together, and they make 1709544, which is the Product of 3496 multiplied by 489 and this 1709544 the Product, contains 3496 the Multiplicand, 489 times.

Practiſe well this firſt Example, and compare it with the Rods as they are Tabulated in Fig. 4 *at the beginning of the Book, as alſo with the foregoing Rules, and you may perform any Multiplication.* However I will give you one or two more Examples, and ſome other ways of *Multiplication.*

Example 2. Let it be required to multiply the ſame Sum 3496, by 261.

```
    3496
     261
  ------
    3496
  20976
  6992
  ------
  912456
```

Set the Numbers down as here is done, then look upon the Rods for the Product of 3496 by 1, and you ſhall find it to be the ſame, wherefore ſet down 3426 under the Line——then look upon the Rods for the Product of 3496 by 6, and you ſhall find it to be 20976, which ſet down under the other Number, one place more towards the left hand.——Again, look in the Rods for the Product of 3496 multiplied by 2, and you ſhall find it to be 6992, which ſet down under the other two.

Laſtly, Draw a Line under them, and add the three Numbers together in order as they ſtand, and the ſum of them will be 912456, which is the Product of 3496 multiplied by 261.

Example 3. Let it be required to multiply the ſame Number 3496 by 520.

```
     3496
      520
  -------
     6992
   17480
  -------
  1817920
```

Set down your Numbers as here you ſee done—— Then becauſe the firſt Figure of your Multiplier towards your right hand is a Cypher, wholly omit it, and multiply 3496 by 52 only, ſo ſhall you find the Product of 3496 by 2, to be 6992, which ſet down: Alſo the Product by 5 will be 17480, which ſet down under the other, one place further; Then draw a Line——and add theſe two ſums together, and they make 181792, to the which if you add a Cypher for the Cypher which you omitted in your multiplier, the Sum will be the 1817920, which is the Product of 3496 by 520.

Example 4. Let it be required to multiply the ſame 3496 by 7003—

```
      3496
      7003
  --------
     10488
  24472..
  --------
  24482488
```

Set down your Numbers as before, and as you ſee here done; Then having Tabulatated 3496, ſee what the Product thereof is upon the Rods, being multiplied by 3 the firſt Figure in your Multiplier and you ſhall find it to be 10488, which ſet down under the Line— Then the two next places of your Multiplier being Cyphers, make two Pricks under the former Number, one under 8, and the other under 4, as you ſee in the Example

ample ; (or inſtead of 2 Pricks you may make two Cyphers,) Then look in the Rods for the Product of 3496 by 7, and you ſhall find it to be 24472, which ſet down under the other Sum, beginning your Number at the fourth place, or beyond the two Pricks or Cyphers. Laſtly, draw a Line and add theſe two Sums together, and their Sum is 24482488, which is the Product of 3496 multiplied by 7003.

Thus have you four Examples in *Multiplication*, in which are included all the Varieties that may at any time happen in that Rule, *viz*. Two where the Multiplier conſiſted all of Figures, as in the firſt and ſecond Examples they did. — Another where the latter place of the Multiplier conſiſted of a Cypher——And this laſt Example where Cyphers were intermixed among the Figures.

And thus much for this kind of *Multiplication*, but before I leave, I will ſhew you

Another Form of Multiplication.

WHereas in the foregoing Form of *Multiplication*, which is the beſt and moſt uſual, (only I inſert this following for variety) You began (your Rods being Tabulated) with that Figure of your Multiplier which ſtands next your right hand, but there is no neceſſity for that, for you may begin with that Figure which ſtandeth next to your left hand, and by ſo doing, and placing your ſeveral Products one place more to the right hand, as you did before, place them to the left hand, thoſe Products added together in the Form they then ſtand, ſhall produce a Product equal to the Former.

Example, For our Example we will take the firſt Example before-going at the beginning of this Section, where it was required to Multiply 3496 by 489. Set the Numbers down as before in that firſt Example, and as you ſee here done.

```
3469
 489
-------
13984
 27968
  31464
-------
1709544
```

Then 3496 being Tabulated, look upon your Rods, for the Product thereof multiplied by 4, (which is the firſt Figure of your Multiplier towards your left hand) and you ſhall find the Product thereof to be 13984, which ſet down.—Secondly, look the Product of 3496 by 8 (your ſecond Figure) and you ſhall find it to be 27968, which muſt not be ſet down as in the other firſt Example but as you ſee it in this, 8 the firſt Figure thereof muſt be ſet one place forwards towards the right hand, as in the other it was ſet a place backward towards the left.—Laſtly, ſeek in your Rods for the Product of 3496 by 9 your laſt Figure, and you ſhall find it to be 31464, which ſet under the other two Numbers, yet one place more to the right hand.—So a Line being drawn under, and theſe three Numbers added together produce 1709544 equal

equal to that in the firſt Example: And that you may the better ſee the difference of the work, I have ſet them one by the other.

As in the firſt Example.	As in this Example,
3496	3496
489	489
31984	13984
27668	27968
13984	31464
1709544	1709544

One Example more in Multiplication, which ſhall be for Advertiſement and direction, I will give, and ſo conclude Multiplication.

I ſaid in the general Rule for working of *Multiplication* (at the beginning of this Section) that it mattered not which of your Numbers were made the Multiplicand, or which the Multiplier, of which I will here give you a Preſident where the leſſer Number ſhall be Tabulated, and the greater Number only ſet down; and I will work it here according to this laſt way of *Multiplication*, and the Example ſhall be as followeth.

Example, Let it be required to multiply 868437 by 3496, and let 3496 (the leſſer Number) be Tabulated.

Let the Numbers be ſet as you here ſee, then 3496 being Tabulated. Begin with the firſt Figure towards the left hand of your Multiplier, which here is 8, and upon your Rods find the Product of 3496 multiplied by 8, which is 27968, ſet that down under the Line——then find the Product of 3496 by 6, the ſecond Figure of your Multiplier, and you ſhall find that to be 20976, ſet this number under the former, one place more towards the right hand—— Again the third Figure of your Product is 8 whoſe Product is 27968, as before, ſet that under the other, ſtill one place more to the right hand.——In this manner do with the other Figures of the Multiplier, as 4 the next Figure, whoſe Product is 13984, which alſo ſet down a place forward.——So alſo the Product of 3 which is 10488,

```
     3496
   868437
 --------
  27968
   20976
    27968
     13984
      10488
       24472
-----------
 3036055752
```

10488, which set down. —And lastly, of 7, which is 24472.——— All these Products being set down in the order as you see them in the Margent, if you add them together, the Sum of them will be 3036055752, which is the Product of 3496 multiplied by 868437, the lesser number being Tabulated.

Other ways of Multiplication I could have added, but these I esteem sufficient.

VI. Division *by the Rods.*

AS in *Multiplication*, so in *Division* there are three *Numbers*, *Terms*, or *Things* required, *viz.*

1. The *Dividend*, or number to be divided.

2. The *Divisor*, or number by which the Dividend is divided, and

3. The *Quotient*, which is the Number issuing from the Dividend's being divided by the Divisor; And this *Quotient* doth always consist of so many *Unites* as the *Divisor* is times contained in the *Dividend*.

Thus much for the *Definition* of *Division*, now let us come to the *Practice* of it by the *Rods*, to perform which this is

THE RULE.

Tabulate the Divisor, (which is always the lesser Number of the two given) and set down the Dividend, and set the Divisor on the left hand, and draw a crooked Line on the right hand for your Quotient, as in common Arithmetick. Then look upon your Tabulated Rods (always) for the Number less than the Number in the first figures of your Dividend, and what figure stands against that Number on the edge of your Tabellet must be the Figure you must put in your Quotient, and that Number you must always substract from the Figures of your Dividend, and to the remainder add another Figure, so proceeding from Figure to Figure till your Division be wholly ended.

Example, Let it be required to divide 1709544 by 3496.

Having Tabulated 3496, set down your Dividend, your Divisor on the left hand thereof, and a crooked Line for the Quotient on the right hand thereof, as by the Rule preceding you were directed, and as you see done in the Example adjoyning.

And because at your first setting down of your Divisor 3496, it would reach (if it were set under your Dividend 1709544) as far

as

as the Figure 5, therefore under the Figure 5 make a prick, to intimate how far you are gone on in your work, and under this prick draw a Line quite under your Dividend; then is your Sum set down ready for work, and will appear as here you see;

3496) 1709544 (

Your Sum thus prepared, ask, how often can you have 3496 in 17095, look in your Tabulated Rods for 17095, which you cannot there find, but the nearest number thereunto amongst the Rods, which is less than 17095 (for you must always take a less number) is 13984, which number stands against the Figure 4 in the Tabellet, wherefore set 4 in your Quotient, and 13984 under the Line, and substract 13984 from 17095, and there will remain 3111, which set over 17095, and so is the first part of your Division ended, and your Work will stand thus.

3111
3496) 1709544 (4

13984

Then make another Prick under 4, the next Figure of your Dividend, so will the remaining number be 31114,——Then look among your Rods for the number 31114 (or the nearest less than it) and the nearest less you shall find to be 27968, which stands against 8 in your Tabellet; put 8 in your Quotient, and set 27968 under 31114, and substract 27968 from 31114, so will there remain 3146, which set over 31114, so is the second part of your Division ended, and your Work will appear thus.

3146
3111
4496) 1709544 (48

13984
27968

Lastly, Make another Prick under the next Figure of your Dividend, which is 4 also, making the remaining number to be 31464, seek among your Tabulated Rods for this number (or the nearest less) but looking you shall find the very number, against which stands on your Tabellet the Figure 9; set 9 in the Quotient, and the

the number 31464 under 31464, and substract it from 31464 the number which stands above the Line, and nothing remains; and being there is never another Figure in your Dividend, your Division is ended, your work will stand thus, and 3496 is contained in 1709544,389 times.

```
            00000
            3146
           3111
   3496) 1709544 (489
              ...
        ——————————
          13984
           27968
            31464
```

Another Example, and by another way of Division.

Let it be required to divide 912456 by 3496, set down your Dividend and Divisor, draw a crooked Line for your Quotient, and also make a Prick under the fourth Figure of your Dividend, and draw a Line under your Dividend, so is your Sum prepared to be divided, and will stand thus.

```
   3496) 912456 (
            .
        ——————————
```

Then your Divisor 3496 being Tabulated, look amongst your Rods for the nearest number to 9124 which is less, and you shall find it to be 6992, against which, stands on your Tabellet the Figure 2, set 2 in the Quotient, and this Number under the Line, and Substract it from 9124, and there will remain 2132, to which number add the next Figure of your Dividend, Namely 5, and it makes 21325, under which number draw a Line, then will your Sum stand thus,

```
   3496) 912456 (2
            ..
        ——————————
         6992
         21325
        ——————————
```

Then among your Rods seek the nearest number to 21325 and you shall find 20976 to be the nearest number less, against which, in your Tabellet stands 6, set 6 in the Quotient, and 20976 under the Line, Substracting it from 21325, which when you have done

there will remain 349, to 349 add the next Figure in your Dividend, which is 6, your laſt Figure, and it makes 3496, under which, draw a Line, and your Work will ſtand as here you ſee.

```
3496) 912456 (26
        ...
      ------
      6992
      21325
      -------
      20976
        3496
      -------
```

This done look amongſt your Rods for the neareſt number to 3496, and you ſhall find the exact number at the top of the Rods, againſt which ſtands the Figure 1 on the Tabellet, ſet 1 in the Quotient, and ſubſtract 3496 from 3496, the remainder is nothing, and ſo is your Diviſion ended, the work ſtanding thus, and 3496 the Diviſor is contained in 912456 the Dividend, 261 times.

```
3496) 912456 (261
        ...
      ------
      6992
      21325
      -------
      20967
        3496
      -------
        3496
        0000
```

A third Example ready wrought by the laſt and beſt way of Diviſion.

I will only ſet it down ready wrought, leaving the practice of it to your ſelf.

Let

Let it be required to divide 73020506 by 3496.

3496) 73020506 (20886$\frac{3050}{3496}$

```
           ·····
           ———————
           6992
            31005
           ———————
           27968
            30370
           ———————
           27968
            24026
           ———————
           20976
             3050
```

This Sum thus divided, produceth in the Quotient 20886, and 3050 remaining, so that the Quotient with the Fraction and all is, 20886$\frac{3050}{3496}$. So that 3496 the Divisor, is contained in 73020506 the Dividend, 20886 times, and 3050 remaining.

This Example well practised, together with them before-going, are sufficient instruction for any Student whatever; and he that can perform these, need not despair of the most difficult that can be proposed. And so I conclude with Division.

VII. *Of the Extraction of* ROOTS.

THe Extraction of *Roots*, which is the difficultest part of Multiplication and Division, is expeditiously and certainly performed by the Rods, for the easie and expedite performance of which, there are two Rods on purpose, one for the Square, the other for the Cube Root, of which I will speak; first, Of their Fabrick: secondly, of their Use.

Of the Fabrick of the Rods for Extracting of Roots.

Of the same matter, and of the same length and thickness of your other Rods, let there be made another Rod, but three times

the breadth of the former, the inscription on the one side serving to extract the Square, and that on the other side for the Cube Root, each of which are divided into three Rows or Columes,

That which serveth for the Square Root, hath in the top or uppermost Square between the Diagonal thereof, these figures 0-1, in the second 0-4, in the third 0-9, in the fourth 1-6, in the fifth 2-5, in the sixth 3-6, in the seventh 4-9 in the eighth 6-4, and in the ninth or lowermost 8-1, which are the Square Numbers belonging to the nine Digits.

In the second Column of the same Rod, in the first Square is inscribed 2, in the second 4, in the third 6, in the fourth 8, in the fifth 10, in the sixth 12, in the seventh 14, in the eighth 16, and in the ninth 18.

In the last or third Column, there are the nine Digits orderly descending, namely, 1, 2, 3, 4, 5, 6, 7, 8 9. This Rod thus made, is fitted for the Square Root.

That which serveth for the Cube Root, hath in the top or uppermost Square of the first Column towards the left hand, between the Diagonal thereof, these Figures, 0-01, in the second 0-08, in the third 0-27, in the fourth 0-64, in the fifth 1-25, in the sixth 2-16, in the seventh 3-43, in the eighth 5-12, and in the ninth 7-29, which are Cube numbers orderly descending.——The second Column of this Rod contains these Square Numbers, 1, 4, 9, 16, 25, 36, 49, 64, 81, orderly descending. ——The third and last Column of this Rod hath in it the nine Digits, 1, 2, 3, 4, 5, 6, 7, 8, 9, orderly descending also.

This Rod thus prepared and inscribed, is fit for extracting of the Square and Cube Roots, a Figure of either side whereof you have at the beginning of the Book: That which serveth for the Square Root having the word *Square* written by the side, that for the Cube Root, hath *Cube* written by the side.

Thus having given you the Fabrick and Inscription of these Rods, I will now shew you their Use; And first,

---

VIII. *The Extraction of the Square Root.*

TO extract the Square Root of any number, you must first prepare it, that is, set down the number on a Paper, then over the first & lowest figure next the right hand, make a point with your Pen, and over the third figure make another, over the fifth another Point; and so forth, over every second figure of the num-

number make a point, always leaving between each Point one figure unpointed, according to the ordinary Rule by the Pen; this being done, you ſhall ſee how many figures will be in the Root for ſo many points as you have, ſo many figures ſhall you bring into the Quotient for the Square Root, of the number given; next draw a Quotient-line, as in Diviſion, and your number is prepared for Operation, and will ſtand as in the Example following, where the number given is 119025, and the Root Square thereof is required. This number being ſet, and Pointed as afore is ſhewed, you may perceive that the Root thereof will be of three figures, becauſe there be three Points over the number given, the two figures belonging to the higheſt Point next the left hand are 11. the two figures belonging to the ſecond Point are 90, and the two figures belonging to the ſecond Point are 25. and the figures for the Root anſwerable to thoſe ſeveral Points, are to be found by the Rods, as followeth.

```
 .  .  .
119025 (
```

Take the Frame, the Rods, and Lamina, and lay them before you; and firſt place the Lamina in the Frame next to the left hand ledge, with that ſide upwards, whereupon are the inſcriptions belonging to the Square Root, and marked at the top with the Letter S, then conſider what is the greateſt Square number in 11. the figures belonging to the firſt Point; the Lamina preſently ſheweth you that the greateſt Square number in 11 is 9, and his Root is 3, for 3 times 3 is 9; therefore put 3 in the Quotient for the firſt figure of the Root, then ſet 9 under the 11 and Subſtract it therefrom, and there will remain 2, this 2 ſet under 9 drawing a Line between them, and to the 2 remaining, bring down the 2 figures under the ſecond Prick, viz. 90, making it 290. and ſo you have gained the firſt Figure of the Root, and the Work will ſtand as in this Example.

```
 .  .  .
119025 (3
 9
——————
 290
```

Having the firſt Figure of the Root; to get the ſecond, and ſo all the reſt in order, you muſt proceed in this manner; double the Root found, which duple Place or Tabulate upon the Rods between the Lamina and the ledge of the Frame; As in this example, the duple of 3, the Root found, is 6, therefore place a Rod that hath in his top or upper Square 6, between the ledge and the Lamina; Then look upon your number given, what Figures, or number it is that belongeth unto the ſecond Point, which you ſee will be 290 in this our Example; Then turn your eye to the Lamina and Rod now Tabulated, and ſearch thereupon what number will (being leſs yet) come neareſt unto 290, the number out of which the ſecond Figure of the Root is to be found, And there you may ſoon ſee it is the number 256, which of any number upon the Rods, leſs than 290 com-

cometh neareſt thereunto, for the next greater number upon the Rods, above 256, is 325, which is greater than 290, and therefore cannot be taken out of it; but 256 is the only number to work withal, againſt which, on the ledge and Lamina, is this Figure 4, this 4 muſt you put in the Quotient for the ſecond Figure of the Root, and then Subſtract 256 out of that 290, and there will reſt 34, this 34 ſet under 256 in its due place, drawing a Line between 256 and 34; then will the Work ſtand as in the Example. If you pleaſe, you may write your numbers to be Subſtracted, under the number out of which Subſtraction is to be made, as I have done here in this Example, for inſtruction ſake, or you may omit that if you pleaſe being you have them before you upon the Rods.

```
  .  .  .
119025 (34
9
------
290
256
------
 34
```

And now for the third figure of the Root, look upon the number given, and there you ſhall ſee that the Remainder 34, with the two figures 25 belonging to the third Point, being all joyned together, make 3425, out of which the third Figure of the Root is to be extracted; To find out what this third Figure ſhall be, double the Root found already, which is thus done very readily; Take forth a Rod that on his top Square hath 8 the duple of 4, the laſt Figure in the Quotient, and this Rod put into the Frame between the Lamina and the Rod that is already Tabulated; This being done, you have no more to do, but to look over the Rods and Lamina for ſuch a number as will come neareſt unto the number 3425, that belongeth to the third Point, and is leſs than it, and the Fig. that you ſee againſt that number ſo found, put in the Quotient for the third Figure of the Root; Thus looking upon the Rods you ſhall at the firſt ſight find the very number it ſelf 3425 that belongeth to the third Point, in the fifth Line of Squares, againſt the figure 5, therefore put 5 in the Quotient for the third figure of the Root, and if you Subſtract 3425, the number now found from 3425, that number belonging to the third Point, there will be no remainder, ſo is the Work done, and the number given, 119025 is a perfect Square number, and the Square Root thereof is 345, which was the thing required to be found.

```
  .  .  .
119025 (345
9
------
290
256
------
  3425
  3425
------
  0000
```

Now

Now if you multiply this 245 into it ſelf, it produceth 119025, the firſt given number, which proveth the work to be truly wrought; for note this evermore, that for the proof of the Extracting the Square Root, you muſt multiply the Root found, by it ſelf; (to the product adding the remain, if any be) and the total will produce the firſt number given, if the work be truly wrought, otherwiſe not.

For a ſecond Example, Let there be given this number 117716237694, and the Root thereof required: Now to perform the Work, firſt write down the Number, and then prepare it with points and a Quotient-line, as you were inſtructed before in the former Example: This being done, the number will appear as here you ſee in the Margine: Now in 11, the Figures belonging to the higheſt point, the greateſt Square Number is 9, whoſe Root is 3, put 3 in the Quotient and ſet 9 under 11, and ſubſtract it from 11, ſo will the remainder be 2, which ſet under 9, and bring down 77 the Figures belonging to the ſecond Prick making it 277, under which draw a Line: Now for the ſecond Figure of the Root, Tabulate the duple of the Root found, and that is no more to do, but to place a Rod that hath 6, the duple of 3, the Root found, in his top Square, between the Lamina and the Ledge, and on the left hand of the Lamina, the ſaid Lamina, being Tabulated with that face upward, that is, for the Square Root; then ſeeing that the Number belonging to the ſecond Point, out of which the ſecond Figure is to be extracted, is in this Example 277, therefore ſearch upon your Tabulated Rod and Lamina for ſuch a Number, as will come neareſt to that 277, which you ſhall quickly find to be 256, and right againſt it, on the right hand Column of the Lamina is the Figure 4, therefore put 4 in in the Quotient for the ſecond Figure of the Root, and ſubſtract 256 from 277 and ſet the remainder 21 under it, and alſo cancel the 256, and ſo have you done with your ſecond point.

```
117716237694  (3
 • • • • • •
——————————————
9
277
——————
```

And now for the third Figure of the Root, obſerve that the 21 remaining with the other 16, two Figures uncancelled belonging to the third Point, being joyned together, make 2116, out of which the ſame third Figure is to be extracted; to perform which Work, take a Rod that carrieth in his upper Square the Figure 8, the duple of 4, the Figure laſt found, and put that Rod into the Frame between

```
1177162237694   (343
• • • • • •
_______________
9
277
_______
256
 2116
_______
 2049
   67
```

tween the Lamina and the other Rod before Tabulated, then look for the Number upon those two Rods, and Lamina, that will nearest take away 2116, the Number belonging to that third Point, and at first sight you shall find that the next Number lesser than 2116 is 2049, and his Figure for the Quotient, 3, therefore put 3 in the Quotient, for the third Figure of the Root and substract 2049, from his respective Number 2116, and there remaineth 67, which set under 2049, and draw a Line under it, so you have done with the third Point, and third Figure, and are to proceed to the fourth.

Where first you see that the 67 last remaining, with the 23, make 6723, the Figures belonging to the fourth point, whereout you must Extract the fourth Figure of the Root. Therefore go on as before, taking such a Rod, as in its top Square hath 6, the duple of 3, the Figure last found, and Tabulate it between the Lamina and the other Rods, and then seek what number there can be found, that will be less than 6723 but at the very first sight you shall see no number upon the Rod so small as that 6723, for

```
1177162237694   (34309
• • • • • • •
_______________
9
277
_______
256
 2116
_______
 2049
  672367
_______
  617481
   5489594
```

the very first number against 1, is 6861, which is greater than 6723, and therefore cannot be taken out of it, so that here you can put no Figure in the Quotient; but must supply the place with a Cypher, therefore put a o in the Quotient for the fourth Figure of the Root, all the rest of the number standing as it did. Now to the next point, which is the fifth, you have this Number belonging 672376 whereout the fifth Figure of the Root is to be found. Now here, in regard your last Figure of the Root found is a o, you must ever in such a case (in stead of putting a Rod, that hath the duple of the Figure found in his top Square,) you must take such a Rod as upon one of his faces carrieth only Cyphers, and Tabulate it between the Lamina and the other Rods already Tabulated; This being done, question the Rods, what Number you must take from that number 672376, or what number it is there, that being less than 672376 yet cometh nearest unto it; and also what the Figure shall be that you must put in the Quotient for

for the respective Figure of the Root; The Rods will suddenly resolve you, that their greatest number less than 672376 is 617481, and its respective Figure for the Root is 9, therefore put 9 in the Quotient for the fifth Figure of the Root, then if you will write 617481 under 672376, and make substraction, your remainder will be 54895, which joyned to the 94, the two last Figures of the Number given, make 5489594, for the number out of which the sixth and last Figure is to be found.

To find this sixth Figure you must Tabulate upon the Rods the duple of the Figure last found between the Lamina, and the Rods already Tabulated; but here, because the duple of 9 is 18, consisting of two Figures, therefore this 18 cannot be Tabulated upon one Rod, as before we did use to do, when the duple was contained of but one Figure, now in this case, (and so of all the like,) First, Tabulate a Rod bearing 8 in its top Square, between the Rods formerly Tabulated and the Lamina, and next to the Lamina, then for the Unite 1, being the highest Figure of the 18, you must Tabulate that upon the last and lowest Rod, formerly Tabulated, which is done by encreasing the Figure in its top Square, one Unite more that it was before; so here the last Rod before Tabulated, carrying only 0; either turn it, or else take it away, and place such a face of that Rod, or of some other, that hath in his upper Square the Unite 1 in room and stead of that face, with Cyphers, so is your Number 18 the duple of 9 Tabulated, this done, look over the Rods, for a Number that will come nearest unto 5489594, in the Eighth line is this Number 5489504, and its respective Figure for the Root, 8: This 8 put into the Quotient, and Substraction being made as is used to be done, there will remain 90, and so is your whole work ended, and the Square Root of 117716237694 is 343098, if you multiply this Root by it self, and to that product add the 90 that remaineth, you shall produce again the first Number given, which argueth that the work is truly wrought.

117716237694 (343098
· · · · · ·
9
277
256
2116
2049
672376
617481
5489594
5489504
90

But now when any thing remaineth, the Extraction being ended, as here it doth, make a Fraction of that remainder as you do in Division, in this manner: Set the Number so remaining after this Extraction is ended, over a Line for Numerator, and for the Denominator, set the duple of the whole Root found, with one ad-

ded thereunto, as here in this example 90 remaineth ; this 90 put over a Line for the Numerator of the fraction, then double 343098 the Root, and it is 686196, to which add one Unite, and then it will be 686197, this ſet down under the Line for Denominator to the fraction, and then the true Root ſquare of the number given will be $343098\frac{90}{686197}$ and will ſtand as in the Example. This is the vulgar, and ordinary way to make a fraction of the remainder.

But the beſt and moſt certain way to attain unto the true value of the fraction remaining, and that too by the Rods, very eaſie and ſpeedy, is to add two, four, or ſix Cyphers, to the remainder, and continue the Work of Extraction, and then your fraction will be in Primes, Seconds, Thirds, &c. that is in 10 parts, 100 parts, and 1000 parts, in the ſame manner as in Diviſion; for note, that for every two Cyphers that be added or adjoyned to the number given, you ſhall have one fractional Figure in the Quotient, which will repreſent the fraction in Decimal parts of an unite, we will add 6 Cyphers to the remiander in this our laſt Example, and it will then be 90000000, and then we will continue the Work, therefore Tabulate 16, the duple of 8, the Figure laſt found ; which to do, put a Rod that carrieth 6, the loweſt Figure of 16 next the Lamina, between it and the Rods afore Tabulated, and then inſtead of that Rod laſt in place next the Lamina, put another Rod that hath in his upper ſquare one unite more than that, as here, change the Rod from 8 to 9, and the Rods are Tabulated, and you are now to look out a number that will neareſt take away 9000, the number belonging to the firſt fraction-point, but the Rods give you none ſo ſmall, therefore put 0 in the Quotient for the firſt Figure of the fraction, and becauſe there is no more to do about this firſt Figure, you are next to Tabulate a Rod with Cyphers next the Lamina, and then ſee for a number that will come neareſt unto 900000, the number belonging to the ſecond point of the fraction, but yet you ſhall have none upon the Rods ſo ſmall, therefore put another 0 in the Quotient for the ſecond fraction Figure: again, Tabulate a Rod with 0 next the Lamina, and you ſhall yet again find no number on the Rods ſo ſmall as 90000000, the number belonging to the third-point, therefore put a third Cypher in the Quotient for the third Figure thereof. Thus have you done with your three points of Cyphers, which you have firſt added, but becauſe you are reſolved to get at leaſt one fraction-Figure, therefore add two Cyphers more to the remainder, making it 9000000000, and alſo Tabulate a Rod with 0 next to the Lamina, and then ſee if you can find a number little enough upon the Rods. And now here at laſt you ſhall find, one ſignificative Figure in the Quotient of your fraction, for the nmber now belonging to the fourth fraction point is 9000000000, conſiſting of 10 places of Figures,

and

and the number Tabulated is also now become to be of 10 places, and withal the highest Figures less than the highest Figures of that number which belongeth to that fourth point, therefore seek upon the Rods for a number less than that 9000000000, and that you shall have in the second line upon the Rods, and it is 6861960004, whose respective figure for the Root is 2; now Substraction being made, there will remain 2138039996. Thus have you gotten one Figure into the Quotient of your fraction, and that in the fourth place Descending, and may be thus expressed fraction-wise $\frac{0002}{10000}$, or thus, $\frac{2}{10000}$ signifying 2 parts of 1000 of an Unite; for note, that so many fractional Points as you bring into the Quotient to produce a new Numerator, the Denominator is always an Unite, with as many Cyphers as you have made fractional Figures. This new found fraction joyned to the whole parts of the Root found, will stand as here in the Example; Or else without a Denominator, thus, which is all one with that other.

9000000000 (.0002
• • • • •

2138039996
6861960004

343098 $\frac{2}{10000}$

343098.0002

These Examples might be sufficient to shew the excellent use of the Rods in Extracting the Square Root of any number; but yet to shew the more variety of works, take one Example more; and if in any thing I be thought too tedious, know, that it is out of a desire of plainness, even of such a plainness as is answerable to that of the Rods.

Let there be given this number, 97419256 and the Square Root thereof required, Write down the number, and prepare it with Points under each second Figure, and a Quotient Line, and then proceed as before, and first look upon the Lamina, what is the greatest Square number there, that can be had out of 97, the two Figures belonging to the first and highest Point; the Lamina sheweth that it is 81, whose Root is 9, put this 9 in the Quotient for the first Figure thereof, and then Substract 81 from 97, and the remain is 16 for the Second Figure, Tabulate 18, the Duple of 9, upon two Rods, between the Lamina and the Ledge, and then upon those two Rods and Lamina seek out the number that comes nearest to 1641, the number to the second point belonging, and that you shall find to be the number 1504, and his respective Figure for the Root 8, which being put into the Quotient, and Substraction made, according to the instructions afore-delivered, the number remaining to the Third point will be 13792, and to find out his proper Figure for the Root, Tabulate 16 the Duple of 8 last found, in this manner, place a Rod that carrieth in his top Square the Figure

6 betwixt the Lamina and the former Rods, and increase the former lowermost Rod one Unite, by changing it from 8 to 9, then shall you see the number upon the Rods nearest unto 13792 is 13769 in the seventh Line, and after Substraction made, there will rest 23, making the number for the least point to be 2356, now to find the respective Figure of that fourth point, Tabulate 14, the Duple of 7 as before you were instructed, and then you shall see at the very first, that no number upon the Rods is so small as that 2356, the number belonging to the last Point, therefore put 0 into the Quotient for the last Figure of the Root, and so have you ended the Work, for the whole part the Root sought for, which in this Example appeareth to be 9870, and the Remainder 2356. But now to make a Fraction of this Remainder, as you were before shewed, set the same 2356 over a line, at the end of the whole part of the Root found 9870, and then duple the Root found, and to that duple add one Unite, and the total will be 19741, which set under the line for Denominator, and then the whole work is finished, and the true Root found answering the demand is $9870\frac{2356}{19741}$, and standeth as in the example it appearth.

97419256 ($9870\frac{2356}{19741}$
• • • •
81
1641
1504
13792
13769
2356

But if you desire to be yet more exact, and would have the truest value and estimate of your fraction; then turn it into a Decimal, and proceed as before; first add to the remainder so many times two Cyphers as you desire to have Figures for your Numerator of your new fraction, that is to say, two Cyphers if you would have but only one Figure, four Cyphers for two Figures, six for three Figures, and so forth, for as many as you would have, and until you think your fraction is small enough: In this Example we add to the remainder three Points of Cyphers, that is, six Cyphers, because we would have three Decimal Figures in our fraction-root; then Tabulate the duple of that figure of the Root last found, but because that is 0, which doth neither increase nor diminish, therefore Tabulate 0, next the Lamina, between it and the other Rods, next see upon the Rods what number there will come nearest to 235600, the number that now belongeth to the highest Point of the fraction-points, and that is only the very first, *viz.* 197401, therefore put 1 in the Quotient, for the first fraction-figure, that will do it, and then make Substraction, and there will remain for the second fraction-point 38199000.

To

To find the ſecond Fraction Figure, Tabulate 2, the duple of 1, and ſeek the Number that cometh neareſt to that remainder 3819900, and that is again only the firſt, therefore put 1 in the Quotient, and make Subſtraction, and then to your third point will belong 184588900, and to find its reſpective Figures for the Root, Tabulate 2,the duple of 1 laſt found, and inquire what number upon the Rods will come neareſt to that 184588900, and that you ſhall find in the ninth line of Squares to be 177662061, therefore put 9 in the Quotient, and when Subſtraction is made, you will have remaining 6926839; thus have you three Figures in the Quotient, which are enough to give the fractions Value in any ordinary Queſtion; if you pleaſe to continue the Work the fourth Figure will be 3, and now is your Fraction turned into a Decimal Fraction, whoſe Numerator is 1193, and his Denominator 10000, and being ſet Fraction-ways, will ſtand thus $\frac{1193}{10000}$: Or it may very well be expreſſed without the Denominator, only with a line, or point of diſtinction thus, 9870, 1193 and ſo the value of this Fraction is 1193 parts of 10000.

The OPERATION at large.

```
97419256.000000 (9870.1193
 . .  . . . . . .
________________________
81
1641
______
1504
  13792
__________
 13769
     235600
        ________
     197401
       3819900
           __________
       197497.4
       184588900
           ____________
       177662061
         6926839
```

If you multiply this Root found 9870.1193 by it ſelf, the Product will be 97419254.99623249, from whence, by a Point or Line cut off 8 places of Figures, according to the Rules of Multiplication in Decimal Arithmetick, and the Number remaining towards

wards the left hand, will want (the Fraction considered,) but a very little more than one Unite of the Number first given, but if you take the pains but to continue the work to one place lower, it will not want an Unite, and so the lower you work the Fraction, the nearer still you come to the exact truth.

IX. *The Extraction of the Cube Root by the Rods.*

TO Extract the Cube Root of any Number, you are first to write down the Number given, whose Root is required, and make a point under the lowest Figure next to the right hand, and another point under the fourth Figure, and so under every third Figure, omitting between every two Points two Figures unpointed, and then so many Points as you have under your Number, so many Figures shall you have in your Root; next draw a Quotient-line, as in the Extraction of the Square Root, this being done, the Number is prepared for Extraction; then go to the Work:

First, seek what is the greatest Cube Number in the Number standing above, or belonging to the highest point next the left hand, which the Lamina will shew upon his left side of that Face for the Cube Root, and in the Column, upon the right side of the same Face is the Root thereof, and when either by your memory or Lamina, you have found the greatest Cube Root in that Number belonging to that first point, then (as in Extraction of the Square Root) substract the greatest Cube Number, to that first point belonging, or that in the Number, to the said first point belonging, can be found from the said Number, &c. then substract it, and set the Remainder under as before in the Extraction of the Square Root, and put the respective Digit Number found in the Quotient for the first Figure of the Root: Now to find the second Figure of the Root, (and so all the rest, how many so ever they be) you must always Triple the Root found, and that Triple multiply again by the Root found, and that last product Tabulate upon the Rods on the left hand of the Lamina, as before in the Extraction of the square Root, you did the duple of the Root found, then look upon those Tabulated Rods and Lamina together, what Number you can find upon them will come nearest to the Number belonging to the Number next following, and less than it, which Number is the Divisor, and the Figure on the right hand of the Lamina is the Figure for the Root answerable to that point, and that Figure put in the Quotient for the respective Figure of the Root, belonging

belonging and anſwerable to that point; but note always that you take the Diviſor ſo often, and no oftener, but that you may yet alſo take another Number from the Number belonging to that point, which other Number is Square of the Digit new found, multiplied into the former Triple, the product add to the Diviſor, with this proviſo, that you place this new product one place higher towards the left hand than is the Diviſor, that is to ſay, ſet the loweſt place of that new product under the ſecond place of the Diviſor, the total of this Addition Subſtract from the Number belonging to the point in action, cancelling the ſaid number, and the remain ſet under, as you uſe to do in Diviſion and Extraction of the Square Root; and ſo proceed to the next point, if you have any more; But to make all plain, we will illuſtrate this by variety of Examples in all the kinds and differences of Works.

Firſt, let 110592 be a number given, and the Cubick Root thereof required; this Cubick Root is thus found.

Firſt, prepare your Number, that is to ſay, write it down, and make a point under 2, the loweſt Figure thereof next the right hand, and one other point under 0, the fourth Figure thereof, leaving two Figures between unpointed, then draw the Quotient-line, and then the number will ſtand ready prepared as in the Example, with two points, whereby it appeareth that the Root will conſiſt of two Figures, which are to be found out according to the former directions; and firſt obſerve that 110 is the number belonging to the firſt point, and upon the Lamina you may alſo obſerve that the greateſt Cube number in that 110 is 64, and his Cube Root 4, therefore put 4 in the Quotient, for the firſt Figure of the Root, and then Subſtract 64 from 110, and there will remain 46, this 46 ſet under 64 and the Work of the firſt point is done, and here you may now obſerve, that the 46 remaining, with the other Figures 592, make 46592, which is the number belonging to the ſecond point, and where out the ſecond Figure is to be found.

```
110592  (4
  .  .
------
64
46592
```

To obtain this ſecond Figure, proceed in this manner, triple 4, the Root found, and it is 12, and that triple multiply by the Root found 4, and the Product is 48, this Product 48 Tabulate upon the Rods on the left hand of the Lamina, between it and the Ledge, then view over theſe Rods and Lamina thus Tabulated, what number there will come neareſt unto 46592 the number belonging to the ſecond point, and be leſs than it, you ſhall ſee the number that cometh neareſt to it, is that in the ninth Line, 43929, and his reſpective Figure for the Root is 9, now Square this 9, and it is 81, this 81 multiply by the former Triple 12, and it yieldeth 972, this 972 add unto 43929, the number found on the Rods (being ſet in Addition, one place higher than is ordinary, as was before ſhewed) and

and the total will be 53949, which if you compare with 46592, the number belonging to the point in action, you shall see it is too great to be taken out of it: Whereby it appeareth that you must not take 9 for your Root, for by your general Rule you must take the Divisor no oftner, but that you may take also the Product made by the square of that new Digit number multiplied into the first triple, out of that number belonging to that second point, therefore take a less number upon the Rods, as the number 38912 in the eighth line, which will surely serve the turn, wherefore put 8 in the Quotient for the second Figure of the Root sought for, now write (if you will) this number 38912, just under 46592, the number belonging to the said second point; then according to the Rule, Square this Digit 8 new found, and it giveth 64, this 64 multiplied by the former triple 12, produceth 768, write down under the former number 38912, setting the lowest Figure 8 of this new product directly under 1, the second figure of 38912, and its second figure 6 under the third figure 9, and so of the other in Order, then add these two numbers together, and the total will be 46592, which is equal to that number above, therefore if Substraction be made, there will nothing remain, and so the Work is ended, whereby you may conclude that 110592, the number given, is a right Cubick number, and that 48 is the Cube Root thereof, which was the thing required to be found.

```
110592   (48
-----------
 64
46592
-----
38912
 768
-----
46592
```

Now if you would at any time prove your Work, whether you have wrought truly or not; multiply the Root found Cubelickly, and add the remainder, when any is to that product, and if the total be the first number given, then the work is truly wrought, or else not, as here in this Example: If you multiply the Root found 48 by 48, it is 2304, and this 2304 multiplied again by 48, produceth 110592, the number first given, and therefore conclude that the Work is truly wrought.

For a second Example, we will take this number 41063625 and seek the Cube Root thereof, first, prepare the number; by writing it down, and making a point under the lowest figure 5, and another under 3, the fourth figure, and another under 1, the seventh figure, and draw the Quotient-line, these three points do declare that the Root will consist of three figures: Now to fall to Work, to find the first of them, consider what the greatest Cube number in 41 is, which appeareth on the Lamina to be 27, and its Root 3, therefore put 3 in the Quotient for the first Figure of the Root, and then substract 27 from 41, and set the remainder 14 under 27, and the work of the first Figure is ended, and the number that belongeth to the second Point is 15063, out of which the second Figure is to be found, behold the Example: To get this second Figure, triple 3, the Root found,

found makes 9, this triple 9 multiply by 3, the Root found giveth 27, this product 27, Tabulate upon the left hand of the Lamina, and then the number that will come neareſt unto that 14063 belonging to the ſecond point now in action, remembring the former Caution, that it be taken no oftner, but that withal there may be taken from thence alſo the product, produced by multiplying the Square of the reſpective Figure into the former Triple, as here the diviſor may be had five times, but by reaſon of that other number, that muſt alſo be taken from thence, cannot be taken alſo, therefore it can be had but only 4 times, wherefore put 4 in the Quotient for the ſecond figure of the Root, and ſet the number 10864 found in that fourth rank of Squares under the number 14063, the number belonging to the ſecond point, then multiply 16 the Square of 4, the digit now found by 9, the Product is 144, this 144 ſet under the former 10864, but according to the former Proviſal, one place higher towards the left hand, and ſet its lower Figure 4 under 6, the Second Figure of 10864, the next 4 under 8, and the uppermoſt being 1 under 0, as you ſee it ſtand in the example; then add theſe two numbers together, and they make 12304, this take from 14063 leaveth 1759, thus is the work of the ſecond point at an end; behold the Example.

```
41063625 (3
 .  .  .
-----------
27
14063
```

```
41063625 (34
 .  .  .
-----------
27
14063
--------
10864
 144
--------
12304
 1759
```

Now for the third figure of the Root, you are firſt to obſerve, that the number belonging to the third and laſt point is 1459625, from whence the third and laſt Figure of the Root is to be extracted, which Figure to find out, triple the Root found 34, and it is 102, and that multiply again by 34, the Root found yieldeth 3468, this product 3468 is the Diviſor, and this Tabulate upon the Rods as before, and joyn the Lamina cloſe to them, then ſeek upon theſe Rods and Lamina what number will come neareſt unto that 1759625, the number belonging to the third point now in action, (remembring the former Caution, but here is no need of that in the work of this point,) you ſhall find the number for the purpoſe, to be the number 1734125 ſtanding in the fifth line and its reſpective figure for the Root 5, therefore put 5 in the Quotient for the third

```
41063625 (345
 .  .  .
-----------
27
14063
--------
10864
 144
--------
22304
1759625
--------
1734125
  2550
--------
1759625
```

third fig. of the Root, now transcribe the number 1734125, from most the Rods into the Paper, just under the 1759625, then to find the other number to be hereunto added; square 5 the figure last found, and it makes 25, this Square 25 multiply by the former triple 102, and the Product is 2550, this set under the other number 1734125, according to the former Proviso, as you see it stand in the Example, and add these two numbers together, and their total will be 1759625, and is equal to that above, belonging to that third point, so that if substraction should be made, there wouldnothing remain which declareth the number given to be a perfect Cubick number, and the Cubick Root thereof to be 345 which was the thing required to be done; if you will multiply this Root 345 Cubickly, it produceth the number first given 41063625, which proveth the work to be truly wrought, the like is to be observed in all other Works of this nature whatsoever.

For a third Example, we will take at an adventure this great number 859271650667, and seek the Cube Root thereof. This number being prepared with points, and a Quotient-line; sheweth by his four points, that his Cube Root will consist of 4 fig. and by the former Directions, the first figure will appear to be 9, for the greatest Cube number in 859, the number belonging to the first point, is 729, which taken from that 859, leaveth 130, which with the 271 between that 130 and the next point, make 130271, out of which the second figure of the Root is to be extracted; this second figure by the former Rules, will be found to be 5, after the work of this second point is ended, there willbe remaining to the third point 189650, and the Root found is 95, this tripled, and also multiplied again by the Root, produceth 27075 for the Divisor, this divisor Tabulated, there is no number to be found upon the Rods so small as is 189650, the number belonging to that third point, therefore put a 0 in the Quotient for the third figure of the Root, and so have you done with that third point,

```
859271650667 (950
.  .  .  .
------------
729
130271
------------
121625
   675
------------
128375
```

Now to find the fourth figure answerable to the fourth point, you need here do no more, but Tabulate two 0 between the Lamina, and the other Rods, without any altering of the Rods already Tabulated, and then seek the fourth figure as before, the reason is, because 0 doth neither multiply nor divide, but only raiseth the places of figures higher towards the left hand, for here if you triple 950, the Root found, it yieldeth 2850, and this multiplied again by 950, the said Root found, produceth 2707500, which is the same with that former Tabulated number, saving only the two Cyphers, and therefore it is, that when there is a 0 in the Quotient,

tient, there needs no more to be done, but to Tabulate two Cyphers between the Lamina and those Rods before Tabulated, when you sought for that last Figure, which happeneth to be a 0. Now upon those Rods thus Tabulated, seek what Number upon those Rods and Lamina will come nearest unto that Number, which belongeth to the fourth point, which is here 1896650667. By viewing the Rods, you shall find that Number in the seventh rank of Squares to be the Number that will serve the turn, *viz.* 1895250343: And its respective Figure to be put into the Quotient 7. To this Number add 139650, the Number made by multiplying 49 the Square of 7, the Digit now found by 2850, the Triple of the Root afore found and the total is 1896646843. This Substracted from that Number to the last point belonging, leaveth remaining over head 3824.

```
859271650667 (950
  .  .  .  .
-----------------
729
130271
------
121625
  675
------
128375
-----------
1895250343
    139650
-----------
1896646843
```

Thus have you finished all your points, and have found the Cube Root of your Number given to be 9507. And being there is a remainder left, it appeareth that the Number first given is not a perfect Cube Number; but the greatest Cube Number therein is 859271646843, and his Cubick Root is 9507. The truth of this Work you may examine by multiplying the Root found Cubickly, which if you do, and add the remainder to the product, you shall produce the first Number given.

Now for the remainder, to make a fraction thereof in such sort, that it may aptly express the nearest Cube Root, I could shew several ways delivered by several Authors, how to bring it nearest to the truth.

But the most absolute and best Rule to get the Cube Root of any Number not Cubick, is this: Add to the remainder so many several points, or Ternaries of Cyphers, as you desire to come nearer to the true Root, (in the same manner as you did in the extraction of the square Root) and then continue on the work for extraction, as you do in whole Numbers, and the Fraction will be turned into a Decimal Fraction, and then so many as you add Ternaries of Cyphers, and the Denominator will have an Unite, with as many Cyphers, as you added Ternaries, or points of Cyphers to the Remainder.

Some Uses of the Square and Cube Root.

Uses of the Square Root.

WHat the *Square* and *Cube Root* are, and how to *extract* them, hath already been taught; and for more ease and expedition, There are *Tables* ready calculated, both of the *Square* and *Cube Root*, from 1 to 1000. We come now to shew some Uses thereof, which in some measure will appear in the *Propositions* following:

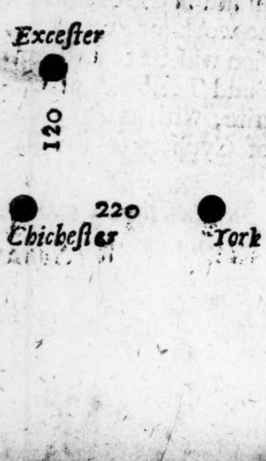
PROPOSITION. I.

Admit the height of the Wall of a Fort or Castle to be scaled, be 30 Foot, and the breadth of the Trench about the Fort be 40 Foot; I demand of what length a Scaling Ladder shall be, justly to reach from the edge or brow of the Trench, to the top of the Wall?

By the 47th of the first Book of *Euclid's Elements* it is demonstrated; that, *the square of the Hypotenusal of all right angled plain Triangles is equal to the Squares of the 2 other sides:* I therefore to resolve this Proposition, square the height of the *Wall*, which is 30, *facit* 900; also I square the breadth of the *Trench* which is 40 *facit* 1600, these two added together make 2500, the Square Root whereof is 50; and so long must a Scaling Ladder be made to reach from the edge of the *Trench* to the top of the *Wall*.

PROPOSITION. II.

There be two Towns, as Chicester *and* York, *which lie North and South one from another, and their distance is* 220 *miles, and* Excester *lieth directly West from* Chichester, 120 *miles; I desire to know the distance of* York *from* Excester?



Square 120, the distance of *Excester* and *Chichester*, it maketh 14400, likewise Square 220, the distance of *York* and *Chichester*, *facit*, 48400; these two Numbers added together make 62800, whose Square Root extracted (or found in the Table) will be 250 ⅗ near, and so many miles is *Excester* distant from *York*.

Use of the Cube Root.

ONE chief use of the *Cube Root*, is to find out a proportion between like Solids; such are Spheres, Cubes, and such like, as in Propositions following.

PROPOSITION. I.

If a Bullet of Brass of 4 *inches Diameter, weigh* 9 *pound, what shall a Bullet of Brass weigh, whose Diameter is* 8 *inches?*

Cube 4, the Diameter of the lesser Bullet, makes 64, likewise Cube the Diameter of the greater Bullet 8, makes 4608. This done, say by the Rule of Proportion; If the Cube 64 give 9 *li.* weight, what shall the Cube number 4608 give? Multiply and divide, you shall have 72; and so many pounds will a Bullet of Brass weigh, whose Diameter is 8 Inches.

PROPOSITION. II.

If a Fathom of Rope of 10 *inches compass about, do weigh* 17 *pound, how much shall a Fathom of Rope weigh, which is but* 8 *inches compass about?*

The Square of 10 is 100, the Square of 8 is 64; wherefore by the Rule of Proportion, say

As 100 (the Square of 10)
Is to 64 (the Square of 8)
So is 17 (the weight of the Fathom of Rope of 10 inches)
To 10 $\frac{88}{100}$ pounds (the weight of the Fathom of Rope of 8 inches about.)

PROPOSITION. III.

If a Ship of 100 *Tun be* 20 *foot broad at the Midship Beam, of what breadth at the Beam shall a Ship (of the like building) be that shall be* 200 *Tun!*

The Cube of 20 is 8000, then by the Rule of Proportion say.

As 100 Tun (the burthen of the Ship given)
Is to 200 Tun (the burthen of the Ship required)
So is 8000 (the Cube of the given Ship's Beam)
To 16000 (the Cube of the required Ship's Beam.)

Now the Cube Root of 16000 is 25 $\frac{1}{5}$ almost, and so long at the Mid-ship Beam must a Ship of the same Model be, whose burthen is 200 Tun.

And let thus much suffice for the Use of *Nepair's Bones*, and so to Instrumental Arithmetick I will put

AN END.

OF
ALGEBRA.

Part IV.

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1700.

ALGEBRA.

PROEME.

IN this *Treatise* of *Algebra*, for the *Symbols* here used; and the *Method* observed; It is that of Mr. *Tho. Harriot*, in such *Æquations* as are proposed in *Numbers*: That of *Descartes* in such *Æquations* as are *Solid*, and not in the *Numbers*. Not, that this Book is taken out of them, neither doth it proceed continually with them; but Disjunctly; as the Author hereof Mr. *Tho. Gibson*, long since, *viz.* in Anno 1655, thought fit to intermix them among other things which are not in them.

Now, forasmuch as *Algebra*, cannot be well attained unto without a competent knowledge in *Geometry*; It will be necessary for the *Reader* to acquaint himself with *Euclid's Elements*; especially the first *Six Books*; and in them principally, with these 48 *Propositions* following.

Propositions of *Euclid*, fit to be known to the *Analist*.

In the *First Book* these *Eleven*. Prop. 6, 13, 14, 15, 18, 19, 28, 32, 43, 47, 48.
In the *Second Book*, these *Twelve*. Prop. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 12, 13.
In the *Third Book*, these *Seven*. Prop. 14, 20, 22, 31, 32, 35, 36.
None in the *Fourth Book*.
In the *Fifth Book*, these *Six*. Prop. 15, 16, 17, 19, 24, 25.
In the *Sixth Book*, these *Twelve*. Prop. 2, 3, 4, 6, 7, 8, 13, 14, 16, 19, 24, 31.

Many more Propositions out of these and the Remaining Books, might be Useful: But these 48, before reckoned, are such as (in my judgment) ought chiefly to be Read and remembred, being very Useful for attaining and Resolving *Æquations*.

DEFINITIONS.

Definition I.

THe unknown Quantity of any Æquation is called generally *Potestas*; or a *Power*, *Quantity*, or *Term*.

Definition II.

A *Rectangle* is in numbers the *Product* of two numbers multiplying one another.

In Geometry it is the *Area*, ſpace, or content of a right angled quadrangular figure, made alſo by multiplication of two lines, which are called the ſides; of which one is the meaſure of the breadth, the other of the length.

Definition III.

A *Rectangled Parallelepipedon* is the product of a Rectangle multiplied by a right line or number: And if that line or number and the Length and breadth of the Rectangle be ſeverally equal it is a Cube, or Die.

Definition IV.

A *Priſme* is a Solid contained within five Superficies of which three are Quadrangular, and the other two being oppoſite, Triangular: Or it is like the top of an ordinary Engliſh houſe cut off by a Plane paſſing through or parallel to the Eaves.

The reſt of this kind I ſhall not define here but refer the Reader to *Euclid*.

The names of the Poteſtates *or* Powers.

1 The firſt *Power* is called a Side, or Root: The latter word *Root* is moſt uſed here; and it is ſignified thus, a.

2. The ſecond *Power* is called a *Square*, and is thus written, aa.

3. The third is called a Cube, and is thus written, $a\,a\,a$. or ſometimes for brevity, a^3

4. The fourth, a *Biquadrat* or Squared Square, anciently a *Zenzi zenzick*, figured thus $z\,z$ now thus, $a\,a\,a\,a$, or for brevity, a^4.

5. The fifth Power is called a *Surſolid*, and is written thus, $a\,a\,a\,a\,a$; or briefly thus a^5.

6. The ſixth a ſquared Cube, or *zenzicube*, written thus, $aaaaaa$ or a^6.

7. The ſeventh, a Second *Surſolid*, and is written $a\,a\,a\,a\,a\,a\,a$, or more ſhort a^7.

8. The eighth is called a Squared Square Squared, or *zenzi zenzi zenzi zenzick*, and is written $a\,a\,a\,a\,a\,a\,a\,a$, or thus a^8, &c.

Conſectary I.

Hence it is manifeſt that theſe Powers uninterrupted, are in continual proportion, the proportion of them being as a, to unity: or the converſe.

Conſectary II.

It is alſo here plain, that every *Power* hath ſo many dimenſions, as the Letters, with which it is written. For a^4 being written with four Letters, if one Letter ſtand for one dimention, that is length or breadth, the other three ariſe by three ſeveral Multiplications, and every Multiplication adds a Dimenſion, in this ſenſe.

A Table of the Powers of the Digits.

Roots of the Powers. 1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	1	1	1
2	4	8	16	32	64	128	256	512
3	9	27	81	243	729	2187	6561	19683
4	16	64	256	1024	4096	16384	65536	262144
5	25	125	625	3125	15625	78125	390625	1953125
6	36	216	1296	7776	46656	279936	1679616	10077696
7	49	343	2401	16807	117649	823543	5764801	40353607
8	64	512	4096	32768	262144	2097152	16777216	134217728
9	81	729	6561	59049	531441	4782969	43046721	387420489

In this *Table*, the *Digits* at the *top* 2, 3, 4, *&c.* ſhew the *Columns* of the *Second*, *Third*, *Fourth Powers*.

The *Digits*, at the *Left-ſide*, ſhew the ſeveral *Roots* or *Firſt Powers*, and their *Proportion* to *Unity*.

CHAP.

CHAP. I.

An Explanation of the Characters and Symbols, used in this Tract.

FIrst, one single Letter of the Alphabet is usually put for any Quantity whatsoever, as well Line as Number; whether known, or unknown.

But for the most part, where any Quantity is sought, there a or some other *Vowel* is put for it; and the other Quantities known, are signified by *Consonants*.

These letters are multiplyed one into another by joyning them together without any prick or comma between, nor doth it import at all which is first or last written: For bcd, bdc, and cbd; are all one.

So a multiplyed by a produceth aa.
And a multiplyed by b, produceth ab.
And ab multiplyed by c, produceth abc.

The like of all others whatsoever, except Fractional Quantities; as, $\frac{ab+fg}{d}$ and $\frac{bc-fb+rc}{b+c}$

If the first of these were to be multiplied by d, it is done by taking away the d under the line, and the product is $ab+fg$.

If the second were to be multiplied by $b+c$, it is done by taking away the Denominator $b+c$, and the Product will be $bc-fb+rc$.

For all Fractions as well in Plain as in Figurate Arithmetick, are nothing else but Quotients of one number divided by another; and are multiplied again by taking away their Divisor and line of Separation.

Division is done in Figurative Arithmetick, most commonly by applying some line of Separation between the Dividend and the Divisor. So $\frac{a}{b}$ is a divided by b, And $\frac{abc}{f}$ signifies that abc is divided by f.

But yet if the Letter f had been found in the Dividend, the application of this Line had not been necessary, for it might have been better done by taking away that Letter out of the Dividend.

So

So afc divided by f quotient is ac
and $ffcc$ divided by fc quotient is fc
by ff quotient is cc
by cc quotient is ff
by ffc quotient is c
by fcc quotient is f
by f quotient is fcc
by c quotient is ffc

And the like may easily be underſtood of all the reſt.

Symbols of	Majority	$>$
	Minority	$<$
	Æquality	$=$
	Addition	$+$
	Subſtraction	$-$
	Root of a quantity	$\surd$
Proportionality continued		$'\ \ ''\ \ '''\ \ ''''$
Proportionality diſjunct		$'\ \ ''\ \ '\ \ ''$

So $b > c$ ſignifies b greater than c
$b < c$ .... b leſs than c
$b = c$ .... b equal to c
$b + c$ .... c added to b
$b - c$ .... c taken from b
$\surd\, 72$ ſignifies the ſquare root of 72, &c.

And $b'\ \ c''\ \ d'''\ \ f''''$ ſignifieth that as b is to c, ſo is c to d, and ſo d to f.

Likewiſe $b'\ \ c''\ \ f'\ \ g''$ ſignifies that as b is to c, ſo is f to g.

Theſe things before expreſſed are almoſt generally received: And uſed not only for brevity in writing, but perſpicuity in proving as will be ſeen hereafter.

Note that whereſoever — is not expreſſed, there + is underſtood, though it be not expreſſed.

Alſo in Trigonometrie. I uſe, s. pzs, for the ſine of an angle pzs, and s. c. zp for the ſine of the Complement of a ſide pz to 90. Alſo, t. zp and t. c. spz for tangent of zp and tangent of the Complement of spz, &c. Alſo for Radius I uſe r.

If the ſign of Addition, namely + ſtand before any quantity, it ſhews that quantity, to be more than nothing; that is ſomething.

But if the ſign of Subſtraction, to wit — ſtand before any quantity; it ſhews that quantity to be leſs than nothing; or a want of the ſaid quantity.

So + 4, ſignifies four of any thing: But — 4, ſignifies a want of four, or four leſs than nothing.

In

In ADDITION.

The addition of a want of any thing, is all one with the ſubſtraction of the ſame thing.

So if to +12 you add —5 it makes +7
And if to +12 you add —16 it makes —4
But if to +12 you add +16 it makes +28

In SUBSTRACTION.

The ſubſtraction of — is all one with adding +
So if from +12 you ſubſtract —5 remain is +17.
And if from +12 you ſubſtract —16 remain. is +28.
Addition of + to + and Subſtraction of — from — is all one with Common Addition and Subſtraction. And generally for both.
In *Addition*, add the quantities together with the ſame ſign.
In *Subſtraction*, add them alſo, but all the ſigns of that which is to be Subſtracted from the other, muſt be changed.

EXAMPLE.

If to +6—2+3, be added +5+1—3 the ſum is +6—2+3 +5+1—3=10.
But if from +6—2+3, be Subſtracted +5+1—3, the remain is +6—2+3—5—1+3=4. This Rule is general, and generally known.

In MULTIPLICATION.

+ multiplied by + ever produceth +
+ multiplied by — ever produceth —
— multiplied by — ever produceth +

More Varieties there are not.

The quantities that are accompanied with theſe ſigns of + & — (in both Multipliers being placed one under another, as in common multiplication) muſt be multiplied every one below into every one above, and then this work is done.

So

So if, $+bb+b-c$, be multiplied by $+f-g$ place them thus.

$$\begin{array}{l} +bb+b-c \\ +f-g \\ \hline \end{array}$$

Saying, $+f$ multiplied into $+bb$ gives	$+bbf$
And $+f$ into $+b$ gives	$+fb$
And $+f$ into $-c$ gives	$-fc$
And $-g$ into $+bb$ gives	$-bbg$
And $-g$ into $+b$ gives	$-bg$
Lastly, $-g$ into $-c$ gives	$+cg$

Which added together is, $\left.\right\} bbf-bbg+fb-fc-bg+cg$

Which is the true product.

In DIVISION.

If the line of separation do not serve the turn, that is, if any desire, (and it may be done) otherwise, it must then be by seeking what quantity may be multiplied by the Divisor to produce the Dividend.

So if $bb+bc-bf-bg-cg+fg$, were to be divided by, $b+c-f$, trial must be made what mixt quantity multiplying $b+c-f$ will produce $bb+bc-bf-bg-cg+fg$.

In which there is this of Compendium, that seeing the Dividend consists of six Members, and the Divisor of three, the quotient must be of two; that is a *Binomial* only.

And because the quantity g is found in the Dividend, and not in the Divisor, it must be in the quotient.

The said quotient therefore must be one of these, $b+g$, $b-g$, or $g-b$.

It cannot be the first, for $+g$, into $-f$ would have produced $-fg$: But in the Dividend it is $+fg$, therefore it must be $-g$.

By the same reason it cannot be the last, as also because $-b$, into $+b$ produceth $-bb$, but it is $+bb$, in the Dividend.

The quotient sought, must therefore be $b-g$.

Some further Rule for saving labour herein might be given: But every one likes that best which he finds out himself. Nor is it my purpose to write a *Book of Algebra*; but to premise so much of the Rudiments thereof, as the Reader may stand in need of in the perusing the following Treatise.

Wherein because Division is seldom needed; If I have a little exceeded already, and shall a little more in treating (but very briefly) of resolving some few Rooted Æquations, I shall ask the Readers pardon for both together.

CHAP.

CHAP. II.

Of Æquations.

AN Æquation is when one or more ſpecial quantities, are equal to one or more other ſpecial quantities, and written with the ſign of equality betwixt them; As $aa = bb$.

This is called a ſimple Square Æquation. And bb, being a known ſquare, the ſquare root thereof being extracted, is equal to a. And that is the thing required.

But, $aa + ba = cc$, and $aa - ba = dd$, and laſtly $-aa + ba = ff$; are all of them of that kind, which are called mixed Æquations, becauſe a (the thing required) is multiplied not only into it ſelf, but into another known quantity, namely into b.

And note that this known quantity in all mixed Æquations is called the *Coefficient.*

Note alſo that the three ſorts of mixed Æquations above expreſſed are all that can happen in Quadratiques: And by ſome one of theſe, all Problems whatſoever tranſcending plain Diviſion, and not reaching Solids, are to be reſolved by finding the Root a, according to theſe *Old Rules.*

In the First, $aa + ba = cc$.

Unto the quantity given namely cc, *add the Square of half the Coefficient, it makes* $+ cc + \frac{bb}{4}$ Which if it be in lines, may be reduced into one Square, *and from the ſide of that Square, take half the Coefficient, and the remainder ſhall be* a. Which was the thing deſired.

In the Second, $aa - ba = dd$.

Unto dd add $\frac{bb}{4}$ as in the former, and the Sum thereof being always in numbers a Square, or in lines to be reduced to a Square as aforeſaid; *Unto the Root or ſide of that Square, add half the Coefficient,* the Sum thereof ſhall be a, or the Root of the Æquation ſought for.

In

In the last, $-aa+ba=ff$.

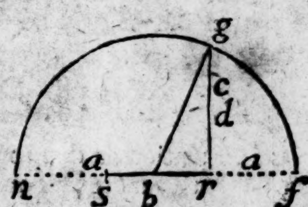
From the Square of half the Coefficient, which is $\frac{bb}{4}$ *take the quantity given*, that is ff, there will remain $\frac{bb}{4}-ff$, which being put into one Square, and the fide thereof known *If that ſide be either added to half the Coefficient, or ſubſtracted therefrom, either the Sum of that addition, or the remain of the ſubſtraction, is equal to a.*

For all Quadratique Æquations of this kind (where aa the greateſt unknown power is wanting) have two Roots, which being both together ever equal to the Coefficient, if upon the Coefficient, as a Diameter, a Semicircle be deſcribed, and the ſide of ff (the quantity given) be applied therein, perpendicular to the Diameter b, two ſegments of b are the two Roots ſought.

For in the Æquation $-aa+ba=ff$, it is by the 14th. of the 6th. of *Euclid*, as followeth.

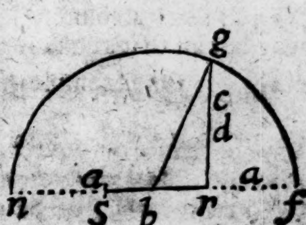
$b-a' \; f'' \; f' \; a''$.

Wherefore either Segment may be a, and the other will be $b-a$, and f a mean betwixt them.



Likewiſe in the two former Æquations, the work may be effected Geometrically and proved alſo by this preſent Scheme.

In which, as the figure intimates, the Perpendicular repreſents the ſide of cc in the firſt Æquation, and the ſide of dd in the ſecond.



Draw a line bg from the center b to the top of the perpendicular, the center b being firſt taken in the middle of the line b, to wit, of the *Coefficient*, for ſo it is uſually called.

And firſt, let the pricked line be put for a, Therefore by the before recited Propoſition,

It is, $a+b' \; c'' \; c' \; a''$. *Euclid.* 6. 14.

 And

And if (as the Rule preſcribeth) to the Square of half b you add the Square of half c, the total ſhall be the Square of the line bg: by the 47th. of the firſt of *Euclid*,

If therefore from the line bg, or (or which is all one) bf, you take the line br, which is half the Coefficient b (for the whole Coefficient b is the ſame with sr) the reſt, namely the pricked line rf, ſhall be equal to a. For,

$$rf = ns = a.$$

In like ſort concerning the ſecond Æquation, $aa - ba = dd$:

If according to the Rule, you add the Squares dd, and $\frac{bb}{4}$ together, it gives the Square of the line bg, to the Root of which, to wit bg, if you add half the Coefficient, to wit, br, or bs the ſum ſhall be fs or nr, either equal to a. And then, as nr, that is a, is to d, ſo is d to rf, or $a - b$, as it ought to be.

I intend anon to write ſomething of *Extraction of Roots*, according to the general Method of reſolving all manner of Æquations of *Powers*, how high or compoſed ſoever. I do not mean to exemplifie them any further then the *Cubique* order. There are Authors enough, whom they that deſire the full of that Artifice, may at their own leaſure in Books conſult.

And now becauſe I ſhall herein make ſome uſe of Æquations, though not higher then Cubiques, or at the moſt the Biquadratique order: I think fit to admoniſh the Reader, that in putting a always for the thing ſought, and working therewith, as if it were known, quite through as the queſtion requires, he ſhall at laſt come to an Æquation but it may be ſuch a one as wants reducing: of which a little.

REDUCTION

Of Æquations is done by adding all that's neceſſary, or ſubſtracting all that's unneceſſary on both ſides the ſign of Æquality: Or by Subſtracting Contradictories if they happen on one and the ſame ſide, untill the Æquation, purg'd of all unneceſſary members, remain with all that's abſolutely known on one ſide, equal to (as little as may be) all that's unknown on the other ſide.

One example of this ſhall ſerve as followeth:

In the Æquation

$$aa - ba + dc + ba = gg + ba - dc.$$

To reduce this, you muſt remember what hath been ſaid before; that the taking away a *Want* of any thing, is all one with the addition of that thing.

There-

Therefore seeing there is on the first side a *Want* of $b\,a$ expressed by $-b\,a$, if you take away that $-b\,a$, you thereby add $b\,a$ on that side.

Wherefore that it may still be an Æquation; you must add $b\,a$, on the other side also.

Then it will be,

$$aa+dc+ba=gg+2ba-dc$$

Again, substract $b\,a$ on each side, then it is,

$$aa+dc=gg+ba-dc$$

Once more, substract $b\,a$ on each side, that you may bring it to that side where $a\,a$ stands.

Then it is,

$$aa-ba+dc=gg-dc.$$

Lastly, (that the Consonants, or known things may come all on one side) substract $d\,c$ on each.

Then will it be,

$$aa-ba=gg-2dc,$$

Take the Rectangle $2\,d\,c$ out of the Square $g\,g$, and let the rest be a Square, namely $f\,f$.

Then it is reduced,

$$aa-ba=ff.$$

Having gone a little about, only for exercise of them that are quite unskilful herein, now they shall see this Reduction might have been quickly done another way, that is, seeing in the Æquation,

$$aa-ba+dc+ba=gg+ba-dc$$

There are in the first parts Contradictories, to wit, $-b\,a$ and $+b\,a$, they (destroying one another) might be taken away both at once,

So it will be,

$$aa+dc=gg+ba-dc.$$

Then if you substract $d\,c$ and $b\,a$ on both sides, it will be reduced to

$$aa-ba=gg-2dc,$$

as it was before. And $g\,g-2\,d\,c$, being put into one Square $f\,f$, the Æquation

$$aa-ba=ff,$$

may be resolved as the Æquation $a\,a-b\,a=d\,d$ was by the second Rule for plain Æquations, a little before expressed.

And as here the Reduction was made by Addition and Substraction only, so sometimes it is made by Multiplication, sometimes by Division; in both or either of which, this is general: that *Whatsoever is done to any one Member, must be done to every Member quite through the Æquation.*

CHAP. III.

Of the resolution of Æquations, according to the general Method composed by Mr. Tho. Harriot.

ALthough (having before shewed Rules for all sorts of mixed squares) it may seem preposterously done hereafter to speak of Simple Squares; yet forasmuch as I pretend not much to Method or Order, and because the general Method of Mr. *Harriot* begins with Squares, I will do so, but only with one Example. That is, Let there be an Æquation of $aa = ff$.

Or let it be exhibited in numbers, $aa = 69169$

First, take notice that all Squares whether Simple or mixed in Numbers, are to be marked with points, the first always over the place of Unity or Unities, and so successively every Binari or second figure.

Cubes with every ternary figure.

Biquadratiques with every quaternary.

Sursolids, every quinquenary, and so forwards.

This Square number so pointed is $\dot{6}9\dot{1}6\dot{9}$

In which because there are three points, there are three figures in the Root.

So that a being a single letter cannot fitly represent that Root, but some trinomial, as is $b + c + d$ should be put equal to a, and the Square thereof should be equal to aa, or 69169.

But because it may be done as well by adding the Gnomons, that is repetition of the second working, (as they are commonly called) so often as the points are more then two; a *Binomial* will serve (with less trouble) to do the same.

Let that *Binomial* be $b + c$.

And put $b + c = a$.

Their Squares shall be therefore equal.

That is, $bb + 2bc + cc = aa$.

That is, $bb + 2bc + cc = 69169$.

The

The Resolution.

The homogeneal number given		69169
First single Root $b = 2$ and $bb = 4.0000$		
Which 4.0000 being substracted from the number given 69169, then there		
Remains of the Number given		29169
Root decuplate $b = 20$		
Divisor $2.b$ 40.00		
The second single root $c = 6$		
$2.bc$	240.00	
cc	36.00	
	276.00	
Substract		276.00
Remains of the Number given		1569
The Root increased $b = 26$		
Root increased and decuplate $b = 260$		
Divisor is $2b = 520$		
The third single Root $c = 3$		
$2bc$	1560	
cc	0009	
Total	1569	
Substract		1569
Remains of the Number given		0000

The Root increased 263, is therefore the true Root, as may be proved by recomposition, or multiplying 263 by 263, for the Product will be 69169, which was the number given.

The Cyphers which are put after in the Divisors and Substracts, are only to fill up the number of places, by which the number given or rather the remaining Points would else exceed.

For the like purpose is used the decuplation of the Roots, as only to supply a place until another figure succeed in Place of the Cypher.

And

And in nothing elſe doth this work differ from the ordinary Extraction of the Square Root, commonly taught and known.

The reaſon depends upon the 4th. *Prop.* of the ſecond Book of *Euclide*, where it is demonſtrated, that *If a right line be divided by chance into two parts, the Square made of the whole, is equal to the Squares of the parts, and to the Rectangle made of the parts twice.*

So it is here as followeth.

The Square of the greater part, that is, of 260 —— $bb = 67600$
The Square of the leſſer part, that is, of 3. ———— $cc = 00009$
The Rectangle of the parts, that is, 260 into 3 twice. — $2bc = 01560$

Equal to the whole Square. ———————— 69169

Nor do theſe letters repreſent ſo naturally the things themſelves in a divided *Superficies* only, but as properly and clearly the parts of *Solid Bodies*, of which, two or three Examples for ſatisfaction.

In which I admoniſh the Reader, to be intent to the ſeveral pointings of the quantities according to their due order, as is before expreſſed, and alſo to the placing of the Diviſors and Subſtracts by Cyphers, as before alſo is intimated: For this to the Ingenious is enough, and a long Verboſity to others will ſcarce be ſo.

Of Cubical Æquations.

Let there be a Cube	$aaa = fff$
Or propoſed in Numbers	$aaa = 41781923$
Put (as before)	$b + c = a$

Then their Cubes alſo ſhall be equal.

That is $bbb + 3bcc + 3bcc + ccc = 41781923$.

The Reſolution.

The Homogeneal Number given	41781923
The firſt ſingle Cubique Root $b = 3$	
And $bbb = 27.000000$	
Subſtract	27.000000
Remains of the Number given	14781923

The

The first Root decuplated $\} b=30$

$3.bb$	2700.000
$3.b$	0090.000
Divisor	2790.000

Second single Root $c=4$

$3.bbc$	10800.000	
$3.bcc$	01440.000	
ccc	00064.000	
	12304.000	
Substract		12304.000

Remains of the Number given	02477923

The Root increased $b=34$

Root increased decuplate $\} b=340$

$3.bb$	346800
$3.b$	001020
Divisor	347820

The third single Root $c=7$

$3.bbc$	2427600	
$3.bcc$	0049980	
ccc	0000343	
Substract	2477923	
		2377923
Remains lastly of the Number given		000

The Root increased $b+c=347$

Which is the true Root of the Cube 41781923, as may be proved by recomposition, that is, by Multiplying 347 by 347, and the Product again by 347, the last Product shall be equal to the Cube which was given to be resolved.

And as above in the Square the Canon of the Resolutions was the letters $bb+2bc+cc$, being the true Square of $b+c$. And those letters did answer exactly to the parts of the Square divided alike in both the Dimensions: So here also the Canon of Resolution, or

the letters $bbb + 3bbc + 3bcc + ccc$, do exactly anſwer to the Parts or Members of a Cube, divided into two parts, alike in all the three Dimenſions, as any one may prove upon a Cube made of ſome ſlender matter, and cut through all three ways, for he ſhall find the whole Cube (ſuppoſed equal to 41781923 as before) juſtly made up of the two Cubes of the two Segments, that is, bbb and ccc, and three Parallelepipedons, whoſe length and breadth are equal to b, and their thickneſs to c, thoſe three are the $3bbc$. And laſtly, three other Parallelepipedons, whoſe length and breadth are equal, to c, and their thickneſs to b, ſuch are the $3bcc$.

See the following Schematiſme.

The Cube of the greater Segment which is 340, bbb	39304000
The three greater Parallelepipedons, $3bbc$	.2427600
The three leſſer Parallelepipedons, or $3bcc$	...46980
The Cube of the leſſer Segment, which is 7, ccc	343
The whole Cube given	41781923

Note, That the greater Segment is the aggregate of all the ſingle Roots except the laſt, being duly valued by a Cypher, as here it is 340, but the leſſer Segment is the laſt ſingle Root only, as here 7,

I have done this to let the Reader ſee, that he may be ſure, let the quantity to be reſolved be great or little whatſoever, if he be careful to make his Canon right, the letters themſelves will direct him how to frame his Diviſors and Subſtracts in order to the final reſolution, eſpecially in theſe unmixed Quantities, where the points limit how far the Subſtract ſhall advance at every operation, beginning firſt at the point next the left hand, not further, and to the Second point only at the ſecond Work, and not otherwiſe in all that follow.

And in Mix'd Æquations, if they be made up of Cube with addition of certain Squares, or certain Roots, or both Squares and Roots, or by Subſtraction of the ſame Canon of the Reſolution muſt ever be made by Multiplying the aſſumed Root $b + c$ in the place of the Queſititious Root a, quite through the Æquation in all the degrees there-

thereof, for ſo ſhall ariſe all the ſeveral parcels of which the ſeveral Subſtracts are orderly to be made.

In a Cubick Æquation, if all the quantities be preſent, there is no need to point any but the Cubicks and Roots: yet I have here diſtinguiſhed the places of the Squares alſo with little Croſſes Obliquely; which labour, when the Workman is intent upon his buſineſs, may well enough be ſpared.

Of the Reſolution of Mixed Cubicks.

Let the Æquation $aaa + daa - ffa = ggg$ be propoſed in Numbers.

As let it be $aaaa + 32aa - 75a = 29282970$

Therefore $d = 32$ and $ff = 75$

And $ggg = 29282970$

Put $b + c = a$

And make the Canon of Reſolution by ſubſtituting $b + c$ in the place of a quite through the ſeveral quantities $aaa + daa - ffa$. The Canon rightly made will be

$$
\begin{array}{l}
+bbb \\
+3bbc + dbb - ffb \\
+3bcc + 2dbc - ffc \\
+\ .ccc + dcc
\end{array}
$$

Theſe ſeveral parcels of the Canon, being rightly ſubſtracted from the homogeneal Number 29282970, the Number ſhall be thereby reſolved, and the Root a found.

☞ Note firſt, That all the parcels in the Canon, which have not the Secondary Root c in them, as $+bbb + dbb$ and $-ffb$, are to be ſubſtracted at the firſt Operation, the other remaining parcels to be all ſubſtracted as often as there ſhall be points left above.

The Reſolution.

$$
\begin{array}{lr}
 & \cdot\ \ \cdot\ \ \cdot\ \ \cdot \\
 & \times\ \times\ \times\ \times \\
\text{The homogeneal Number given} & 29282970 \\
 & \cdots \\
\text{The first single Root } b = 2 & \\
+bbb \quad 8.000000 & \\
+bbb \quad 128.0000 & \\
+928.0000 & \\
-ffb\ ..\ 150.00 & \\
+92650.00 & \\
\text{Substract} & 92650.00 \\
 & \cdot\ \ \ \cdot \\
 & \times\ \ \ \times \\
\text{Remains of the Number given} & 20017970 \\
 & \cdot\ \cdot
\end{array}
$$

Remains of the Number given 20017970

The first Root decuplate $b = 20$

$3bb$	1200.000
$3b$	..60.000
$2db$	.1280.00
d	...32.00
	13912.00
$-ff$	....75.0

Divisor 1390450

The second single Root $c = 9$

$3bbc$	10800.000
$3bcc$	.4860.000
ccc	..729.000
$2dbc$	.11520.00
dcc	..2592.00
	178002.00
$-ffc$	....675.0
	17793450

Substract 17793450

Remains of the Number given 02224520

The Root increased $b = 29$
Root increased decuplate $b = 290$

Remains of the Number given 02224520

$3bb$	252300
$3b$	...870
$2db$	.18560
$-ff$	75

Divisor 2 71655

Third

Third single Root $c=8$

$$\begin{array}{rl} 3bbc & .2018400 \\ 3bcc & ...55680 \\ ccc & .....512 \\ 2dbc & ..148480 \\ dcc & ....2048 \\ \hline & 2225120 \\ -ffc & ....600 \\ \hline & 2224520 \end{array}$$

Substract lastly 2224520

Remains of the Number given 000

Whereby it appears that the whole Root 298 is the true Root whereby this Æquation is explicable, as may be proved also by recomposition.

$$\begin{array}{rl} \text{For } bbb= & 24389000 \\ 3bbc= & .2018400 \\ 3bcc= & ...55680 \\ ccc= & .....512 \\ dbb= & .2691200 \\ 2dbc= & ..148480 \\ dcc= & ....2048 \\ \text{In all}= & 29305320 \end{array}$$

From which substract $ffb+ffc=-...22350$

Remains 29282970

Which was the whole Homogeneal Number given.

NOTE.

Whereas in composing the Divisor all the gradual quantities are used, as in the former example, $3b$ and d, as well as $3bb$ and $2db$, it is to be noted that in practice, those smaller particles $3b$, &c. May be comitted; the other without them ministring light enough for choosing the Secondary Roots.

Having now instanced in an Example where all the powers were present, in these one or two that follow, to make the Work shorter, I shall leave out one or other of them.

In the Æquation $aaa+ffa=ggg$.

Propounded in Numbers $aaa+320406\,a=8348132$, It sometimes happens that the Coefficient abounds with more binarie figures than the Homogeneal doth with ternaries, in such a case that there may be room made to begin the Extraction. The Coefficient must be devolved to the next point further to the right hand, or to the

 second,

second, third, fourth, or further, if need require, and there the Work is to begin. The Coefficient is always the known quantity which Multiplies any of the unknown inferior quantities.

Example of Devolution.

$aaa + 320406.a = 8348132$

Put $b + c = a$

The Canon will be $\left.\begin{cases} bbb + 3bbc \\ + 3bcc + ccc \\ + ffb + ffc \end{cases}\right\} = 8348132$

Resolution.

The Homogeneal number given 8348132

The first single Root $b = 2$

$+ bbb$ 0008.000

$+ ffb$ 640812.0

$= 6416.12.0$

Substract 641612.0

Remains of the Number given 1932012

The first Root decuplate $b = 20$

$3bb$ 00001200

ff 00320406

Divisor 321606

The second single Root, $c = 6$

Remains of the number given 1932012

$3bbc$ 7200

$3bcc$ 2160

ccc 216

ffc .1922436

1932012

Substract 1932012

Remains of the number given 000

Where-

Wherefore the whole Root is equal to the Root increaſed, 26, as may be proved in manner as beforeſaid.

It ſometimes happens alſo in the Æquation
$aaa - ffa = ggg$ Put into Numbers.
As $aaa - 105000.a = 203125$.

That the Coefficient abounds with more binarie figures than the Homogeneal with ternaries: Wherefore that there may be place for the Reſolution, put before the Homogeneal, toward the left hand, ſo many Cyphers as will afford that to receive as many Cubical points, as the Coefficient doth Quadratical: And at the firſt empty point, as it were by anticipation, begin the Reſolution. In which there is this of Compendium, that the firſt Square Root extracted out of the Coefficient, is either equal to the firſt ſingle Root of the Homogeneal ſought, or leſs than it by Unity.

But if the Æquation had but two Dimenſions, As $aa - 254a = 65024$, then the firſt figure of the Coefficient, namely 2, is the firſt Root.

Example of Anticipation.

The Homogeneal Number given $+0203125$

$b + c = a$

The Canon is $\left\{\begin{array}{l} bbb + 3bbc + 3bcc + ccc \\ \quad - ffb - ffc \end{array}\right.$

The Reſolution.

The firſt ſingle Root $b = 3$
$+ bbb$ 27.000000
$- ffb$ 3150.0000

Subſtract the difference which is $\}$ -4500.00

Remains of the Number given $+4703125$

The firſt Root decuplate $b = 30$
$+ 3bb$ 2700.000
$- ff$ 105000.0

Divisor 165000.0

3bbc

$3bbc$	5400.000
$3bcc$	.360,000
ccc	...8.000
$-ffc$	21000.00
	+366800.0

Substract +366800.0

Remains of the Number given +1035125

The Root increased $b=32$

Root increased decuplate $b=320$

$3bb$	307200
$-ff$	105000
Divisor	202200

The third single Root $c=5$

$3bbc$	1536000
$3bcc$	..24000
ccc	125
$-ffc$	.525000
	1035125

Substract 1035125

Remains of the Number given 000

Which sheweth that the Root increased $b+c=325$, is the true Root of the Æquation, And it may be proved by recomposition as formerly.

In the Æquation $-aaa+ffa=ggg$, Which is explicable by two Roots, as shall be shewed in the next Chapter, *Section* 5, to find them both. Put the Æquation into Numbers.

$$\text{As} -aaa+52416\,a=1244160$$

$$\text{Therefore } ff=52416 \text{ and } 1244160=ggg$$

$$\text{Put } b+c=a$$

$$\text{Therefore } \left.\begin{array}{l} -bbb+ffb \\ -3bbc \\ -3bcc+ffc \\ ccc \end{array}\right\}=1244160$$

Extracti-

Extraction of the greater Root.

The Homogeneal Number given 1244160

The first single Root $b=2$

ffb	104832.00
$-bbb$	8.000000
	+ 24832.00

Substract + 24832.00

Remains of the Number given — 1239040

The first Root decuplate $b=20$

ff	52416.0
$-3bb$	1200.000

Divisor — 67584.0

The second single Root $c=1$

Remains of the number given — 1239040

ffc	.52416.0
$-3bbc$	1200.000
$-3bcc$	. .60.000
$-ccc$	. . .1.000
	—736840

Substract —736840

Remains of the number given —502200

The Root increased and decupled $b=210$

ff	52416
$-3bb$	132300

Divisor —79884

The

The third single Root $c=6$

$+ffc$	314496
$-3bbc$	793800
$-3bcc$	. 22680
$-ccc$	. . . 216
	-502200

Substract -502200

Remains of the number given 000

Root increased $b+c=216$, which is the true Root sought.

II. *Eduction of the lesser Root by Devolution.*

The Homogeneal Number given 1244160

$b=2$

ffb	104832.0
$-bbb$	$-$. . .8.000
	$+1040320$

Substract $+1040320$

Remains of the Number given $+203840$

The Root increased and decupled $b=20$

ff	.524.16
$-3bb$	. .1200

Divisor. 51216

The second single Root $c=4$

ffc	209664
$-3bbc$	. .4800
$-3bcc$	. . .960
$-ccc$	64
	$+203840$

Substract $+203840$

Remains of the Number given 000

The Root increased $b+c=24$

Where-

Wherefore 24 is the true Root ſought, as may be proved by recompoſition, as hath been ſhewed before.

So this Æquation is explicable by two Roots, that is, 216, and 24.

VIETA, *Lib. de Recognitione æquationum*, *Chap.* 18. *Prop.* 2. ſaith, That in the Æquation $-aaa+ffa=ggg$, the Coefficient ff is compoſed of three proportional Squares, and the Homogeneal ggg is made by Multiplication of the aggregate of the two firſt, or the two laſt, (for all is one) into the ſide of each other, and the Root a may be the ſide either of the firſt or third. This (or the ſame in ſubſtance) ſaith that Noble Author, And it is evident, for make

$$cc'+dd''+bb'''=ff$$

And put $c=a$

Therefore $ccc+ddc+bbc-ccc=ddc+bbc$

Or put $b=a$

It is $bbb+ddb+ccb-bbb=ddb+ccb$

Both which are manifeſt.

COMPENDIUM 1.

Hence it may be ſhewed, that either of the quæſititious Roots, as a, being found and called c, the other Root e may be found by a Quadratique Æquation only. For ſuppoſing

$ee+ce=ff-cc$, Then

It is $ee+ce+cc=ff$.

And cc' ce'' ee''' *Euclide* 6. 23.

But by conſtruction cc' dd'' bb'''. And $cc+dd+bb=ff$. So then $bb=ee$ and $b=e$.

But it was ſhewed before that b might be a Root of this Æquation $-aaa+ffa=ggg$ And therefore e alſo is a Root of the ſame, and the Compendium is proved.

Example also in Numbers.

In the last Æquation	$aa=cc=46656$
And	$ff=52416$
From which take	$cc=46656$
	05760
Remains	$ff-cc==5760$
But	$ee= 576$
And	$ce=5184$
	5760
The Sum is	$ee+ce==5760$

Therefore $ee+ce=ff-cc$, Which was, &c.

In the Æquation $-aaa+faa=ggg$, the Coefficient f is composed of three proportional Lines, and ggg is equal to a Solid made by a Square (whose side is equal to the two first, or the two last) multiplied into the remaining Line: And the aggregate of the first and second may be a, and the aggregate of the second and third shall be e. Put $1'\ 2''\ 4'''$

And suppose $-aaa+7aa=36$

Then a may be 3, and e is 6. *Vieta, de Recognit. Cap.* 18. *Prop.* 6.

COMPENDIUM 2.

And therefore the Root a found, and called c, the Root e may be found by a plain Æquation; for suppose the middle proportional y, it is $f-y-c'\ y''\ c'''$.

And $fc-cy-ec=yy$ Or, $yy+cy=fc-cc$. And making $fc-cc=xx$, it is $yy+cy=xx$. And the Root y being found by the first Rule of *Chap.* 2, It is lastly (making $c'\ y''\ d''$) $y+d=e$.

I will here add a few Rules (grounded upon Mr. *Harriots* 6 Sections) by which the Reader may easily perceive the Fabrique of Æquations, their Roots, increment and decrement, Multiplication and Division of them, and their Number in any Æquation as followeth.

CHAP.

CHAP. IV.

RULE I.

Every Æquation being composed of some known and some unknown quantities hath its Original by Roots composed of a quantity known and of one other quantity unknown, and these Roots Multiplied together produce certain particular Members with + and — respectively signed (for in every Æquation both these signes are present) which orderly placed make up the Æquation. As the Æquation $aa - ba - ca + bc = 0$. is made by multiplying $a - b = 0$ by $a - c = 0$. And because it was at first $a - b = 0$ therefore $a = b$ and the Like of c. And from hence it follows that where the first term (or highest power) in a quadratique Æquation is signed — there the Æquation hath two Roots, as here by Substracting on both parts $+ aa - ba - ca$, the Æquation will be $bc = - aa + ba, + ca$, and must have 2 Roots.

1. These compound quantities so multiplying I shall call *Binomials*, whether $a + b$ or $a - b$. not having any need in this Treatise to distinguish betwixt *Binomials* and *Residuals*.

2. The Æquation $aa - ba + ca = bc$, If it be, $b < c$. put $c - b = d$. then the Æquation will be, $+ aa + da = bc$, and is of the first kind mentioned in *Chap.* 2. but if it be $b > c$, put $b - c = f$ and the Æquation will be $+ aa - fa = bc$, and is like the second sort in the same Chapter.

The Original of the Æquation $aa - ba + ca - bc = 0$ here proposed, is $+ a - b = 0$ multiplied by $+ a + c = 0$, that is $a = b$ by $a = - c$. This æquation hath but one true Root, which is b, and one false, which is c.

3. By this which hath been said it is plain that some æquations have as many Roots as Dimensions, some not so many, but none can have more; for the number of dimensions being the same with the number of Multipliers (if all diverse) can be but all Roots. Nor can the æquation be divided by any other thing than one of those *Binomials* by whose Multiplication it was made.

But if the Multipliers how many soever be still the same, there can be but one Root. For let $+ a - b = 0$ be Multiplied *Biquadratically*, the product is $+ aaaa - 4baaa + 6bbaa - 4bbba + bbbb$. where it is plain there can be no other Root but b. I mean none greater or less than it; because in truth here are 4 Roots, but every one singularly equal to b.

For if there may, let it be d, and let d be greater or less than b, it imports not which. And seeing $d = a$, Substitute d in the Place of a, quite through the Æquation, it will be

$dddd - 4bddd + 6bbdd - 4bbbd + bbbb = 0$. Which if $d > b$, or else $d < b$, is at the first sight impossible: For the difference between the + and — is always equal to the power of the difference between b and d, which power is here a Biquadrat, therefore $d = b$. And again seeing this Æquation may be derived by putting b equal to d, for substituting b in the place of d quite through, It will be

$$+2bbbb + 6bbbb = 4bbbb + 4bbbb$$

Which is manifest, therefore again $b = d$, which is contrary to the supposition, therefore b is the only Root of this Æquation, for indeed, the Æquation proposed being made only of multiplications of $a - b = 0$ cannot be divided, that is resolved, by any other Binomial then $a - b$, of which it was made.

4. Hence it is that the last term in every Æquation may be called the Homogeneal, because it is naturally made by multiplication of the Roots of the Æquation, though the Coefficients in some ordinary Æquations are disguised with other Characters, which happens by Addition or Substraction of them, to reduce the canonical Æquation to fewer Numbers, whereby the redundancy of the Signs + and — is to be taken away, this is to be seen above in this Rule, where the Æquation
$+aa - ba + ca - bc = 0$ is reduced to $+aa + da - bc = 0$, by making $d = c - b$ and $+aa + ba + ca - bc = 0$, reduced to $+aa - fa - bc = 0$, by making $b - c = f$
Where the Coefficient d or f, is not a part of the Homogeneal bc, but a difference by which b is greater or less than c: By help of which difference, the Æquation which consisted canonically of four Members, hath now but three.

5. And this Reduction is useful, for as M. *Des Cartes* saith, and which may be seen true by the way of Multiplication above shewed, every Æquation hath so many true Roots as the Signes + and — therein are changed, which in the canonical Æquation
$+aa - ba + ca - bc = 0$, are changed three times, whereas the Æquation hath not three true Roots, but one true and one false that is b and c, and the common Æquation reduced changeth the Signs but once, that is from $+da$ to $-bc$ in the former; or from $+aa$ to $-fa$ in the latter; And from thence it may be known that the Æquation hath but one true Root, The like consideration ought to be in others.

And whereas the said *Des Cartes* doth often mention false Roots, it is to be noted that such are less than nothing, as $+a + b = 0$: Or $+a = -b$, and if any true Root, as $+a - c = 0$ be multiplied by this $+a + b = 0$, there will arise an Æquation $+aa + ba - ca - bc = 0$ where the Sign + follows twice, the Sign — twice, and they are once changed, which should intimate (according to *Des Cartes*) two false Roots, and one true: For he saith, So

many

many times as $+$ or $-$ come twice together, so many false Roots there are, this Æquation therefore must be reduced, by making $b - c = d$ if $b > c$, or else if $b < c$ then make $c - b = f$, so it will be either $+aa + da - bc = 0$, Or $+aa - fa - bc = 0$ which confirms that which *Des Cartes* saith of twice $+$ or $-$: Namely, that there are as many false Roots in the Æquation, as $+$ or $-$ come twice together, and so many true Roots as $+$ and $-$ are changed.

And where the Roots are all false, the Æquation is impossible, as $a + b = 0$ multiplied by $a + c = 0$, produceth $aa + ba + ca + bc = 0$ which cannot be. And therefore when there is an Æquation pretended like $aa + ba + ca = - bc$, present judgement may be made.

6. The same *Des Cartes* saith also that all the false Roots in any Æquation, may be turned to true ones, and the true ones to false, by changeing the Signs of the second, fourth, and every even term. And this is evident, for of the Æquation $a^4 - 2a^3 + 10aa - 30a - 87 = 0$ by such change is made $+a^4 + 2a^3 + 10aa + 30a - 87 = 0$ where the first had three true Roots, and but one false, the latter hath three false and but one true. This æquation was taken at all adventures, to serve for an Example only, whereas any other whatsoever will do the like.

RULE II.

The unknown Roots of an Æquation may be increased or decreased, by supposing another unknown quantity $+$ or $-$ the decrement or increment, and of that Binomial composing the Æquation as it was before of the first unknown Quantity: And if this increment be put equal to such a part of the Coefficient of the second Term, as Unity is of the dimensions of the first Term (if the Signs of the first and second be both $+$ or both $-$) or if the Decrement be made equal to such a part of the said Coefficient as Unity is of the dimensions as aforesaid, (If the Signs of the first and second Term be one $+$ the other $-$) then by such increase or decrease of the Root the second Term of the Æquation shall be taken away, and annulled.

EXAMPLE.

In the æquation $+aaa + baa - bbc = 0$, the Root a may be increased by making $e - q = a$, and Substituting $e - q$ in the place of a quite through the æquation, and thereby shall arise a new æquation:

$$\left.\begin{array}{r} +eee - 3qee + 3qqe - qqq \\ +bee - 2bqe + bqq \\ -bbc \end{array}\right\} \begin{array}{l} = +aaa \\ = +baa \\ = -bbc \end{array}$$

Which

Which is equal to the former as you see agreeing in the particulars, and the Root e being found, a may be had by casting away q from e.

And because the Number of the dimensions of the first Term aaa is 3, if according to the latter part of the Rule the quantity b be proportioned, by making $3'\ \ 1''\ \ b'\ \ q''$ then $b = 3q$ and $+bee$ will destroy $-3qee$, and so the second Term ee will be quite taken out of the æquation as is manifest, for the æquation so purged will be $+eee - 3qqe + 2qqq - bbc = 0$ And by Substracting on each part $+2qqq - bbc$ having first made $bbc - 2qqq = ddd$, it will be then $+eee - 3qqe = ddd$. The manner of such reduction of Solids, shall follow in the next Chapter.

In like sort the Root a, might have been decreased by any quantity, as x, which if it be proportioned to b as aforesaid, would take away the second Term of an æquation, where the Signs of the first and second Terms are not like; as in the æquation $+aaa - baa - bbe = 0$, by putting $3x = b$, and $e + x = a$: The Problem will be fully performed by making $e + x$ the Root of the new æquation, as before was $e - q$, observing the same order in composing the particulars, due respect had to the Signs $+$ and $-$, where they ought to be altered.

The former reduced æquation $+e^3 - 3qqe = ddd$ might be further reduced (if need require) to $+e^3 - bqe = ddd$.

NOTE.

This augmentation and diminution of the Roots in such manner as to take away the second Term of any æquation, is of excellent use in such æquations as have three or four dimensions, and cannot by any division with any Binomial made of $a +$ or $-$ some other known quantity, as b, c, or the like, be reduced to fewer dimensions, whereby it is certain that such an æquation is Solid, and cannot by any artifice already, or likely to be invented, be resolved by Ruler and Compass, but by any of the Conique Sections it may; in this case it is either necessary or extreamly facilitating, to take away the second Term (if there be any) from the æquation, as shall be seen hereafter in its place.

RULE III.

The unknown Root of any Æquation may be mulitplied (or divided by any known Quantity multiplying (or dividing) the second Term of the Æquation by the said Quantity, the third by the Square, the fourth by the Cube thereof, and so forward continually in this order, as often as there are Terms in it, having first assumed another unknown Quantity, so multiplex to the said unknown Root, as is required.

E X-

EXAMPLE.

In the Cubical *Æquation* $a^3 + baa + cca - bcd = 0$: Let it be required to multiply the Root a by 4.

Assume $e = 4a$ and write

$$eee + 4bee + 16cce - 64bcd = 0$$

Which is an æquation, and the Root e is Quadruple to a, as may be proved thus.

Put $a = 4$ $b = 3$ $c = 2$ and $d = 21\frac{1}{3}$

$$\text{Then}\quad \left.\begin{array}{rr} aaa = & 64 \\ baa = & 48 \\ cca = & 16 \\ \hline \text{In all} & 128 \end{array}\right\} \text{But } bcd = 128$$

Therefore $a^3 + baa + cca - bcd = 0$

Again, Put $e = 16$ All else the same still.

$$\text{Then}\quad \left.\begin{array}{rr} eee = & 4096 \\ 4bee = & 3072 \\ 16cce = & 1024 \\ \hline \text{In all} & 8192 \end{array}\right\} \text{But } 64bcd = 8192$$

Therefore $e^3 + 4bee + 16cce - 64bcd = 0$

And $e = 16 = 4a$ which was to be proved.

The utility of this Rule will appear in reducing æquations affected with Fractions, to whole Numbers, by multiplying the Roots by the denominator or denominators of the Fraction, for by such means the Coefficient of the second Term is multiplied by the same as before, multiplying a by 4, multiplying also b by the same Number 4. And many times by this Rule æquations may be freed from Surd Numbers also; especially if such be found in the second Term, as is easie to be seen by trial, for if there be an æquation so affected,

As $aaa + \surd 8\, aa + \frac{29}{24} a - 4\surd 2 = 0$

Put $e - \surd 8 = a$

And write $+ eee + 8ee + 9\frac{2}{3} e - 128 = 0$

So the Surds are vanished.

But if yet it be required to avoid the Fraction $9\frac{2}{3} e$, then make $y = 3e$. And multiplying 8 by 3, $9\frac{2}{3}$ by 9, and 128 by 27, there will be a new third æquation.

$+yyy$

$$+yyy+24yy+87y-3456=0$$

Which conſiſts of entire Numbers, having one true Root which is 9, and the Root of the middle æquation was 3, which is the third thereof, and the Root of the firſt æquation was $3\sqrt{8}$ And now I hope this Rule and the uſe of it is plain enough.

NOTE I.

It may be noted, that if the Surds in the ſecond and laſt Terms of the firſt æquation, to wit, $aaa+\sqrt{8}aa+\frac{29}{24}a=4\sqrt{2}$ had been utterly incommenſurable, the reduction had not been ſo feaſible. For although $4\sqrt{2}$ multiplied by the Cube of $\sqrt{8}$ that is by $8\sqrt{8}$ produced $32\sqrt{16}$. which is equal to the intire number 128, yet if it had been $2\sqrt{3}$ or $2\sqrt{5}$, or any ſuch primes to be multiplied by $8\sqrt{8}$ the product would have been $16\sqrt{24}$ or $16\sqrt{40}$ though this laſt may (by the note after the Conſectary in *Chap.* 6.) be reduced by multiplying it again by $\sqrt{40}$ unto the intire number 640. Neverthele ſs this ſecond multiplication by a Surd, renders the æquation inexplicable, at leaſt by the precedent Rule.

NOTE II.

It may be further noted, that if inſtead of $e=4a$ one would put $e=fa$ lines not being ſo liquid as Numbers, the æquation would then be $eee-fbee+ffcce-fffbcd=0$ increaſing the dimenſions of the leſſer Terms, for remedy whereof three lines are to be found in proportion one to another as are the magnitudes $fb.\ ffc.\ fffb.$ of which let the firſt line be ſuppoſed to contain *Unity* as often as the ſuperficies fb doth (for which purpoſe *Unity* muſt be a line ſet, and agreed on before.) The Names of theſe lines when found may be called g, h, k, and the æquation may be written

$$+eee+gee+hee-kbc=0.$$

NOTE III.

But it is again to be noted, that where the lines f, b, and c, are commenſurable in length the three lines k, h, g, may be very eaſily found, for then they may be ſignified by Numbers and if f be put for *Unity* then $e=a$ and the work fruſtrate, but where the ſaid lines are incommenſurable in length this Reduction is always hard if not impoſſible: For theſe incommenſurable lines do moſt commonly repreſent ſuch Surd Numbers as cannot by any Reduction be compared.

RULE

RULE IV.

The Æquation $aaa - 3bba = 2ccc$, *or any other like it, by putting* $\frac{cc + bb}{e} = a$ *may if* $c > b$ *be brought to* $eee = ccc + ddd$ *or if* $c = b$ *to* $eee = ccc$, *or lastly, if* $c < b$ *then to* $eee = ccc + \sqrt{-dddddd}$: *Which last may be called an impossible Æquation.*

Put $e' \; b'' \; \frac{bb'''}{e}$ And because a is equal to the sum of the Extreams, which are $e + \frac{bb}{e}$ therefore,

From thence it will be

$$\left.\begin{array}{l} \frac{+e^6 + 3bbe^4 + b^4ee + b^6}{eee} = +aaa \\ \text{And } \frac{-3bbe^4 - 3bbbbee}{eee} = -3bba \end{array}\right\} = +2ccc$$

Therefore rejecting the contradictories, and multiplying all by eec, it is, $+e^6 + b^6 = 2c^3c^3$.

Therefore $+e^6 - 2cccc^3 = -bbbbbb$.

And $+e^6 - 2c^3e^3 + c^6 = +c^6 - b^6$.

Therefore, (for $e^3 - c^3 = \sqrt{e^6 - 2c^3c^3 + c^6}$) $+e^3 = ccc + \sqrt{c^6 - b^6}$.

If now in the first case c be greater than b,

then put $c^6 - b^6 = d^6$.

Then it will be $eee = ccc + \sqrt{dddddd}$ That is, $eee = ccc + ddd$. Which is the Æquation promised in the first case.

Secondly, If b be equal to c, then $c^6 - b^6 = 0$ And it will easily follow, seeing (as is shewed above) that $e^6 - 2c^3e^3 + c^6 = 0$, therefore the Root of it $e^3 - c^3 = 0$, that is $eee - ccc$ the second æquation prescribed.

Lastly, by the third case, seeing c is less than b,

Put $cccccc - bbbbbb = -dddddd$:

Then it will be $eee = ccc + \sqrt{-dddddd}$ the æquation prescribed in the third case, and (because of the inexplicability of $\sqrt{-dddddd}$) impossible.

COMPENDIUM.

Whereas Mr. *Harriot* saith *Propter* $\sqrt{-d^6}$ *inexplicabilitatem*, &c. The said quantity $\sqrt{-d^6}$ is not explicable because $-d^6$ ariseth by multiplying $+d^3$ by $-d^3$ betwixt which two there is no mean; for no one thing can produce d^6 but d^3 only, and $-d^6$ is not produced by $+d^3$ or $-d^3$ because by both, this therefore may serve for a *Compendium* to save labour which might else be lost, in seeking that which is impossible to be found.

NOTE.

I use b^6 for $bbbbbb$, and b^4 for $bbbb$, and b^3c^3 for $bbbccc$, and the like, (as *Des Cartes* hath done) only for abridgment, as in the Definition of the Powers is already shewed. And $ccc + \sqrt{\overline{c^6 - b^6}}$ with that line over to distinguish betwixt $\sqrt{c^6 - b^6}$ as one quantity, and $\sqrt{c^6}$ taken by it self and $-b^6$ taken apart also, for by such mistakes may great errors succeed.

I will add no more Rules, these 4 may be multiplied by any one that doth not find these sufficient for his purpose, at his own pleasure.

CHAP. V.

Of Reduction of Solids.

HAving spoken in *Chap.* 4 *Rule* 2. of making $bbc - 2qqq = ddd$, and in *Rule* 4. of $c^6 - b^6 = dddddd$, I think it not amiss here to shew how such Addition and Substraction of Solids may be performed.

And it may be noted that ddd, is for brevity sake there usurped for ggc, or some other solinomial rectangle *Parallilpipedon*, equal to the *Binomial* rectangle Solid $bbc - 2qqq$, for if this *Binomial* could (by plain Geometry) be given in a *Cube*, as is ddd, something else might be done which here I will not speak of.

Now therefore seeing $b = 3q$ as there it is, the æquation may be written

$$9qqc - 2qqq = ddd, \text{ or rather}$$
$$9qqc - 2qqq = ggc.$$

Make

Make $\frac{qq}{c} = f$, therefore $2qqq = 2qfc$,

Secondly, make $9qq - 2qf = gg$, from thence it is plain that $bbc - 2qqq = ggc$, which was first to be done.

Thirdly, to reduce $c^6 - b^6$ into one intire Solid, though not into a Squared Cube as d^6, as is usurped by Mr. *Harriot* for brevity in writing, or facility in reasoning, *Pag.* 100, supposing that done which cannot be done by streight lines and Circles hitherto.

Now therefore seeing $c^6 - b^6$ is produced by multiplication of $ccc + bbb$ into $ccc - bbb$.

Make $\frac{cc}{b} = f$, and $\frac{bb}{c} = g$, and $f + g = q$ and $f - g = p$, therefore $bcf = ccc$, and $bcg = bbb$, and $bcq = ccc + bbb$. Secondly also $bcp = ccc - bbb$: And therefore $bbccpq = cccccc - bbbbbb$, which was secondly to be done.

Example in Numbers.

Put $b = 2$ and $c = 3$: Then $ccc^3 = 729$, and $bbbbbb = 64$, and the then $cccccc - bbbbbb$, that is $729 - 64 = 665$, which is produced by multiplying $27 + 8$ by $27 - 8$, that is, 35 by 19. Now make $f = \frac{9}{2}$, and $g = \frac{4}{3}$, then $f + g = 5\frac{5}{6} = q$, and $f - g = 3\frac{1}{6} = p$. And $bcq = 35$, and $bcp = 19$. And lastly $bbccpq = 665$.

Moreover, if you make $pq = xx$, the Solid is further reduced to $bbccxx$, which although it be not a Squared Cube, yet it hath a square Root, namely bcx, which may be of good use in many cases to resolve Æquations into Analogismes, of which kind of Demonstration, by help of *Euclide* 6. 14. some notice is taken before in *Chap.* 2.

NOTE.

The three Cases of the Æquation $a^3 - 3bba = 20^3$, mentioned in the beginning of the fourth *Rule* of the last *Chap.* are called by Mr. *Harriot*, the first *Hyperbolical*, the second *Parabolical*, the third *Elliptical*, because of some similitude between them and those Sections, of which three Cases, the first is resoluble by a Conique Section, the second by a Circle, and the third not at all.

Multiplication and *Division* of Solids is altogether as easie as Addition or Substraction, for if one would divide ccc by bb, make $\frac{cc}{b} = x$, and again make $\frac{cx}{b} = z$, and then z is the Quotient required.

Example

Example in Numbers.

Put $b=2$ and $c=3$, then $\frac{ccc}{bb}=6\frac{3}{4}$, to find which make $\frac{cc}{b}=x=\frac{9}{2}$, then $cx=\frac{27}{2}$, and $\frac{cx}{b}=\frac{27}{4}=z=\frac{ccc}{bb}=6\frac{3}{4}$, as it should be.

Again, if c^5 should be divided by b, it is now $\frac{ccc}{bb}=z$, and multiplying by b it is $\frac{ccc}{b}=bz$.

Again, multiplying by cc it is $\frac{c^5}{b}=bccz$, and $bccz$ is the Quotient required.

But if it be required to bring the Quotient to a Biquadrat, make $bz=dd$, then $ccdd=bccz$ and make $cd=ff$, then the Quotient will be $ffff$.

Multiplication is naturally so easie that there needs no more be said of it, than what hath been said already in *Chap.* 1.

Now, of Æquations consisting of 3 terms in continual proportion as $a^4+bbaa=c^4$, or secondly $a^6-bbba^3=c^6$, or lastly let it be $-a^8+bbbba^4=c^8$, let them first be proposed in Numbers as $a^4+2aa=24$, if by Rule 1. of *Chap.* 2. it be wrought, it will be found $\surd 25-1=aa$, and $aa=4$ or $a=2$.

Otherwise if the Square of half the Coefficient be added on both parts, then

$a^4+bbaa+1=25$.

And their square Roots also are equal; that is $aa+1=5$ and $aa=4$ or $a=2$ as before, and the latter may prove the former.

2. In the second, let it be $a^6-10aaa=459$ add 25 to each part, then it is

$aaaaaa-10aaa+25=484$.

Now each part of the Æquation is a Square and their Roots also are equal, that is $aaa-5=22$, that is $aaa=27$, and $a=3$.

3. Lastly, If $-a^8+700a^4=46875$ from the Square of $\frac{700}{2}$, that is, from 122500 take the Homogeneal 46875, there remains 75625, whose square Root is 275; And either $350+275$: Or $350-275$, that is either 625 or 75 is equal to $aaaa$, and $a=5$: Or $\surd qq.\ 75=a$, which Character $\surd qq.$ signifies the Biquadraticall Root.

NOTE.

NOTE.

The first and last of these three Æquations, may be done as well in Lines as Numbers (by the said three Rules of *Chap.* 2.) and so any Æquation of 4, 8, 16, or 32 dimensions, but Æquations of 6, 12, or 24 Dimensions, cannot be effected so, because there is ever one or more Cubique Roots to be extracted, which without two means cannot be done.

For if it may, then I say, that two means between any two lines may thereby be found, for in the second Æquation $a^6 - bbbaaa = c^6$ by Rule 2 *Chap.* 2 $c^6 + \frac{1}{4}b^6$ is a square, make $\frac{cc}{b} = d$, then $bd = cc$, and $bbdd = c^4$, and $bbccdd = cccccc$, then make $\frac{1}{4}\frac{bb}{c} = f$, therefore $fc = \frac{1}{4}bb$, and $fcb^4 = \frac{1}{4}b^4$. Now because $\frac{bb}{c} = 4f$, make $b = 4f$, then $fcccbb = \frac{1}{4}b^6$.

Make $fc = ll$, then $ccbbll = \frac{1}{4}b^6$. Again, make $bb + bb = mm$, and $dd + ll = nn$. And then it will be .. $ccmmnn = bbccdd + ccbbll$, that is, $c^6 + \frac{1}{4}b^6$, to the square Root hereof cmn, add $\frac{1}{2}bbb$: Thus make $\frac{1}{2}\frac{bb}{m} = p$; then $mp = \frac{1}{2}bb$, and $bmp = \frac{1}{2}bbb$. Make $\frac{bp}{n} = q$, then $mnq = \frac{1}{2}bbb$. Lastly, make $c + q = x$, then it is $cmn + \frac{1}{2}bbb = mnx = aaa$, by *Chap.* 2. *Rule* 2. Now if m, n, and x be proportional, then the middlemost is equal to a, but that is uncertain, and cannot be made otherwise: But by making $rr = mn$ it will be $rrx = a^3$ and a will then be the lesser of two means between r and x if $r < x$ or the greater mean, if $r > x$. And so if r and x had been given, and required to find 2 means between them by *Retrogradation* orderly, one might come to the said Æquation $a^6 - bbbaaa = c^6$ of which if the Root a be found, two means are also found between r and x which was to be proved.

CHAP.

CHAP. VI.

Of Surd Numbers.

RULE I.

THE square Root of any Number being multiplied by that Number, produceth the square Root of the Cube of the Number.

For $\surd a$ multiplied by a produceth $a\surd a$, but $a\surd a = \surd aaa$ for taking *Equimultiplices* they will be equal, as if the first, namely $a\surd a$ be multiplied still by $\surd a$, the Product is $a\surd aa$, that is aa. And if $\surd aaa$ be multiplied by $\surd a$ it produceth $\surd aaaa$ that is aa also, wherefore $a\surd a = \surd aaa$: And therefore $3\surd 3 = \surd 27$ either of which is the Cube of $\surd 3$, and the like of all others.

RULE II.

Surd Numbers are multiplied and divided like whole Numbers, the Product retaining still the Character of the Root.

That is, $\surd 2$ multiplied by $\surd 3$, produceth $\surd 6$, and so of all others.

RULE III.

A Rule for Squaring Binomial Surds.

Multiply the Quantity to which the Sign $\surd$ belongs, into the Square of the Coefficient; and the Product is the Square required.

Example in Numbers.

If the Square of $3\surd 7$ be required: Multiply 7 into 9, the Product is 63; the Square required.

Or, if the Square of $4\surd 9$ be demanded; 9 into 16, the Product is 144; which is the Square demanded: The like of all others. And this shews, That all such Surds are commensurable in Powers.

NOTE.

Where I shall have occasion (if any be) to speak of a Cubique Root, I shall sign it thus, $\surd c.$ and the Biquadratique Root thus $\surd qq$.

RULE IV.

To multiply, divide, add or Substract the Roots of Surd Numbers.

And first of MUL-

MULTIPLICATION.

Besides that which hath been said in the last Rule above, these Roots of Surds may be multiplied and divided, and known by other names, so as sometimes the Products, or Quotient shall be rational. First therefore any square Root doubled is the square Root of the quadruple, as

$2\sqrt{5} = \sqrt{20}$ and $2\sqrt{20} = \sqrt{80}$.
$3\sqrt{5} = 45$. $4\sqrt{5} = \sqrt{80}$. $5\sqrt{5} = \sqrt{125}$.
$2\sqrt{10} = \sqrt{40}$. $3\sqrt{10} = \sqrt{90}$.
$4\sqrt{10} = \sqrt{160}$, and $5\sqrt{10} = \sqrt{250}$, &c.

infinitely still multiplying the Numerator, 2, 3, 4, 5, &c. into it self, and the product into the Surd Number, as if $3\sqrt{10} = \sqrt{90}$, it ariseth from 3 times 3 into the Surd Number $\sqrt{10}$: And the like of all others whatsoever.

For put $\sqrt{a} = \sqrt{10}$, to be multiplied by another Number, as by $a = 10$, the product is $a\sqrt{a} = 10\sqrt{10}$, which by the first Rule is $\sqrt{aaa} = \sqrt{1000}$, that is, the Numerator 10 into it self making 100, which multiplied again by the Surd $\sqrt{10}$, gives $\sqrt{1000}$.

And if it had been at first $\sqrt{a} = \sqrt{10}$, multiplied by any other Number, as $e = 3$, the product must by the same method be $e\sqrt{a} = e\sqrt{10}$ that is (by the same reason as the former) $\sqrt{eea} = \sqrt{ee\,10} = \sqrt{90}$.

And it is plain, that if any Root be multiplied by

$$\left.\begin{matrix}2\\3\\4\\5\\6\end{matrix}\right\}\ \text{The Product shall be the Root of the}\ \left\{\begin{matrix}\text{Quadruple.}\\\text{Noncpule.}\\\text{Sedecuple.}\\\text{Vigintiquintuple.}\\\text{Trigintisextuple.}\end{matrix}\right.$$

And so forward infinitely, according to the proportion of the Squares of the Multipliers.

Also by Decuplation, as if $5\sqrt{5} = \sqrt{125}$, then $5\sqrt{50} = \sqrt{1250}$: Or if $4\sqrt{4} = \sqrt{64}$ then $4\sqrt{40} = \sqrt{640}$. And (as above) if $4\sqrt{10} = \sqrt{160}$, then $4\sqrt{100} = \sqrt{1600}$.

Also by Subdecuplation, if $2\sqrt{10} = \sqrt{40}$, then $2\sqrt{1} = \sqrt{4}$: Or if $5\sqrt{20} = \sqrt{500}$, then $5\sqrt{2} = \sqrt{50}$: And (according to that aforesaid) $3\sqrt{37} = \sqrt{333}$, and $3\sqrt{36} = \sqrt{324}$, that is, the square Root of 3 times 3 times 36.

And this may often be of use, not only in Numbers but Species, and is therefore to be had in memory by him that would be ready in Multiplication of Surd Numbers, or Surd Quantities.

Furthermore it may be useful to remember that in *Reciprocal Surds* as $4\sqrt{5}$ and $5\sqrt{4}$ these two have that proportion one to another as 4 hath to a mean betwixt 4 and 5.

As for Example $4\sqrt{9}$ hath that proportion to $9\sqrt{4}$ as hath 4 to 6, which is a mean betwixt 4 and 9, for $4\sqrt{9} = 12$, and $9\sqrt{4} = 18$,

$=18$, but $4'$ $6''$ $12'$ $18''$ or more generally $a\sqrt{e'}$ $e\sqrt{a''}$ a' $\sqrt{ae''}$ for multiply the Means, it is $ae\sqrt{a}$ and multiply the Extreams it is $a\sqrt{aee}$, and divide each of them by a the first is $e\sqrt{a}$ the other is $\sqrt{aee}$, but by the former part of this Rule $e\sqrt{a}=\sqrt{aee}$ wherefore this is proved.

CONSECTARY.

Hence it is evident that Roots of themselves inexplicable may be so multiplied as the Product may be rational: For if $\sqrt{20}$, be multiplied by $4\sqrt{5}$ the Product will be $4\sqrt{100}=40$.

For $2\sqrt{5}=\sqrt{20}$ and $2\sqrt{20}=\sqrt{80}$, therefore $4\sqrt{5}=\sqrt{80}$, but $\sqrt{80}$ multiplied by $\sqrt{20}$ gives $\sqrt{1600}=40$,

I need say nothing of *Division*, for that is no more but by the same steps to go back again, as $\sqrt{1600}$ divided by $\sqrt{80}$ Quotient is $\sqrt{20}$. And so of the rest which hath been said in Multiplication.

NOTE

These things being so, it will not be hard to find some Number to compare with any *Surd* Number so as to make that work rational and exprimible which seemed not so: For there is not any Surd Number can be given which may not by some multiplication be made a rational Number: For let it be $\sqrt{5}$, $\sqrt{7}$, $\sqrt{8}$, or any of these as $\sqrt{7}$ multiply it first by $\sqrt{7}$ that produceth 7, but multiply $\sqrt{7}$ by any square Number whatsoever, as by 4 omitting the Sign $\sqrt{}$, it gives 28, than again multiply $\sqrt{7}$ by $\sqrt{28}$ it produceth $\sqrt{196}=14$.

For this is all one as to multiply one Square Number by another which must needs produce a Square Number.

So here the Square Number 4 was multiplied by 7 and after by 7, that is by 49, which multipliers cannot produce any other than a Square Number, to wit 196 *Euclid*. 9. 1.

And whatsoever hath hitherto been said of Quadratiques, may serve for cubiques also; due respect always had to the degree of the quantity and Root, for any $\sqrt{c.}$ multiplied by 2 gives

$8\sqrt{c.}$ by 3 it gives $27\sqrt{c.}$ by 4 it gives
$64\sqrt{c.}$ that is $2\sqrt{c.}\,8=\sqrt{c.}\,64$ and
$3\sqrt{c.}\,8=\sqrt{c.}\,216$, and
$3\sqrt{c.}\,27=\sqrt{c.}\,729$ the proportion still increasing as the Cube of their Multipliers.

And the like consideration had, this may be applicable to *Biquadratiques*, or any higher order.

And still whatsoever hath been said of Multiplication, serves in a retrograde way for division also.

RULE

RULE IV.
For ADDITION.

Surd Roots are usually Added and Substracted by the Signs $+$ and $-$ as the Square Root of 2 added to the Square Root of 8, Sum is $\surd 2 + \surd 8$ or Substracted rest is $\surd 8 - \surd 2$.

But these may be added into one Sum, for seeing 8 is quadruple to 2 therefore $2 \surd 2 = \surd 8$. And the Sum is $3 \surd 2$ and the remain is $\surd 2$. Likewise the *Reciprocal Surds* $8 \surd 2 = 2 \surd 8$, are capable of Addition, Substraction, Multiplication or Division; for they are being added $3 \surd 32$ that is $\surd 288$; Substracted, $\surd 32$, Multiplied $\surd 4096$; divided $\surd 4$; but such as are neither commensurable nor reciprocal cannot be amassed into one Sum.

And the Sum of the former Addition of $\surd 8 + \surd 2$ being already reduced to $3 \surd 2$ may be yet further reduced to $\surd 18$, for $3 \surd 2$ is equal to the square Root of three times two as hath been more than once shewed.

And generally when the Surds given are denominated by Numbers in quadruple proportion, as $\surd 2$ to $\surd 8$, and $\surd 3$ to $\surd 12$, &c. the lesser and the greater twice being added together, as 2 to 16, or 3 to 24, the square Root of the Sum is equal to the Sum of the two square Roots given to be added; that is, $\surd 2 + \surd 8 = \surd 18$, and $\surd 3 + \surd 12 = \surd 27$.

The reason is, $\surd 1 + \surd 4 = \surd 9$, which 9 is composed of the lesser once and the greater twice, that is, as often as the $\surd 1$ is contained in the $\surd 4$.

But if the Numbers be prime one to another, they must be added or substracted by the Signs $+$ and $-$, for these Rules reach not to primes.

And having said this little to acquaint such as are wont to be afraid of operations where Surds are present, with this which will render some things easie which perhaps seemed hard, and others which were hard, less difficult. I will now leave this rugged Subject, and recreat a little with a few easie *Propositions*; the performing of which may serve to recal into Use and Practice that which hath been spoken of Solids in the former Chapter.

CHAP. VII.

PROB. I.

Any right line being given, to divide it into two parts, so as the Rectangle of the whole and one of the parts; may be to the Square of the other part, in such proportion as is betwixt any two right lines given.

Let the right line given be b.
The segment to be squared a.
Then the other Segment is $b-a$.
And let the two lines given be r and s.

Then $bb-ba'$ aa'' r' s''.
And $raa=sbb-sba$. per 16. 6. *Euclide.*
That is, $raa+sba=sbb$.

Make $\frac{sb}{r}=d$, and divide all by r.

Then it is, $aa+da=db$. Make $db=ff$.
Then lastly it is $aa+da=ff$. And a is easily found by *Rule* 1. of *Chap.* 2.

And if it had been required to have had the Rectangle + or — some other plain to have had any limited proportion to the Square aa, the work had been almost the same, with some small addition.

PROB. II.

To make a Scalenon *Triangle, of which the Base, Perpendicular, and proportion of the other Sides shall be given.* (I account that the Base which subtends the divided angle.)

Let the base given be b, the perpendicular c, and the proportion of the other sides, as r to s.
Of which let r be the lesser.
And for the lesser Segment of the Base put a:

Therefore by supposition,

$$r' \quad s'' \quad \sqrt{cc+aa}' \quad \sqrt{cc+bb-2ba+aa}''$$

So that the Squares of them are also proportional,

This is,

$$rr' \quad ss'', \; cc+aa' \quad cc+bb-2ba+aa''.$$

And by multiplying the means and extreams,

It is,

$$ssaa+sscc=rrcc+rrbb-2rrba+rraa.$$

That is,

$$ssaa-rraa+2rrba=rrcc+rrbb-sscc.$$

$$\text{Make } \frac{ss}{r}=x.$$

And divide all the Æquation by r, Then it is,

$$xaa-raa+2rba=rcc+rbb-xcc.$$

Secondly, make $x-r=f$, and $gg=bb+cc$.
Then it is, $faa+2rba=rgg-xcc$.

Again

Again make $\frac{rg}{f}=h$, and $\frac{xc}{f}=k$, and divide $faa+2rba=rgg-xcc$ by f.

Then it will be,

$$aa+\frac{2rb}{f}a=hg-ck.$$

Lastly, make $\frac{2}{f}=q$, and $hg-kc=mm$.

The Æquation finally reduced will be then $aa+qa=mm$, and a may be found by the first Rule for square Æquations. *Chap.* 2.

PROB. III.

Any Number being given, to find two other Numbers, so as all the three may constitute a Rectangle Triangle.

Unto the Square of the Number given add Unity, the half of the Sum shall be the Hypothenuse, or from the said Square take Unity, the half of the remain shall be the middle side.

For let the Number given be a, the Square is aa, to which adding Unity, the Sum is $aa+1$, the half whereof is $\frac{1}{2}aa+\frac{1}{2}$, for the Hypothenuse.

Secondly, from aa take Unity, the rest is $aa-1$, the half whereof is, $\frac{1}{2}aa-\frac{1}{2}$ for themiddle side.

But the lesser side (by supposition) is a.

The Square of the lesser side is aa.

The Square of the middlemost is $\frac{1}{4}aaaa-\frac{1}{2}aa+\frac{1}{4}$

Both those Squares are $\frac{1}{4}aaaa+\frac{1}{2}aa+\frac{1}{4}$:

But the Square of the Hypotenuse, *viz.* of $\frac{1}{2}aa+\frac{1}{2}$

is equal to these, that is, $\frac{1}{4}a^4+\frac{1}{2}aa+\frac{1}{4}$.

Therefore by the 48. of the first of *Euclide* the Proposition is proved.

COROLLARY.

Hence it is plain, that the two greater sides of any Rectangle Triangle differ by Unity, for if two Squares differ by 2, their halves differ by 1.

NOTE.

If it be required to have all the three sides in whole Numbers, then the lesser side must be an odd Number.

PROB. IV.

The difference of the sides of a Rectangle, with the Area and Diagonal in one Sum, being given in Numbers, to find out the Sides.

Let the difference of the ſides be 7

And the Area and Diagonal together 73,

And put the leſſer ſide equal to a.

Then the greater is $a + 7$.

Theſe two multiplied produce $aa + 7a$, which is equal to the Area.

And therefore $73 - aa - 7a$ is the Diagonal

The Square of which is $\left.\begin{array}{l} + 5329 - 146aa \\ + aaaa + 14aaa + \\ + 49aa - 1022a \end{array}\right\}$

Which reduced and rightly ordered, Is

$$+ aaaa + 14aaa - 97aa - 1022a + 5329$$

Which by the 47. of the firſt of *Euclide*, is equal to the two Squares of the other ſides a, and $a + 7$, whoſe ſquares are aa, and $aa + 14a + 49$.

That is, $+ aaaa + 14aaa - 97aa - 1022a + 5329 = 2aa + 14a + 49$.

That is, $+ aaaa + 14aaa - 99aa - 1036a + 5280 = 0$.

That is, $- aaaa - 14aaa + 99aa + 1036a = 5280$.

In which Æquation, becauſe $aaaa$ hath four Dimenſions, and the Homogeneal 5280, but four Places, the Root a cannot conſiſt of more than one Place, or Figure, which muſt be found out by trying every one of the nine Digits, if need be, and will be found at laſt to be 5, therefore the other ſide is $5 + 7 = 12$, the Area 60, and the Diagonal 13.

But if a had been more or leſs than 5 yet (except ſomething elſe lead a readier way) it is good to try 5 at firſt, if it be too little then 7, if that alſo too little, then 9, ſo there will be no need to try the even Numbers, 6, 8, &c. for if 5 be too little and 7 too great, it muſt be 6, the like reaſon will ſerve for 8, 4, 2, ſo that he which gueſſeth moſt unfortunately, needs not try above four or five Digits, which is no great matter, the like happening ſometimes in ſeeking the Quotient in plain Diviſion, for no man is ſure to gueſs right at firſt.

But that we may exemplifie this in bigger Numbers, where a may conſiſt of two or more places.

Let the difference of ſides be 71

The Area and diagonal together 1177.

Working as in the former example, there will ariſe an Æquation which being reduced and ordered as before, will be

$$- aaaa - 142aaa - 2685aa + 167276a = 1380288.$$

And putting $b + c = a$:

Then the Canon of the reſolution will be

$$\begin{array}{l} - bbbb - 4bbbc - 6bbcc - 4bccc - cccc \\ - 142bbb - 426bbc - 426bcc - 142ccc \end{array}$$

$$= 2685$$

$-2685bb - 5370bc - 2685cc$
$+167276b + 167276c.$

To be orderly ſubſtracted from the Homogeneal Number given 1380288, as followeth.

The number given +1380288

The firſt ſingle Root $b = 1.$

$-bbbb$	1.0000
$-142bbb$	142.000
$-2685bb$	2685.00
In all	$-$4205.00
$+167276b$	167276.c

Subſtract (the difference of + and $-$) +1252260

Remains of the Number given +0128028

The firſt Root decuplate $b = 10.$

$-4bbb$	4000
$-6bb$	0600
$-4b$	0040
$-426bb$	42600
$-426b$	4260
$-5370b$	53700

Then $-$ 105200 is all the $-$

And + 167276 is all the +

Diviſor + 62076 is their difference.

The ſecond ſingle Root $c = 3$

Remains of the number given +128028

$-4bbbc$	12000
$-6bbcc$	.5400
$-4bccc$	.1081
$-cccc$	...81
$-426bbc$	127800
$-426bcc$	.38340
$-142ccc$	..3834
$-5370bc$	161100
$-2685cc$	24165
In all	$-$373800
$+167276c =$	501828
All the $-$ being	373800
The difference is	+ 128028

Which

Which being taken from the remains of the Number given $+ 128028$, there remains finally nothing, so that the given Æquation is justly resolved by the Root $b + c = 13$.

The lesser side a is therefore 13, to which if the difference given namely, 71, be added, the middle side 84 is thereby composed.

Again, if to that middle side 84 be added Unity, the Hypotenusa of a right angled triangle is composed, whose three sides are 13, 84, 85.

The Superficies of this Triangle is half the Parallelogram or Rectangle required.

For 84 multiplied by 13, gives 1092 for the Area of the Rectangle, to which adding 85 the Diagonal, composeth the Number 1177, as was required in the Proposition.

COMPENDIUM.

Seeing the two greater sides of any Rectangle Triangle, exceed one another by Unity (as by the former Corollary) the difference betwixt the two lesser sides being given, the difference betwixt every two sides is also given.

So that putting a for the lesser side of the Rectangle, the greater side is $a + 71$, and the diagonal $a + 72$, whose Square is $+ aa + 144a + 5184$, to which the two Squares of the sides, being $aa + aa + 142a + 5041$, are equal: That is,

$$2aa + 142a + 5041 = aa + 144a - 5184$$

And Substracting from each part

$$aa + 144a + 5041$$

There will remain $+ aa - 2a = 143$.

And a will be found 13, by the second Rule of *Chap.* 2.

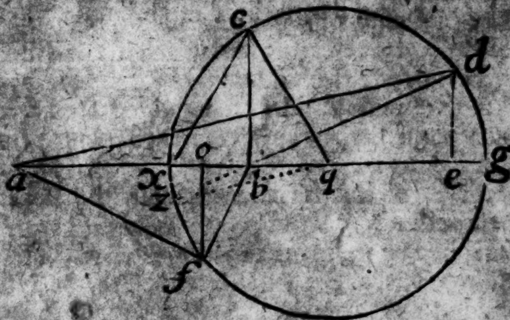
RESUMPT.

In the second *Probleme* of this Chapter it hath been shewed how upon a Base and Perpendicular and Proportion of the remaining sides given, to describe a Triangle.

It is there to be understood of an Acute-Angled Triangle, in which the Perpendicular falls within the Triangle.

Now therefore let it be otherwise.

As



As in this figure, of the Triangle adb, let there be given, the Base $ab = b$, the Perpendicular $de = c$, and the Proportion of the other sides, namely $bd = r$, and $da = s$; whereas in the second Probleme, the lesser Segment of the base was called a, and the greater $b - a$. Now here the line be may be called a, and ae shall be $b + a$, and the rest of the work will be like that before in *Prob.* 2.

NOTE.

And it may be noted, that if from a and b (the ends of the line ab) be drawn other binarie Lines, how many soever, so as they hold the same proportion as s to r and concur in other Points, as c, f, g, &c. Those Points are all in the circumference of a Circle whose Center is in the line ab, produced towards g.

For upon ab describe the rectangle abc whose two sides ac, cb, may be as s or r, and divide the Angle acb into two equal parts by the right line cx, and draw cq perpendicular to ac in c, then the Angle $xcq = 90 - xca$ likewise the Angle $cxq = 90 - xcb$, but $xcb = xca$ therefore $xcq = cxq$; and $cq = xq$.

And because of the similitude of the Triangles acq and cbq it is $aq' \; cq'' \; bq'''$ that is $aq' \; xq'' \; bq'''$.

Now by supposition $ag' \; bg'' \; ac' \; bc''$.

Therefore by composition also, it will be $aq + xq' \; xq + bq'' \; xq' \; bq''$

But $ac' \; bc'' \; aq' \; xq''$.

And $xq' \; bq'' \; aq' \; xq''$.

Therefore $ag' \; bg'' \; aq + xq'' \; xq + bq''$.

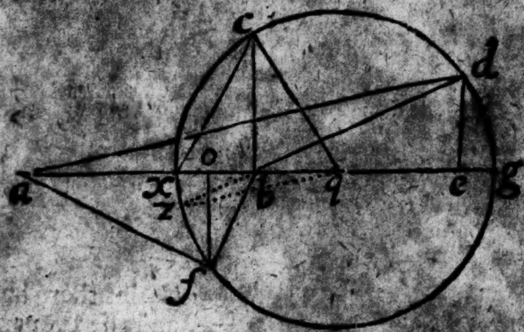
And therefore $ag = aq + xq$. And $xq = gq$.

But it hath been proved that $xq = cq$.

Therefore $xq = cq = gq$.

Again,

Again, upon the Center q, and the ſpace xq, cq, or gq, deſcribe the Circle xcq, producing db to the Circumference in z, and draw



qz; and becauſe it hath been proved above, that $ag' \; bg'' \; ac' \; bc''$.

That is $ag' \; bg'' \; xq' \; bq''$, therefore alſo by Diviſion $ag - bg' \; bg'' \; xq - bq' \; bq''$.

That is $ab' \; bg'' \; xb' \; bq''$.

Therefore the Rectangles $abq = xbg$. *Eucl.* 6. 16.

And becauſe by Suppoſition

$ac' \; bc'' \; ad' \; bd''$

And $ac' \; bc'' \; qc' \; qb''$.

Therefore $ad' \; bd'' \; qc' \; qb''$, but $qc = qz$, therefore $ad' \; bd'' \; qz' \; qb''$, and the Angles abd, zbq being equal, the Triangles abd, zbq are Equiangular, *Eucl.* 6. 6.

And therefore $ab' \; db'' \; bz' \; bq''$ *Eucl.* 6. 4. and the Rectangles $dbz = abq$. *Eucl.* 6. 16.

But it was now proved, that $abq = xbq$, therefore $xbq = dbz$.

So that the Points x, z, q, being in the Circumference of the Circle zxq, the Point d muſt be in the ſame Circumference, *Eucl.* 3. 35.

The like Proof may ſerve to ſhew that the Point f is in the ſame Circumference; which is all that was to be proved.

This Circumference, however deſired by the Ancients, and affected by Modern *Mathematicum*, ſeems yet to have little Uſe, more than to help the Conſtruction of the Triangle, which (but now I ſhewed) may be done without it.

FINIS.